ELEMENTARY INSTRUCTIONS for operating the SLIDE RULE

KEUFFEL & ESSER CO.
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MANNHE IM TYPE

The Slide Rule has application in the fields of engineering, architecture, estimating, cost accounting, statistics, physics, chemistry, astronomy, merchandising,—wherever quick calculations are necessary.

It is used primarily for multiplication and division and the related operations of proportion, percentage, and combined multiplication and division.

Problems involving reciprocals, squares, square roots, cubes, cube roots, and trigonometric functions are also solved by means of the Slide Rule; but the beginner is advised to confine his study to the simple operations of multiplication and division on the $C$ and $D$ scales.

Before attempting to perform the calculations, the student should practice the reading of the scale until he has acquired accuracy in locating numbers without hesitation.

The $C$ and $D$ scales are alike and are numbered from 1 to 10, the spaces between the whole numbers decreasing steadily toward the right, as is brought out in the following diagram:

![Diagram of Slide Rule Scales]

Fig. A.
TO LOCATE THREE-Figure NUMBERS ON THE C AND D SCALES, there are three steps of procedure in the following sequence:

**STEP I.** Read the first significant figure. (The first significant figure of a number is the first numeral that is not zero. Thus, 2 is the first significant figure of the numbers 0.0024, 24.0, 0.024, or 2.40.) If the first significant figure is 1, the number will lie between the main divisions 1 and 2. If it is 2, the number will lie between 2 and 3. If 3, between 3 and 4, etc. (See Fig. I.)

**EXAMPLE.** The number 246 lies between the main division 2 and 3 (as indicated by the bracket in Fig. I, a skeleton scale showing only the main divisions) since the first significant figure is 2.

![Fig. I. MAIN DIVISIONS](image)

**STEP II.** The second figure locates the number on the secondary divisions in a similar manner.

**EXAMPLE.** In the number 246 the second figure 4 indicates that the location is between the 4th and 5th secondary divisions beyond the second main division, as indicated by the bracket in Fig. II, which is a skeleton scale with only the secondary divisions filled in. Note that there are ten of these secondary divisions to each main division.

![Fig. II. SECONDARY DIVISIONS](image)

**NOTE:** On the 10 in. rule, owing to the lack of space, the secondary divisions between the 1st and 2nd main divisions are the only ones which are numbered.

**STEP III.** In a like manner the third figure locates the number on the third set of divisions, which appear in Fig. III—the Slide Rule Scale in its actual form.

**EXAMPLE.** Since the number 246 lies between the main divisions 2 and 3, where the subdivision is in fifths, the third figure locates the number finally on the third small space past the 4th secondary division beyond the main division 2, as indicated by the arrow in figure III.

![Fig. III. COMPLETE SCALE](image)

**IMPORTANT NOTE:** Were it practical to manufacture a rule with ten subdivisions to every secondary division, it would certainly be done, so that each space would have a single value as have the secondary divisions. However, since the spaces grow smaller toward the right end of the scale, it is practically impossible to subdivide the secondary divisions into tenths throughout the scale. Therefore:

(a) The spaces between the secondary divisions lying between Main Divisions 1 and 2 are divided in tenths; so each of these subdivisions has a single value.

![Diagrams](image)

(b) The spaces between the secondary divisions between Main Divisions 2 and 3 and 3 and 4 are divided in fifths; so each subdivision has a double value.

![Diagrams](image)

c) The spaces between the secondary divisions for the remainder of the scale are only divided in half; so each subdivision has a value of five.

![Diagrams](image)

**STEPS I, II and III are condensed in Fig. IV, below, showing the location of the number 246.**

(1ST): 246 LIES BETWEEN 2ND AND 3RD MAIN DIVISIONS

(2ND): LIES BETWEEN 4TH & 5TH SECONDARY DIVISIONS

(3RD): FINAL LOCATION 246

![Fig. IV](image)
In locating a number it is advisable for the beginner to use the hairline on the indicator (glass runner) to follow each step.

For example, to locate the number 478, employ the following three steps:

I. First significant figure 4 (indicates that number lies between 4 and 5). Set indicator at 4 (Fig. V.).

II. Second figure 7 (indicates number lies between 7th and 8th secondary divisions). Move indicator to 7th secondary division (Fig. VI.).

III. Third figure 8 (indicates the number lies 3/5ths of the distance between the single subdivision (half) and the next secondary division (Fig. VII.).
Numbers containing a single digit are located at the main divisions, as—

![Fig. VIII.](image)

Two digit numbers are located like the three digit numbers, but are finally located on the secondary divisions instead of the final subdivisions, as—

![Fig. IX.](image)

**Numbers containing a large number of digits need only be set to the third or fourth place;** since the percentage of error introduced in the result is so minute, as to be insignificant in the majority of problems,—especially ratio and percentage calculations, combined multiplication and division, and multiplications involved in estimating and appraising.

Thus, 187,575 would have to be called 187,600 and set as follows:

![Fig. X.](image)

The student should practice setting and reading until he feels confident that he can do so accurately and without hesitation. Then he is ready to give his attention to the solution of simple multiplication problems.

**MULTIPLICATION**

**Rule:** To multiply two factors together, set the index of the C scale (either the right or left end figure one) adjacent to one of the factors on the D scale and read the answer on D under the other factor on C.
2 \times 3 = x

(1ST) SET C INDEX  \quad (3RD) UNDER 3 ON C

(2ND) TO 2 ON D  \quad (4TH) READ 6 ON D (ANS.)

Fig. XI.

18 \times 26 = x

(1ST) SET C INDEX  \quad (3RD) UNDER 26 ON C

(2ND) TO 18 ON D  \quad (4TH) READ 458 ON D (ANS.)

Fig. XII.

72 \times 51 = x

(3RD) UNDER 51 ON C  \quad (1ST) SET C RIGHT INDEX

(4TH) READ 3672  \quad (2ND) TO 72 ON D

Fig. XIII.

Note that in making settings to solve the last problem that the right index must be used, instead of the left, as was used in the first two problems. *If the slide projects too far to the right when the left index is used, use the right index.*

**THE DECIMAL POINT**

**Important:** No mention has been made as to the method of determining the position of the decimal point in the last problems, since it has been apparent at a glance. In most cases, however, the operator should substitute round numbers for those appearing in the problem and determine the correct position of the decimal point by approximation.
Thus \( 2.47 \times 34.2 = x \)

Fig. XIV.

Make the setting in the regular way, and read the answer 845. Substitute 2 for 2.47 and 30 for 34.2 and note that the answer would be approximately 60. Therefore the answer must be 84.5 which is nearer to the approximation than 845 or 8.45.

When the student has operated the slide rule for some time, he will learn to make these approximations mentally and almost instantaneously.

MULTIPLICATION OF THREE OR MORE FACTORS

Example: \( 642 \times 3.5 \times .0164 = x \)

The first two factors are multiplied together, as previously indicated.

Fig. XV.

No note need be taken of the product of these two numbers as we are interested only in the final product.
The $C$ index is moved to the product of the first two and the final reading is made on $D$, under the third factor on $C$. Thus—

![Diagram](image)

(Fig. XVI.)

Approximating, $600 \times 3 \times .01 = 18$, it is certain that the final answer must be 36.9 and not 369 or 3.69.

Any number of factors can be multiplied together in a similar manner. The decimal point in multiplications such as these can be quickly determined by the approximation method, which has already been explained.

**Note:** Problems of this type can be solved with a single setting of the slide by using the $CI$ scale. See page 13.

**DIVISION**

Division is the reverse of multiplication; refer to Fig. XI, showing $2 \times 3 = 6$. The same setting shows $6;3 = 2$.

**Rule:** To divide one number by another, set the divisor on the $C$ scale to the dividend on the $D$ scale and read the quotient on the $D$ scale, under the $C$ index.

**Example:**

$$875 \div 35 = x$$

![Diagram](image)

(Fig. XVII.)
As in multiplication, the decimal point should be set by approximation. Substituting round numbers, in the last problem, we see that 900 divided by 30 equals 30. Therefore the answer must be 25, as this is closer to 30 than 250 or 2.5.

SOLUTION OF PROBLEMS INVOLVING BOTH MULTIPLICATION AND DIVISION.

Problems involving both multiplication and division can be worked out on the slide rule with great rapidity, whereas considerable time would be required for solution by the arithmetic method. Note, in solving a problem of this type, that it is not necessary to read the answer for each step, since we are interested only in the final answer.

Example: \[ \frac{840 \times 648 \times 426}{790 \times 611} = x \]

The best method for solving problems of this type is to perform division first; then multiplication; and to continue in this order as far as possible.

To 840 on \( D \) set 790 on \( C \) (Division).

Move indicator to 648 on \( C \) (Multiplication).

(1st) To 840 on \( D \)

(2nd) Set 790 on \( C \)

(3rd) Move indicator to 648 on \( C \)

Move 611 on \( C \) to indicator line (Division).
Move indicator to 426 on C (Multiplication).
Read 480 on D (Answer).

SQUARES AND SQUARE ROOTS

The A and D scales are so arranged that if the indicator is set over a number on D, its square will be found on the A scale under the indicator line.

Rule: To find the square of a number, set the indicator to the number on the D scale and read its square under the indicator line on the A scale. Optionally, set the indicator to the number on the C scale and read its square under the indicator on the B scale.

Example: Find the square of 43.8.

The decimal point is set in the same manner as in multiplication and division. Squaring 40, the nearest round number to 43.8, we get 1600. Therefore the answer must be 1920 and not 192.0 or 19200.

Rule: To find the square root of a number, the reverse process is used. Set the indicator at the number on the A scale and read the square root on D, under the indicator line. (Optionally, read from B to C).

Important: Always use the left half of the A scale for numbers with an odd number of figures before (to the left of) the decimal point and the right half for those with an even number of figures before the decimal point.
Example: Find the square root of 625.

Use left half of scale (odd number of figures.)

(1ST) SET INDICATOR TO 625 ON A (LEFT HALF)

(2ND) READ 25 ON D

Fig. XXI.

Find square root of 6250. As this number contains an even number of figures to the left of the decimal point, we use the right side of the A scale.

(1ST) SET INDICATOR TO 625 ON A (RIGHT HALF)

(2ND) READ 791

Fig. XXII.

By approximation, we see that the answer is 79.1

Note: To find the square root of a decimal number, if the number has no zeros or an even number of zeros after the decimal point use the right half of A. If it has an odd number of zeros use the left half.

**PROPORTION**

Problems in proportion are encountered daily, and offer one of the most common uses for the slide rule. Among problems of this type are those which call for—

(1) The conversion of—

Yards to meters
Dollars to pounds
Knots to miles
Meters to feet, etc.

(2) The determination of weight of one quantity when the weight of another quantity is known.
It will be found that when the slide is set so that 2 on C coincides with 4 on D, that all readings on C bear to the coinciding reading on D a ratio of 2:4 or 1:2.

Making this into a general rule we can say — with any setting of the slide, all coinciding readings on scales C and D are in the same ratio to each other.

**Example:** If we know that 2.7 quarts of a liquid weigh 4 lbs. and we wish to determine the weight of 1.4 quarts, we set 2.7 on C scale adjacent to 4 on the D scale and under 1.4 read the answer 2.07.

![Fig. XXIII.](image)

**Example:** If we desired to convert a number of different readings in square meters to square yards, we could set 1 on C to 1.196 on D (1 square meter = 1.196 square yards) and under any reading in square meters on C, we would find the corresponding reading in square yards on D.

In this instance, the value of a square meter expressed in square yards could only be set with difficulty on the slide rule. As this occurs in these transpositions frequently, a table of settings which can be easily made has been worked out and will be found printed on the back of the Mannheim type slide rules.

For the conversion of square meters to square yards, we find the setting given as 51 to 61; so we would set 51 on the C scale to 61 on the D scale and read in the same manner as above described.

![Fig. XXIV.](image)
USING THE CI SCALE

Like the C and D scales, the CI scale is a single logarithmic unit, but on the CI scale the logarithmic values increase from right to left, instead of from left to right, as on the C and D scales.

With this arrangement the values of CI and C are the reciprocals of each other, (giving proper consideration to the decimal points). Note that 2 is opposite 5, as 0.5 is the reciprocal of 2, also 0.2 is the reciprocal of 5.

Since dividing by the reciprocal of a number is the same as multiplying by the number itself, any number read on the CI scale may be used in multiplication in the same way that the corresponding number on the C scale is used in division.

To multiply two numbers using the CI and D scales, we place the two numbers opposite each other on the CI and D scales respectively, and read the answer at the index of the C scale. This plan saves time because it is never necessary to shift indexes to bring the answer on the scale as it sometimes is in using the C and D scales.

Example: \( 19 \times 6 = x \)

![Image of CI scale diagram](image)

Another advantage of the CI scale is that it permits multiplying three factors with a single setting of the slide.

From the above example, we see that without changing the setting of the slide, we are in a position to multiply by a third factor.
Example: $19 \times 6 \times 1.71 = x$

We first multiply $19 \times 6$ as above and then opposite 171 on $C$, read 195 on $D$. (See illustration on preceding page).

A third advantage of the $CI$ scale is its ability to give a table of quotients where the dividend remains constant and the divisor is a series of numbers.

Example. 732 is to be divided in turn by 14, 23, 32, 41 and 50.

![Fig. XXVI.]

To 732 on $D$ set the index of the $CI$ scale; then opposite 14, 23, 32, 41, and 50 on the $CI$ scale, we read 52.3, 31.8, 22.87, 17.85, and 14.64 respectively on the $D$ scale.

**CUBES AND CUBE ROOTS**

The $K$ and $D$ scales are so arranged that if the indicator is set over a number on $D$, its cube will be found on the $K$ scale under the indicator line.

**Rule:** To find the cube of a number, set the indicator to the number on the $D$ scale and read its cube under the indicator line on the $K$ scale.

**Example:** Find the cube of 4.38. Opposite 4.38 on $D$ read 84 on $K$.

![Fig. XXVII.]
Cubing 4, the nearest round number to 4.38, is $4 \times 4 \times 4 = 64$. Therefore the answer must be 84, and not 8.4 or 840.

**Rule:** To find the cube root of a number, set the indicator to the number on the K scale and read the cube root on D, under the indicator line.

**Important:** The K scale consists of three sections. The cube roots of whole numbers with 1 or 4 digits, and the cube roots of decimal quantities with 2 or 5 zeros following the decimal point, are found by using the left-hand section. The cube roots of whole numbers with 2 or 5 digits, and the cube roots of decimal quantities with 1 or 4 zeros following the decimal point are found by using the middle section. The cube roots of whole numbers with 3 to 6 digits, and the cube roots of decimal quantities with no zero or 3 zeros following the decimal point are found by using the right-hand section. Thus: the cube roots of 0.008, 0.000008, and 8000 are found on D opposite the left-hand 8 of K; the cube roots of 0.00008, 0.08, 80 and 80000 are found on D opposite the middle 8 of K; and the cube roots of 0.0008, 0.8, 800 and 800,000 are found on D opposite the right-hand 8 of K.

**Example:** Find the cube roots of 6.25, 62.5 and 625.0. Use 625 on each of the three sections of K as shown below, reading the answers on the D scale.

![Fig. XXVIII.](image)

**The Decimal Point:** In cube root the decimal point is placed as follows:

Where a decimal fraction has 3, 4 or 5 zeros after the decimal point, its cube root will have one zero between the decimal point and the answer as given on the D scale.

Where a decimal fraction has no zeros, 1 or 2 zeros after the decimal point, its cube root will be the answer as given on the D scale, preceeded by the decimal point.

Where a whole number consists of 1, 2 or 3 digits, its cube root will have one digit before the decimal point.

Where a whole number consists of 4, 5 or 6 digits, its cube root will have two digits before the decimal point.
TRIGONOMETRY

The slide rule has been adopted by many High Schools for use in connection with their Trigonometry work. It can be used for the actual solution of triangles, but is more often used to check answers obtained by other methods.

For this work, the slide is reversed and set so that the $S$ scale is adjacent to the $A$ scale, and the $T$ scale adjacent to the $D$ Scale. In this position, problems of multiplication, division and proportion, in which one factor is the sine or tangent of an angle, can be quickly solved. The method is the same as used when both factors are numbers.

**Rule:** The numerical values of the sines of the angles appearing on the $S$ Scale can be found by closing the rule and reading from $S$ to $A$. (See Fig. XXIX.)

**Important:** All natural sines read on the left half of the $A$ scale have one zero to the left of the first significant figure and to the right of the decimal point. The natural sines read on the right half of the $A$ scale have the decimal point just before the first significant figure.

This must be borne in mind in determining the final location of the decimal point in problems making use of the sine and tangent scale.

**Rule:** The natural tangents of various angles are read in a similar manner, using the $T$ and $D$ scale. The natural tangents of all angles read in this way on the $D$ scale have the decimal point just before the first significant figure.

Angles below $5^\circ 43'$, as will be noted, cannot be read on the $T$ scale. However, as the natural tangents of angles below $5^\circ 43'$, for all practical purposes, are the same as the natural sines of like angles, the left half of the $S$ scale can be used for tangents as well as for sines.

The natural tangents of angles greater than $45^\circ$ should be found by using the formula $\tan x = \frac{1}{\tan (90^\circ - x)}$.

![Image of slide rule with trigonometric values](image)

Fig. XXIX,
SIMPLE PROBLEMS MAKING USE OF
THE S AND T SCALES

Example: Multiplication. \(4 \times \sin 11^\circ = x\)

![Diagram](image)

Fig. XXX.

Example: Division. \(\frac{3}{\tan 11^\circ} = x\)

![Diagram](image)

Fig. XXXI.

Example: Proportion. \(\frac{3}{\sin 9^\circ} = \frac{x}{\sin 30^\circ}\)

![Diagram](image)

Fig. XXXII.
CHECKING AND SOLVING TRIANGLES

The following is a typical right angle triangle problem, with one side and adjacent angle known.

Given \( A = 32^\circ 30' \)
\( c = 14.7 \)

To find: Sides
\( a \) and \( b \).

Solution by logarithms gives the following results:
\[
\begin{align*}
   a &= 7.9 \\
   b &= 12.4
\end{align*}
\]

These answers are generally checked in the following manner.
\[
\begin{align*}
a^2 &= c^2 - b^2 = (c + b)(c - b) \\
2 \log a &= \log (c + b) + \log (c - b) \\
1.79508 &= \log 27.1 + \log 2.3 \\
        &= 1.43297 + .36173 \\
        &= 1.79470
\end{align*}
\]

This checks, as can be seen, to only three figures, but this is as much as can be expected since \( c, a \) and \( b \) are given to only 3 figures.

In comparison with the above check, which takes about 10 minutes, that on the slide rule requires only a few seconds.

The sine formula is a proportion which expresses the relation between the known and unknown quantities, and is very convenient for slide rule use.

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \frac{14.7}{(\sin 90^\circ)}
\]

Set 90° on \( S \) to 14.7 on \( A \)

Over 32° 30’ (A), will be found 7.9’ (a) 
Over 57° 30’ (B), will be found 12.4 (b).
If we had desired to do so, we could have solved the problem in the first place in exactly the same manner, but it is the general practice in High School work to have the student use the logarithmic method first.

When functions other than the sine or tangent are encountered, make use of the following formulae to express them in terms of sine or tangent.

\[
\begin{align*}
\cos x &= \sin (90^\circ - x) \\
\cot x &= \frac{1}{\tan x} \\
\sec x &= \frac{1}{\sin (90^\circ - x)} \\
\csc x &= \frac{1}{\sin x}
\end{align*}
\]

Thus, if we have— \(3.4 \times \csc 14^\circ = y\)

make the problem read \(y = \frac{3.4}{\sin 14^\circ}\)

and solve as follows:

![Diagram](image)

**Fig. XXXV.**

**LOGARITHMS**

With the slide reversed, as in Fig. XXIX, note the scale of equal parts \((L)\) between the \(S\) and \(T\) scales. To find the logarithm (mantissa) of any number from 1 to 10, align the indexes, set the indicator to the number on scale \(D\) and read its logarithm on scale \(L\), to three places.

For occasional reference, logarithms are read with the slide in its usual position (the scale of equal parts underneath), by setting the number on \(C\) above the right index of \(D\) and reading the value of the mantissa on the \(L\) scale under the hairline on the underside of the rule.

**Example:** To find the logarithm of 40, set 4 on \(C\) over the right index of \(D\); underneath read 602 on \(L\). Placing the decimal point, and prefixing the characteristic as usual, \(\log 40 = 1.602\).