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SLIDE RULES
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WITH TRIGONOMETRICAL SCALES
DIVIDED TO REPRESENT
DEGREES AND MINUTES

INSTRUCTIONS MANUAL

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NEW YORK HOBOKEN, N. J.

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SLIDE RULE MANUAL

No. 4168 SLIDE RULE

WITH TRIGONOMETRICAL SCALES DIVIDED TO REPRESENT DEGREES AND MINUTES

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SLIDE RULE MANUAL

POLYPHASE DUPLEX®

POCKET SLIDE RULE

MANUAL

No. 68 2059

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4168

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Explanation of the Various Covers for the Various Issues of this Manual

The 4168 was released under four different model numbers at various times in its production history.

Initial release was a model number 10000. This was probably only produced during 1948. After 15 years of collecting I have never seen one but we do have a manual.

During 1949–1952 the same rule was known as model number 9068.

In 1952 its model number was changed to 4168 and it carried this designation until all slide rule model numbers were changed to 68 xxxx numbers in 1962. In 1962 the model number was changed to 68 1555.

The manuals of all four are identical except for the covers. These scans are of the 68 1555 manual.

Clark McCoy

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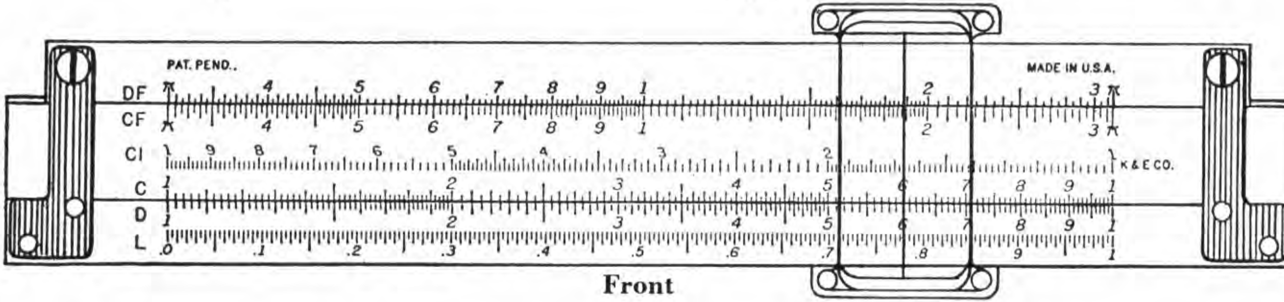
with trigonometrical scales divided to represent degrees and minutes



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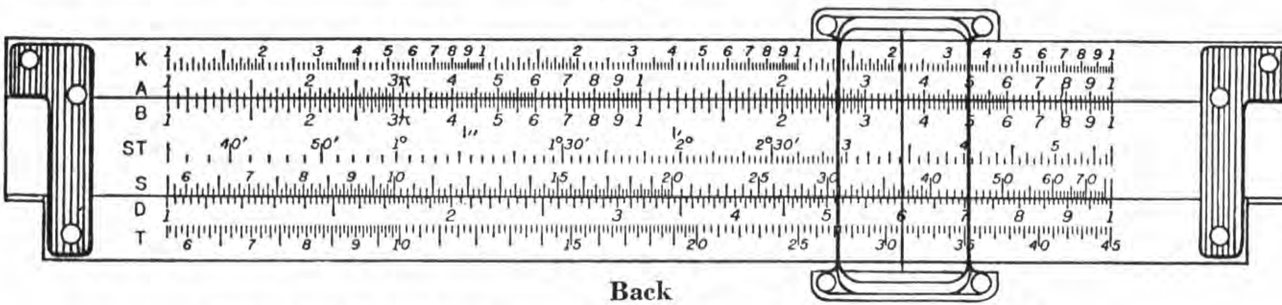

Back

Note that all numbers slant in the direction of increasing scale values.

PREFACE

This slide rule manual has been written for study without the aid of a teacher. For this reason one might suspect that the treatment is superficial. On the contrary, however, the subject matter is so presented that the beginner uses two general principles while he is learning to read the scales and perform the simpler operations. The mastery of these two principles gives the power to devise the best settings for any particular purpose, and to recall settings which have been forgotten.

These principles are so simple and so carefully explained and illustrated both by diagram and by example that they are easily mastered. In Chapter II, they are applied to simple problems in multiplication and division; in Chapters III and IV they are used to solve problems involving multiplication, division, square and cube root, trigonometry, and logarithms of numbers.

More and more today machines are doing the work which was carried out laboriously in earlier periods by hand methods. One of the simplest of these modern instruments is the slide rule. Its great value consists in the fact that a large number of operations can be carried out on it in a fraction of the time required by other methods. The results obtained, being accurate approximately to three figures, are good enough for a great many purposes. In any case, they serve as a check on work done by other methods. The slide rule, perfected by three centuries of use, easy to understand, easy to operate, has come to be widely used in recent years and its popularity is increasing rapidly.

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CHAPTER I

MULTIPLICATION AND DIVISION

1. Introduction. This pamphlet is designed to enable any interested person to learn to use the slide rule efficiently. The beginner should keep his slide rule before him while reading the pamphlet, should make all settings indicated in the illustrative examples, and should compute answers for a large number of the exercises. The principles involved are easily understood but a certain amount of practice is required to enable one to use the slide rule efficiently and with a minimum of error.

2. Reading the scales.* Everyone has read a ruler in measuring a length. The number of inches is shown by a number appearing on the ruler, then small divisions are counted to get the number of 16th's of an inch in the fractional part of the inch, and finally in close measurement, a fraction of a 16th of an inch may be estimated. We first read a primary length, then a secondary length, and finally estimate a tertiary length. Exactly the same method is used in reading the slide rule. The divisions on the slide rule are not uniform in length, but the same principle applies.

Figure 1 represents, in skeleton form, the fundamental scale of the slide rule, namely the *D* scale. An examination of this actual

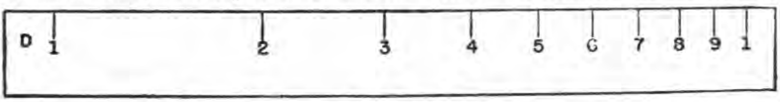


FIG. 1.

scale on the slide rule will show that it is divided into 9 parts by primary marks which are numbered 1, 2, 3, . . . , 9, 1. The space between any two primary marks is divided into ten parts by nine secondary marks. On the ten-inch rule these are numbered between 1 and 2 but not elsewhere. On the five-inch rule no secondary marks are numbered.

*The description here given has reference to the ten-inch slide rule. However anyone having a five-inch rule or a rule of different length will be able to understand his rule in the light of the explanation given.

Fig. 2 shows the secondary marks lying between the primary marks of the *D* scale. On this scale each italicized number gives the reading to be associated with its corresponding secondary mark. Thus,

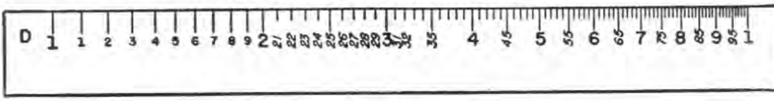


FIG. 2.

the first secondary mark after 2 is numbered 21, the second 22, the third 23, etc.; the first secondary mark after 3 is numbered 31, the second 32, etc. Between the primary marks numbered 1 and 2 the secondary marks are numbered 1, 2, . . . , 9. Evidently the readings associated with these marks are 11, 12, 13, . . . , 19. Finally between the secondary marks, see Fig. 3, appear smaller or tertiary marks which aid in obtaining the third digit of a reading. Thus

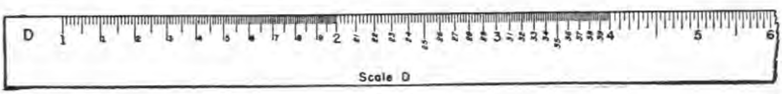


FIG. 3.

between the secondary marks numbered 22 and 23 there are 4 tertiary marks. If we think of the end marks as representing 220 and 230, the four tertiary marks divide the interval into five parts each representing 2 units. Hence with these marks we associate the numbers 222, 224, 226, and 228; similarly the tertiary marks between the secondary marks numbered 32 and 33 are read 322, 324, 326, and 328, and the tertiary marks between the primary marks numbered 3 and the first succeeding secondary mark are read 302, 304, 306, and 308. Between any pair of secondary marks to the right of the primary mark numbered 4, there is only one tertiary mark. Hence, each smallest space represents five units. Thus the tertiary mark between the secondary marks representing 41 and 42 is read 415, that between the secondary marks representing 55 and 56 is read 555, and the first tertiary mark to the right of the primary mark numbered 4 is read 405. The reading of any position between a pair of successive tertiary marks must be based on an estimate. Thus a position half way between the tertiary marks associated with 222 and 224 is read 223, and a position two fifths of the way from the tertiary mark numbered to the 415 next mark is read 417. The principle illustrated by these readings applies in all cases.

Consider the process of finding on the D scale the position representing 246. The first figure on the left, namely 2, tells us that the position lies between the primary marks numbered 2 and 3. This region is indicated by the brace in figure a. The second figure from the left, namely 4, tells us that the position lies

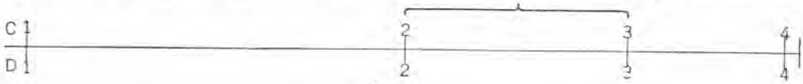


FIG. a.

between the secondary marks associated with 24 and 25. This region is indicated by the brace in Fig. b. Now there are 4 marks between the secondary marks associated with 24 and 25. With these

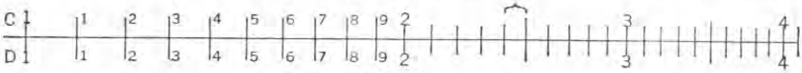


FIG. b.

are associated the numbers 242, 244, 246, and 248 respectively.

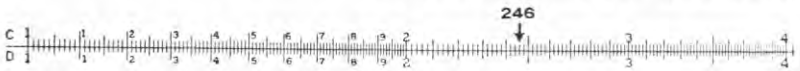


FIG. c.

Thus the position representing 246 is indicated by the arrow in Fig. c. Fig. abc gives a condensed summary of the process.

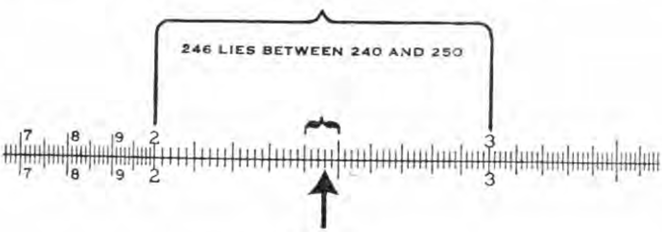


FIG. abc.

It is important to note that the decimal point has no bearing upon the position associated with a number on the C and D scales. Consequently, the arrow in Fig. abc may represent 246, 2.46, 0.000246, 24,600, or any other number whose principal digits are 2, 4, 6. The placing of the decimal point will be explained later in this chapter.

For a position between the primary marks numbered 1 and 2, four digits should be read; the first three will be exact and the last one estimated. No attempt should be made to read more than three digits for positions to the right of the primary mark numbered 4.

While making a reading, the learner should have definitely in mind the number associated with the smallest space under consideration. Thus between 1 and 2, the smallest division is associated with 10 in the fourth place; between 2 and 3, the smallest division has a value 2 in the third place; while to the right of 4, the smallest division has a value 5 in the third place.*

The learner should read from Fig. 4 the numbers associated with the marks lettered A, B, C, . . . and compare his readings with the

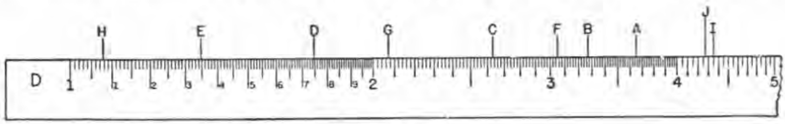


FIG 4.

following numbers: A 365, B 327, C 263, D 1745, E 1347, F 305, G 207, H 1078, I 435, J 427.

3. Accuracy of the slide rule. From the discussion of §2, it appears that we read four figures in a result on one part of the D scale and three figures on the remaining part. This means roughly an attainable accuracy of 999 parts in 1000. The errors made in reading a scale are nearly proportional to the length of it associated with a given range of numbers. Hence we associate with a five-inch D scale an accuracy of 499 parts in 500, with a twenty-inch D scale an accuracy of 1999 parts in 2000, and with a hundred-inch D scale an accuracy of 9999 parts in 10000. The accuracy obtainable with a ten-inch rule, or even a five-inch rule, is sufficient for many purposes; in any case the slide rule result serves as a check.

4. Definitions. The central sliding part of the rule is called the *slide*, the other part the *body*. The glass runner is called the *indicator* and the line on the indicator is referred to as the *hairline*.

The mark associated with the primary number 1 on any scale is called the *index* of the scale. An examination of the D scale shows that it has two indices, one at the left end and the other at the right end.

Two positions on different scales are said to be *opposite* if, without moving the slide, the hairline may be brought to cover both positions at the same time.

* On the D scale of the five-inch rule the smallest interval between 1 and 2 has a value of 2 in the in the third place, between 2 and 5 a value of five in the third place, and between 5 and 10, a value of 10 in the third place, or 1 in the second place.

5. Multiplication. The process of multiplication may be performed by using scales C and D. The C scale is on the slide, but in other respects it is like the D scale and is read in the same manner.

To multiply 2 by 4,

to 2 on D set index of C,
push hairline to 4 on C,
at the hairline read **8** on D.

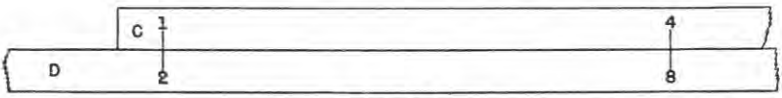


Fig. 5 (a).

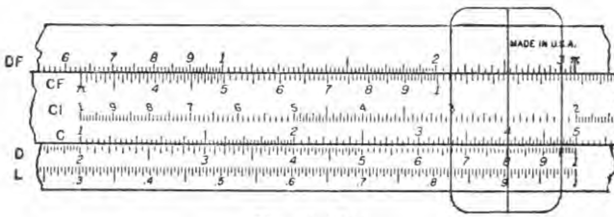


Fig. 5 (b).

Fig. 5(b) shows the rule set for multiplying 2 by 4 and Fig. 5(a) shows the same setting in skeleton form. To multiply 3×3,

to 3 on D set index of C,
push hairline to 3 on C,
at the hairline read **9** on D.

See Fig. 6(a) for the setting in skeleton form and Fig. 6(b) for a photograph of the setting.

Fig. 6 (a).

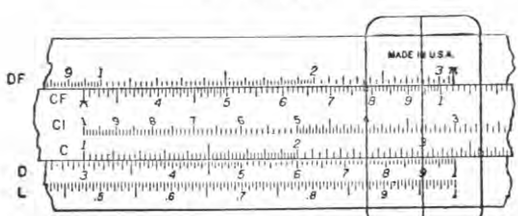


Fig. 6 (b).

To multiply 1.5×3.5, disregard the decimal point and
to 15 on D set index of C,
push hairline to 35 on C,
at the hairline read **525** on D.

By inspection we know that the answer is near to 5 and is therefore **5.25**.

To find the value of 16.75×2.83 (see Fig. 7(a) and Fig. 7(b))

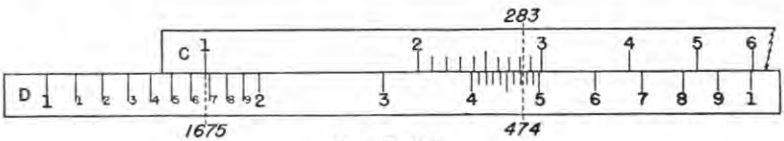


FIG. 7 (a).

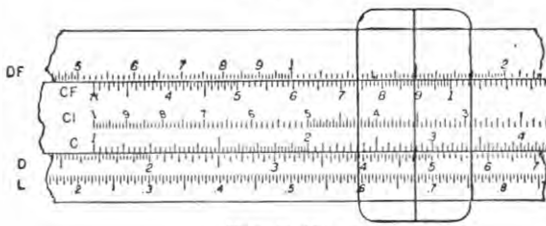


FIG. 7 (b).

disregard the decimal point and
to 1675 on D set index of C,
push hairline to 283 on C,
at the hairline read **474** on D.

To place the decimal point we approximate the answer by noting that it is near to $3 \times 16 = 48$. Hence the answer is **47.4**.

These examples illustrate the use of the following rule.

Rule. *To find the product of two numbers, disregard the decimal points, opposite either of the numbers on the D scale set the index of the C scale, push the hairline of the indicator to the second number on the C scale, and read the answer under the hairline on the D scale. The decimal point is placed in accordance with the result of a rough calculation.*

EXERCISES

1. 3×2.
2. 3.5×2.
3. 5×2.
4. 2×4.55.
5. 4.5×1.5.
6. 1.75×5.5.
7. 4.33×11.5.
8. 2.03×167.3.
9. 1.536×30.6.
10. 0.0756×1.093.
11. 1.047×3080.
12. 0.00205×408.
13. $(3.142)^2$.
14. $(1.756)^2$.

6. Either index may be used. It may happen that a product cannot be read when the left index of the C scale is used in the rule of §5. This will be due to the fact that the second number of the product is on the part of the slide projecting beyond the body. In this case reset the slide using the right index of the C scale in place of the left, or use the following rule: *when a number is to be read on the D scale opposite a number of the C scale and cannot be read, push the hairline to the index of the C scale inside the body and draw the other index of the C scale under the hairline.** The desired reading can then be made.

If, to find the product of 2 and 6, we set the left index of the C scale opposite 2 on the D scale, we cannot read the answer on the D scale opposite 6 on the C scale. Hence, we set the right index of C opposite 2 on D; opposite 6 on C read the answer, **12**, on D.

Again, to find 0.0314×564,

to 314 on D set the right index of C,
push hairline to 564 on C,
at the hairline read **1771** on D.

A rough approximation is obtained by finding $0.03 \times 600 = 18$. Hence the product is **17.71**.

EXERCISES

Perform the indicated multiplications.

1. 3×5.
2. 3.05×5.17.
3. 5.56×634.
4. 743×0.0567.
5. 0.0495×0.0267.
6. 1.876×926.
7. 1.876×5.32.
8. 42.3×31.7.

7. Division. The process of division is performed by using the C and D scales.

To divide 8 by 4 (see Figs. 8(a) and 8(b)),

push hairline to 8 on D,
draw 4 of C under the hairline,
opposite index of C read **2** on D.

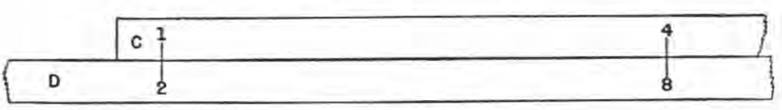


FIG. 8 (a).

*This rule, slightly modified to apply to the scales being used, is generally applicable when an operation calls for setting the hairline to a position on the part of the slide extending beyond the body.

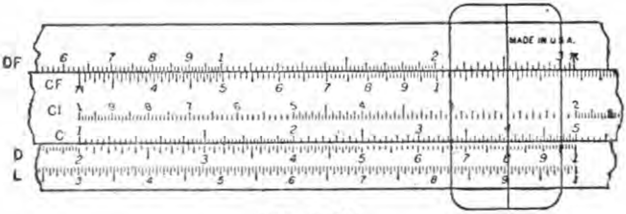


FIG. 8 (*b*).

To divide 876 by 20.4,

push hairline to 876 on D,
draw 204 of C under the hairline,
opposite index of C read **429** on D.

The rough calculation $800 \div 20 = 40$ shows that the decimal point must be placed after the 2. Hence the answer is **42.9**.

These examples illustrate the use of the following rule.

Rule. *To find the quotient of two numbers, disregard the decimal points, opposite the numerator on the D scale set the denominator on the C scale, opposite the index of the C scale read the quotient on the D scale. The position of the decimal point is determined from information gained by making a rough calculation.*

EXERCISES

Perform the indicated operations.

1. $87.5 \div 37.7$.
2. $3.75 \div 0.0227$.
3. $0.685 \div 8.93$.
4. $1029 \div 9.70$.
5. $0.00377 \div 5.29$.
6. $2875 \div 37.1$.
7. $871 \div 0.468$.
8. $0.0385 \div 0.001462$.
9. $3.14 \div 2.72$.
10. $3.42 \div 81.7$.

8. Simple applications. Percentage. Rates. Many problems involving percentage and rates are easily solved by means of the slide rule.

One per cent (1%) of a number N is $N \times 1/100$; hence 5% of N is $N \times 5/100$, and, in general, $p\%$ of N is $pN/100$. Hence to find 83% of 1872

to 1872 on D set right index of C,
push hairline to 83 on C,
at the hairline read **1554** on D.

Since $(83/100) \times 1872$ is approximately $\frac{80}{100} \times 2000 = 1600$, the answer is **1554**.

To find the answer to the question "M is what per cent of N?" we must find $100\ M \div N$. Thus, to find the answer to the question "87 is what per cent of 184.7?" we must divide $87 \times 100 = 8700$ by 184.7. Hence

push hairline to 87 on D,
draw 1847 of C under the hairline,
opposite index of C read **471** on D.

The rough calculation $\frac{9000}{200} = 45$ shows that the decimal point should be placed after the **7**. Hence the answer is **47.1%**.

For a body moving with a constant velocity, distance = rate times time. Hence if we write d for distance, r for rate, and t for time, we have

$$d = rt, \text{ or } r = \frac{d}{t}, \text{ or } t = \frac{d}{r}.$$

To find the distance traveled by a car going 33.7 miles per hour for 7.75 hours, write $d = 33.7 \times 7.75$, and

to 337 on D set right index of C,
push hairline to 775 on C,
at hairline read **261** on D.

Since the answer is near to $8 \times 30 = 240$ miles, we have $d = $ **261** miles.

To find the average rate at which a driver must travel to cover 287 miles in 8.75 hours, write $r = 287 \div 8.75$, and

push hairline to 287 on D,
draw 875 of C under the hairline,
opposite the index of C read **328** on D.

Since the rate is near $280 \div 10 = 28$, we have $r = $ **32.8** miles per hour

EXERCISES

1. Find (*a*) 86.3 per cent of 1826.
   (*b*) 75.2 per cent of 3.46.
   (*c*) 18.3 per cent of 28.7.
   (*d*) 0.95 per cent of 483.

2. What per cent of
   (*a*) 69 is 18?
   (*b*) 132 is 85?
   (*c*) 87.6 is 192.8?
   (*d*) 1027 is 28?

3. Find the distance covered by a body moving
   (*a*) 23.7 miles per hour for 7.55 hours.
   (*b*) 68.3 miles per hour for 1.773 hours.
   (*c*) 128.7 miles per hour for 16.65 hours.

4. At what rate must a body move to cover
   (*a*) 100 yards in 10.85 seconds.
   (*b*) 386 feet in 25.7 seconds.
   (*c*) 93,000,000 miles in 8 minutes and 20 seconds.

5. Find the time required to move
   (*a*) 100 yards at 9.87 yards per second.
   (*b*) 3800 miles at 128.7 miles per hour.
   (*c*) 25,000 miles at 77.5 miles per hour.

9. Use of the scales *DF* and *CF* (folded scales). The *DF* and the *CF* scales are the same as the *C* and the *D* scales respectively except in the position of their indices. The fundamental fact concerning the folded scales may be stated as follows: *if for any setting of the slide, a number M of the C scale is opposite a number N on the D scale, then the number M of the CF scale is opposite the number N on the DF scale.* Thus, if the learner will draw 1 of the *CF* scale opposite 1.5 on the *DF* scale, he will find the following opposites on the *CF* and *DF* scales

CF	1	2	4	5	6	6.67	
DF	1.5	3	6	7.5	9	1	

and the same opposites will appear on the *C* and *D* scales.

In accordance with the principle stated above, if the operator wishes to read a number on the *D* scale opposite a number *N* on the *C* scale but cannot do so, he can generally read the required number on the *DF* scale opposite *N* on the *CF* scale. For example to find 2×6,

to 2 on *D* set left index of *C*,
push hairline to 6 on *CF*,
at the hairline read **12** on *DF*

By using the *CF* and *DF* scales we saved the trouble of moving the slide as well as the attendant source of error. This saving, entering as it does in many ways, is a main reason for using the folded scales.

The folded scales may be used to perform multiplications and divisions just as the *C* and *D* scales are used. Thus to find 6.17×7.34,

to 617 on *DF* set index of *CF*,
push hairline to 734 on *CF*,
at the hairline read **45.3** on *DF*;

or

to 617 on *DF* set index of *CF*,
push hairline to 734 on *C*,
at the hairline read **45.3** on *D*.

Again to find the quotient 7.68/8.43,

push hairline to 768 on *DF*,
draw 843 of *CF* under the hairline,
opposite the index of *CF* read **0.912** on *DF*;

or

push hairline to 768 on *DF*,
draw 843 of *CF* under the hairline,
opposite the index of *C* read **0.912** on *D*.

It now appears that we may perform a multiplication or a division in several ways by using two or more of the scales *C*, *D*, *CF*, and *DF*. The sentence written in italics near the beginning of the article sets forth the guiding principle. A convenient method of multiplying or dividing a number by π (= 3.14 approx.) is based on the statement: any number on *DF* is π times its opposite on *D*, and any number on *D* is $1/\pi$ (= 0.318) times its opposite on *DF*. For example:

push hairline to 5 on *D*,
at the hairline read **15.71** ($= 5\pi$) on *DF*,
push hairline to 5.32 on *DF*,
at hairline read **1.693** ($= 5/\pi$) on *D*.

EXERCISES

Perform each of the operations indicated in exercises 1 to 11 in four ways; *first* by using the *C* and *D* scales only; *second* by using the *CF* and *DF* scales only; *third* by using the *C* and *D* scales for the initial setting and the *CF* and *DF* scales for completing the solution; *fourth* by using the *CF* and *DF* scales for the initial setting and the *C* and *D* scales for completing the solution.

1. 5.78×6.35.
2. 7.84×1.065.
3. $0.00465 \div 73.6$.
4. $0.0634 \times 53{,}600$.
5. $1.769 \div 496$.
6. $946 \div 0.0677$.
7. 813×1.951.
8. $0.00755 \div 0.338$.
9. $0.0948 \div 7.23$.
10. $149.0 \div 63.3$.
11. $2.718 \div 65.7$.
12. $783\,\pi$.
13. $783 \div \pi$.
14. $0.0876\,\pi$.
15. $0.504 \div \pi$.
16. $1.072 \div 10.97$.

CHAPTER II

THE PROPORTION PRINCIPLE AND COMBINED OPERATIONS

10. Introduction. The *ratio* of two numbers a and b is the quotient of a divided by b or a/b. A statement of equality between two ratios is called a proportion. Thus

$$\frac{2}{3}=\frac{6}{9},\ \frac{x}{5}=\frac{7}{11},\ \frac{a}{b}=\frac{c}{d}$$

are proportions. We shall at times refer to equations having such forms as

$$\frac{2}{3}=\frac{x}{5}=\frac{9}{y}=\frac{10}{z}\text{, and } \frac{a}{b}=\frac{c}{d}=\frac{e}{f}$$

as proportions.

An important setting like the one for multiplication, the one for division, and any other one that the operator will use frequently should be practiced until it is made without thought. But, in the process of devising the best settings to obtain a particular result, of making a setting used infrequently, or of recalling a forgotten setting, the application of proportions as explained in the next article is very useful.

11. Use of Proportions. If the slide is drawn to any position, the ratio of any number on the D scale to its opposite on the C scale is, in accordance with the setting for division, equal to the number on the D scale opposite the index on the C scale. In other words, *when the slide is set in any position, the ratio of any number on the D scale to its opposite on the C scale is the same as the ratio of any other number on the D scale to its opposite on the C scale.* For example

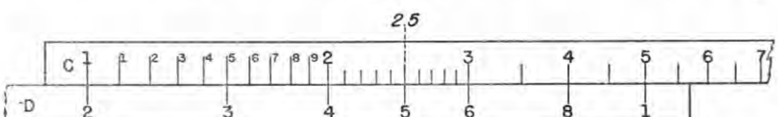


FIG. 1.

draw 1 of C opposite 2 on D (see Fig. 1) and find the opposites indicated in the following table:

C (or CF)	1	1.5	2.5	3	4	5	
D (or DF)	2	3	5	6	8	10	,

and draw 2 of C over 1 on D and read the same opposites. The same statement is true if in it we replace C scale by CF scale and D scale by DF scale. Hence, *if both numerator n and denominator d of a ratio in a given proportion are known, we can set n of the C scale opposite d on the D scale and then read, for an equal ratio having one part known, its unknown part opposite the known part.* We could also begin by setting d on the C scale opposite n on the D scale. It is important to observe that *all the numerators of a series of equal ratios must appear on one scale and the denominators on the other.* For example, let it be required to find the value of x satisfying

$$\frac{x}{56} = \frac{9}{7}, \tag{1}$$

Here the known ratio is 9/7. Hence

push hairline to 7 on D,
draw 9 of C under the hairline,
push hairline to 56 on D,
at the hairline read **72** on C.

or

push hairline to 9 on D,
draw 7 of C under the hairline,
push hairline to 56 on C,
at the hairline read **72** on D.

The CF and DF scales could have been used to obtain exactly the same settings and results.

For convenience we shall indicate the settings for solving (1) as follows:

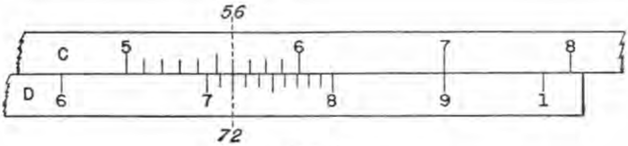


FIG. 2.

C (or CF)	set 7	opposite 56	(2)
D (or DF)	to 9	read x (= 72)	

C (or CF)	set 9	read x (= 72)	(3)
D (or DF)	to 7	opposite 56	

The letter at the beginning of a line indicates that all numbers on that line are to appear on the scale designated by that letter, and any pair of numbers set opposite each other in the frame are to appear as opposites on the slide rule.

To find the values of x, y, and z defined by the equations

$$\frac{C}{D}: \qquad \frac{3.15}{5.29} = \frac{x}{4.35} = \frac{57.6}{y} = \frac{z}{183.4},$$

we observe that 3.15/5.29 is the known ratio, and

push hairline to 529 on D,
draw 315 of C under the hairline;
opposite 435 on D, read x = **2.59** on C;
opposite 576 on C, read y = **96.7** on D;
opposite 1834 on D, read z = **109.2** on C.

The positions of the decimal points were determined by noticing that each denominator had to be somewhat less than twice its associated numerator because 5.29 is somewhat less than twice 3.15. The setting may appear simpler when written in the following form analogous to (2):

C	set 3.15	read x (= 2.59)	opposite 57.6	read z (= 109.2)	(4)
D	to 5.29	opposite 4.35	read y (= 96.7)	opposite 183.4	

When an answer cannot be read, apply the italicized rule of §6. Thus to find the values of x and y satisfying

$$\frac{C}{D}: \qquad \frac{x}{587} = \frac{14.56}{97.6} = \frac{5.78}{y},$$

to 976 on D set 1456 of C; then, since the answers cannot be read, push the hairline to the index on C, draw the right index of C under the hairline and

opposite 587 on D, read $x =$ **87.6** on C;
opposite 578 on C, read $y =$ **38.7** on D.

Here the positions of the decimal points were determined by observing that each denominator had to be about six times the associated numerator.

When a result cannot be read on the C scale nor on the D scale it may be possible to read it on the CF scale or on the DF scale. Thus, to find x and y satisfying the equations

$$\frac{C\ (\text{or } CF)}{D\ (\text{or } DF)}: \qquad \frac{4.92}{x} = \frac{1}{3.23} = \frac{y}{13.08},$$

to 323 on D set left index of C;
opposite 492 on CF, read $x =$ **15.89** on DF;
opposite 1308 on DF, read $y =$ **4.05** on CF.

A slight inspection of the scales will show the value of the statement: If the difference of the first digits of the two numbers of the known ratio is small use the C and D scales for the initial setting; if the difference is large use the CF and DF scales. Since in the next to the last example, the difference between the first digits was great, the CF and DF scales should have been used for the initial setting. This would have eliminated the necessity for shifting the slide.

EXERCISES

Find, in each of the following equations, the values of the unknowns.

1. $\frac{2}{3} = \frac{x}{7.83}$

2. $\frac{x}{1.804} = \frac{y}{25} = \frac{1}{0.785}$

3. $\frac{x}{709} = \frac{246}{y} = \frac{28}{384}$

4. $\frac{x}{0.204} = \frac{y}{0.0506} = \frac{5.28}{z} = \frac{2.01}{0.1034}$

5. $\frac{x}{2.07} = \frac{3}{61.3} = \frac{z}{1.571}$

6. $\frac{8.51}{1.5} = \frac{9}{x} = \frac{235}{y}$

7. $\frac{17}{x} = \frac{1.365}{8.53} = \frac{4.86}{y}$

8. $\frac{x}{y} = \frac{y}{7.34} = \frac{3.75}{29.7}$:

9. $\frac{x}{49.6} = \frac{z}{y} = \frac{y}{3.58} = \frac{1.076}{0.287}$

12. Forming proportions from equations. Since proportions are algebraic equations, they may be rearranged in accordance with the laws of algebra. For example, if

$$x = \frac{ab}{c}, \tag{1}$$

we may write the proportion

$$\frac{x}{1} = \frac{ab}{c}, \tag{2}$$

or we may divide both sides by a to get

$$\frac{x}{a} = \frac{ab}{ac}, \text{ or } \frac{x}{a} = \frac{b}{c}, \tag{3}$$

or we may multiply both sides by c/x to obtain

$$\frac{cx}{x} = \frac{cab}{xc}, \text{ or } \frac{c}{1} = \frac{ab}{x}. \tag{4}$$

Rule (*A*). *A number may be divided by 1 to form a ratio.* This was done in obtaining proportion (2).

Rule (*B*). *A factor of the numerator of either ratio of a proportion may be replaced by 1 and written as a factor of the denominator of the other ratio, and a factor of the denominator of either ratio may be replaced by 1 and written as a factor of the numerator of the other ratio.* Thus (3) could have been obtained from (1) by transferring a from the numerator of the right hand ratio to the denominator of the left hand ratio.

For example, to find $\frac{16 \times 28}{35}$, write $x = \frac{16 \times 28}{35}$, apply ($B$) to

obtain $\frac{C}{D}$: $\frac{x}{16} = \frac{28}{35}$,

and push hairline to 35 on D,
draw 28 of C under the hairline;
opposite 16 on D, read $x =$ **12.8** on C.

To recall the rule for dividing a given number M by a second given number N, write $x = \frac{M}{N}$, apply rule (A) to obtain $\frac{D}{C}$: $\quad \frac{x}{1} = \frac{M}{N}$,

and push hairline to M on D,
draw N of C under the hairline;
opposite index of C, read x on D.

To recall the rule for multiplication, set $x = \frac{M\ N}{1}$, apply rule (B) to obtain $\frac{D}{C}$: $\quad \frac{x}{M} = \frac{N}{1}$,

and to N on D set index of C;
opposite M on C, read x on D.

To find x if $\frac{1}{x} = \frac{864}{(7.48)\ (25.5)}$, use rule ($B$) to get $\frac{7.48}{x} = \frac{864}{25.5}$ make the corresponding setting and read $x = \mathbf{0.221}$. The position of the decimal point was determined by observing that x must be about $\frac{1}{40}$ of 8, or 0.2.

EXERCISES

Find in each case the value of the unknown quantity.

1. $y = \frac{86 \times 70.8}{125}$.
2. $y = \frac{147.5 \times 8.76}{3260}$.
3. $y = \frac{0.797 \times 5.96}{0.502}$.
4. $\frac{37 \times 86}{y} = 75.7$.
5. $498 = \frac{89.3x}{0.563}$.
6. $0.874 = \frac{3.95 \times 0.707}{x}$.
7. $2580y = 17.9 \times 587$.
8. $0.695 = \frac{0.0879}{x}$.
9. $\frac{1}{386} = \frac{0.772}{2.85y}$.
10. $3.14y = 0.785 \times 38.7$.

13. Equivalent expressions of quantity.* When the value of a quantity is known in terms of one unit, it is a simple matter to find its value in terms of a second unit. Thus to find the number of square feet in 3210 sq. in., write

$$\frac{1}{144} = \frac{\text{no. of sq. ft.}}{\text{no. of sq. in.}} = \frac{x}{3210},$$

*Tables of equivalents may be found in Engineer's Manuals and in other places.

since there are 144 sq. in. in a square foot; hence

to 144 on D, set index of C;

opposite 3210 on D, read $x = 22.3$ on C;

that is, there are 22.3 sq. ft. in 3210 sq. in.

Again consider the problem of finding the number of nautical miles in 28.5 ordinary miles. Since there are 5280 ft. in an ordinary mile and 6080 ft. in a nautical mile, write

$$\frac{5280}{6080} = \frac{\text{no. of naut. mi.}}{\text{no. of ord. mi.}} = \frac{x}{28.5},$$

make the corresponding setting and read $x = \mathbf{24.7}$ naut. mi.

EXERCISES

1. An inch is equivalent to 2.54 cm. Find the length in cm. of a rod 66 in. long.

2. The number of meters in a given length is to the number of yards as 171 is to 187. Find the number of meters in a 300 yd. distance.

3. If 7.5 gal. water weigh 62.4 lbs., find the weight of 86.5 gal. water.

4. 31 sq. in. are approximately 200 sq. cm. How many square centimeters in 36.5 sq. in.?

5. If one horse-power is equivalent to 746 watts, how many watts are equivalent to (a) 34.5 horsepower, (b) 5280 horsepower, (c) 0.832 horsepower?

6. If one gallon is equivalent to 3790 cu. cm., find the number of gallons of water in a bottle which contains (a) 4250 cu. cm., (b) 9.68 cu. cm., (c) 570 cu. cm. of the liquid.

14. The *CI* (reciprocal) scale. The reciprocal of a number is obtained by dividing 1 by the number. Thus, $\frac{1}{2}$ is the reciprocal of 2, $\frac{2}{3}$ $(= 1 \div \frac{3}{2})$ is the reciprocal of $\frac{3}{2}$, and $\frac{1}{a}$ is the reciprocal of a.

The reciprocal scale *CI* is like the *C*, *D*, and *CF* scales, respectively, with the exception that it is inverted, i.e. the numbers represented by the marks on this scale increase from right to left. A very important consideration may be stated as follows: *When the hairline is set to a number on the C scale, the reciprocal (or Inverse) of the number is at the hairline on the CI scale; conversely, when the hairline is set to a number on the CI scale, its reciprocal is at the hairline on the C scale.* By means of the hairline, the operator can read the opposites indicated in the diagram:

CI	1	2	4	5	8	9	
C	1	0.5 (= 1/2)	0.25 (= 1/4)	0.2 (= 1/5)	0.125 (= 1/8)	0.1111 (= 1/9)	

By using the facts just mentioned, we can multiply a number or divide it by the reciprocal of another number. Thus to find $\frac{28}{7}$, we may think of it as $28 \times \frac{1}{7}$ and

to 28 on D set index of C;
opposite 7 on CI read 4 on D.

Again to find 12×3, we may think of it as $12 \div \frac{1}{3}$ and

push hairline to 12 on D,
draw 3 of CI under the hairline;
opposite index of C, read **36** on D.

When the CI scale is used in multiplication and division, the position of the decimal point is determined in the usual way.

EXERCISES

1. Use the CI scale to find the reciprocals of 16, 260, 0.72, 0.065, 17.4, 18.5, 67.1:

2. Using the D scale and the CI scale, multiply 18 by 1/9 and divide 18 by 1/9.

3. Using the D scale and the CI scale, multiply 28.5 by 1/0.385 and divide 28.5 by 1/0.385. Also find 28.5/0.385 and 28.5×0.385 by using the C scale and the D scale.

4. Using the D scale and the CI scale, multiply 41.3 by 1/0.207 and divide 41.3 by 1/0.207.

15. Proportions involving the *CI* scale. The CI scale may be used in connection with proportions containing reciprocals. Since any number $a = 1 \div \frac{1}{a}$ and since $\frac{1}{a} = \frac{1}{a} \div 1$, we have

Rule C. *The value of any ratio is not changed if any factor of its numerator be replaced by 1 and its reciprocal be written in the denominator, or if any factor of its denominator be replaced by 1 and its reciprocal be written in the numerator.* Thus $\frac{a}{b} = a\left(\frac{1}{b}\right) = \frac{1}{b\ (1/a)}\cdot$ Hence if $\frac{x}{a} = bc$, we may write $\frac{x}{a} = \frac{b}{(1/c)} = \frac{c}{(1/b)}$; if $ax = bc$, we may write $\frac{x}{(1/a)} = \frac{b}{(1/c)} = \frac{c}{(1/b)}$. A few examples will indicate the method of applying these ideas in computations.

To find the value of y which satisfies $\frac{y}{4.27} = 0.785 \times 3.76$, apply Rule C to get $\frac{D}{C}$: $\qquad \frac{y}{4.27} = \frac{0.785}{(1/3.76)}\cdot$

Since when 3.76 of CI is under the hairline, 1/3.76 of C is also under the hairline

push hairline to 785 on D,
draw 376 of CI under the hairline;
opposite 427 on CF, read $y =$ **12.60** on DF.

The position of the decimal point was determined by observing that y was near to $4 \times 1 \times 4 = 16$.

To find the value of y which satisfies $7.89\ y = \frac{0.0645}{0.381}$, use Rule ($C$) to obtain $\frac{D}{C}$: $\qquad \frac{y}{(1/7.89)} = \frac{0.0645}{0.381}$,

and push hairline to 645 on D,
draw 381 of C under the hairline;
opposite 789 on CI, read $y =$ **0.0215** on D.

The position of the decimal point was determined by observing that .06 is about $\frac{1}{6}$ of 0.38, that y is therefore about $\frac{1}{6}$ of $\frac{1}{8}$, or about 0.02.

To find the values of x and y which satisfy $57.6x = 0.846y = 7$, use Rule (C) to obtain

$$\frac{D}{C}: \qquad \frac{x}{(1/57.6)} = \frac{y}{(1/0.846)} = \frac{7}{1}, \qquad (a)$$

and to 7 on D set index of C,
opposite 576 on CI, read $x = \mathbf{0.1215}$ on D,

exchange indices (see § 6.),

opposite 846 on CI, read $y = \mathbf{8.27}$ on D.

EXERCISES

In each of the following equations find the values of the unknown numbers:

1. $3.3x = 4.4y = \dfrac{75.2}{1.342}$.
2. $76.1x = 3.44y = \dfrac{111}{22.8}$.
3. $1.83x = \dfrac{y}{24.5} = (162)\ (1.75)$.
4. $\dfrac{0.342}{x} = \dfrac{y}{4.65} = (189)\ (0.734)$.
5. $5.83x = 6.44y = \dfrac{12.6}{z} = 0.2804$.
6. $3.42x = \dfrac{1.83}{y} = \dfrac{17.6}{z} = (2.78)\ (13.62)$.

16. Combined multiplication and division.

Example 1. Find the value of $\dfrac{7.36 \times 8.44}{92}$.

Solution. Reason as follows: first divide 7.36 by 92 and then multiply the result by 8.44. This would suggest that we

push hairline to 736 on D,
draw 92 of C under the hairline;
opposite 8.44 on C, read **0.675** on D.

Example 2. Find the value of $\dfrac{18 \times 45 \times 37}{23 \times 29}$.

Solution. Reason as follows: (a) divide 18 by 23, (b) multiply the result by 45, (*c*) divide this second result by 29, (*d*) multiply this third result by 37. This argument suggests that we

push hairline to 18 on D,
draw 23 of C under the hairline,
push hairline to 45 on C,
draw 29 of C under the hairline,
push hairline to 37 on C,
at the hairline read **449** on D.

To determine the position of the decimal point write $\frac{20 \times 40 \times 40}{20 \times 30}$ = about 50. Hence the answer is 44.9.

A little reflection on the procedure of Example 2 will enable the operator to evaluate by the shortest method expressions similar to the one just considered. He should observe that: the D scale was used only twice, once at the beginning of the process and once at its end; *the process for each number of the denominator consisted in drawing that number, located on the C scale, under the hairline; the process for each number of the numerator consisted in pushing the hairline to that number located on the C scale.*

If at any time the indicator cannot be placed because of the projection of the slide, apply the rule of §6, *or carry on the operations using the folded scales.*

Example 3. Find the value of $1.843 \times 92 \times 2.45 \times 0.584 \times 365$.

Solution. By using Rule C of §15, write the given expression in the form

$$\frac{1.843 \times 2.45 \times 365}{(1/92)\ (1/0.584)}$$

and reason as follows: (*a*) divide 1.843 by (1/92), (*b*) multiply the result by 2.45, (*c*) divide this second result by (1/0.584), (*d*) multiply the third result by 365. This argument suggests that we

push hairline to 1843 on D,
draw 92 of CI under the hairline,
push hairline to 245 on C,
draw 584 of CI under the hairline,
push hairline to 365 on C,
at the hairline read **886** on D.

To approximate the answer we write 2(90) (5/2) (610) 300 = 81,000. Hence the answer is **88,600**.

Example 4. Find the value of $\dfrac{0.873 \times 46.5 \times 6.25 \times 0.75}{7.12}$.

Solution. The following arrangement in which the difference between the number of factors in the numerator and the number in the denominator is no greater than 1 is obtained by applying Rule (C) of §15:

$$\frac{0.873 \times 46.5 \times 0.75}{7.12 \times (1/6.25)}.$$

This may be evaluated by (a) dividing 0.873 by 7.12, (b) multiplying the result by 46.5, (c) dividing the second result by (1/6.25), (d) multiplying the third result by 0.75. Hence

push hairline to 873 on D,
draw 712 of C under the hairline,
push hairline to 465 on C,
draw 625 of CI under the hairline,
push hairline to 75 on CF,
at the hairline read **267** on DF.

To approximate the answer write $\dfrac{1 \times 42 \times 6 \times 1}{7} = 36$. Hence the answer is 26.7.

A consideration of the examples of this article indicates that the operator should rewrite the expression to be evaluated so that its numerator shall contain the same number of factors, or one more factor than its denominator.

EXERCISES

1. $\dfrac{1375 \times 0.0642}{76{,}400}$.

2. $\dfrac{45.2 \times 11.24}{336}$.

3. $\dfrac{218}{4.23 \times 50.8}$.

4. $\dfrac{235}{3.86 \times 3.54}$.

5. $2.84 \times 6.52 \times 5.19$.

6. $9.21 \times 0.1795 \times 0.0672$.

7. $37.7 \times 4.82 \times 830$.

8. $\dfrac{65.7 \times 0.835}{3.58}$.

9. $\dfrac{362}{3.86 \times 9.61}$.

10. $\dfrac{24.1}{261 \times 32.1}$.

11. $\dfrac{75.5 \times 63.4 \times 95}{3.14}$.

12. $\dfrac{3.97}{51.2 \times 0.925 \times 3.14}$.

13. $\dfrac{47.3 \times 3.14}{32.5 \times 16.4}$.

14. $\dfrac{3.82 \times 6.95 \times 7.85 \times 436}{79.8 \times 0.0317 \times 870}$.

15. $187 \times 0.00236 \times 0.0768 \times 1047 \times 3.14$.

16. $\dfrac{0.917 \times 8.65 \times 1076 \times 3152}{7840}$.

CHAPTER III

SQUARES AND SQUARE ROOTS, CUBES AND CUBE ROOTS

17. Squares. The square of a number is the result of multiplying the number by itself. Thus $2^2 = 2 \times 2 = 4$.

The A scale is so designed that *when the hairline is set to a number on the D scale, the square of the number is found under the hairline on the A scale.* Similarly, if the hairline be set to a number on the C scale its square may be read at the hairline on the B scale. Note that the rule can be turned at will to enable the user to refer from one face to the other. For example, if one hairline of the indicator is set to 2 on C, the number $4 = 2^2$ will be found at the other hairline on scale B.

To gain familiarity with the relations between these scales the operator should set the hairline to 3 on the D scale, and read 9 at the hairline on the A scale; set the hairline to 4 on D, read 16 at the hairline on A; etc. To find 278^2, set the hairline to 278 on D, read 773 at the hairline on A. Since $300^2 = 90{,}000$, we write 77,300 as the answer. Actually $278^2 = 77{,}284$. The answer obtained on the slide rule is accurate to three figures.

The area of a circle may be conveniently found when its radius is known by using the A, B, C, and D scales. If π represents a mathematical constant whose value is approximately 3.14, and r represents the radius of a circle, then the area A equals πr^2. Similarly if d represents the diameter of a circle then its area is given by the formula $A = (\pi/4)d^2 = 0.785\, d^2$ nearly. Hence to find the area of a circle,

> to index of A set $\pi/4$ ($= 0.785$ approx.) on B;
> opposite any diameter on D, read area on B.

Note that a special mark toward the right end of the A and B scales gives the exact position of $\pi/4$. Thus to find the area of a circle of diameter 17.5 ft.

> to index on A set $\pi/4$ on B right;
> opposite 175 on D, read **241** on B.

Therefore the area is **241** sq. ft.

EXERCISES

1. Use the slide rule to find, accurate to three figures, the square of each of the following numbers: 25, 32, 61, 75, 89, 733, 452, 2.08, 1.753, 0.334, 0.00356, 0.953, 5270, 4.73×10^6.

2. Find the area of a circle having diameter (*a*) 2.75 ft.; (*b*) 66.8 ft.; (*c*) 0.753 ft.; (*d*) 1.876 ft.

3. Find the area of a circle having radius (*a*) 3.46 ft.; (*b*) 0.0436 ft.; (*c*) 17.53 ft.; (*d*) 8650 ft.

18. Square roots. The square root of a given number is a second number whose square is the given number. Thus the square root of 4 is 2 and the square root of 9 is 3, or, using the symbol for square root, $\sqrt{4} = 2$, and $\sqrt{9} = 3$.

The *A* scale consists of two parts which differ only in slight details. We shall refer to the left hand part as *A left* and to the right hand part as *A right*. Similar reference will be made to the *B* scale.

Rule. *To find the square root of a number between* 1 *and* 10, *set the hairline to the number on scale A left; and read its square root at the hairline on the D scale. To find the square root of a number between* 10 *and* 100, *set the hairline to the number on scale A right and read its square root at the hairline on the D scale.* In either case place the decimal point after the first digit. *A* similar statement relating to the *B* scale and the *C* scale holds true. For example, set the hairline to 9 on scale *A* left, read 3 $(=\sqrt{9})$ at the hairline on *D*, set the hairline to 25 on scale *B* right, read 5 $(=\sqrt{25})$ at the hairline on *C*.

To obtain the square root of any number, *move the decimal point an even number of places to obtain a number between* 1 *and* 100; *then apply the rule written above in italics; finally move the decimal point one half as many places as it was moved in the original number but in the opposite direction.** The learner may also place the decimal point in accordance with information derived from a rough approximation.

For example, to find the square root of 23,400, move the decimal point 4 places to the left thus getting 2.34 (a number between 1 and

*The following rule may also be used: If the square root of a number greater than unity is desired, use *A* left when it contains an odd number of digits to the left of the decimal point, otherwise use *A* right. For a number less than unity use *A* left if the number of zeros immediately following the decimal point is odd; otherwise, use *A* right.

10), set the hairline to 2.34 on scale A left, read 1.530 at the hairline on the D scale, finally move the decimal point $\frac{1}{2}$ of 4 or 2 places to the right to obtain the answer 153.0. The decimal point could have been placed after observing that $\sqrt{10,000} = 100$ or that $\sqrt{40,000} = 200$. Also the left B scale and the C scale could have been used instead of the left A scale and the D scale.

To find $\sqrt{3850}$, move the decimal point 2 places to the left to obtain $\sqrt{38.50}$, set the hairline to 38.50 on scale A right, read 6.20 at the hairline on the D scale, move the decimal point one place to the right to obtain the answer **62.0.** The decimal point could have been placed by observing that $\sqrt{3600} = 60$.

To find $\sqrt{0.000585}$, move the decimal point 4 places to the right to obtain $\sqrt{5.85}$, find $\sqrt{5.85} = 2.42$, move the decimal point two places to the left to obtain the answer **0.0242.**

EXERCISES

1. Find the square root of each of the following numbers: 8, 12, 17, 89, 8.90, 890, 0.89, 7280, 0.0635, 0.0000635, 63,500, 100,000.

2. Find the length of the side of a square whose area is (*a*) 53,500 ft.^{2}; (*b*) 0.0776 ft.^{2}; (*c*) 3.27×10^7 ft.^{2}

3. Find the diameter of a circle having area (*a*) 256 ft.^{2}; (*b*) 0.773 ft.^{2}; (*c*) 1950 ft.^{2}

19. Evaluation of simple expressions containing square roots and squares. When the hairline is set to a number on the proper one of the two B scales, its square root is automatically set to the hairline on the C scale. Consequently we may multiply and divide numbers by square roots of other numbers or we may find the value of the unknown in a proportion involving square roots.

For example, to find $3\sqrt{3.24}$ set index of C to 3 on D, push hairline to 3.24 on left B scale, read 540 at the hairline on D. Since $3\sqrt{3.24}$ is nearly equal to $3\sqrt{4} = 6$, the answer is **5.40.**

It will be convenient at times to indicate solutions by forms like the following which indicates a setting for evaluating $3\sqrt{3.240}$:

B		opposite 3.24 (left)
C	set 1	
D	to 3	read **5.40**

Note that each number is written on the same line as the letter naming *the scale on which it is to be set, and that numbers in the same vertical column are to be set opposite each other.*

The same plan is used below to indicate a setting for evaluating $85 \div \sqrt{4290}$:

B	set 4290 (right)	
C		opposite 1
D	to 85	read **1.297**

Computations involving square roots may be considered from the standpoint of multiplication and division or from that of proportions. Thus to find the value of $\frac{28\sqrt{375}}{369}$, we may compute it directly by the multiplication and division principle or we may write $x = \frac{28\sqrt{375}}{369}$, apply Rule B §12 to obtain

$$\frac{C}{D}: \quad \frac{x}{28} = \frac{\sqrt{375} \rightarrowtail (B\,\text{left})}{369},$$

and solve the proportion to obtain the answer $x =$ **1.470.**

To find $x = \frac{347}{7.92 \times \sqrt{0.0465}}$, write the equation in the form $x = \frac{347\ (1/7.92)}{\sqrt{0.0465}}$ and perform the operations indicated in the following diagram:

B	set 0.0465 (left)	
CI		opposite 7.92
D	to 347	read $x =$ **203**

The position of the decimal point was obtained from the fact that $320 \div (0.2)\ 8 = 200$.

A second method consists in applying Rule B §12 to get $7.92x = \frac{347}{\sqrt{0.0465}}$, then applying Rule C §15 to obtain

$$\frac{x}{(1/7.92)} = \frac{347}{\sqrt{0.0465}},$$

and solving this proportion to find $x =$ **203**.

When the hairline is set to a number on the D scale it is automatically set to the square of the number on the A scale, and when set to a number on the C scale it is automatically set to the square of the number on the B scale. Hence by using the A and B scales as fundamental scales, many expressions involving squares can be evaluated conveniently. Thus to find $x = \frac{(24.6)^2 \times 0.785}{4.39}$, write

$$\frac{A}{B}: \qquad \frac{x}{0.785} = \frac{(24.6)^2}{4.39},$$

and

push hairline to 246 on D,
draw 439 of B (either left or right) under the hairline,
push hairline to 785 on B (left or right),
at the hairline read **108.2** on A.

The decimal point was placed in the usual manner. Of course this computation could have been carried out on the C and D scales, but one will find it convenient at times to use the setting just indicated.

EXERCISES

1. $42.2\sqrt{0.328}$.
2. $1.83\sqrt{0.0517}$.
3. $\sqrt{3.28} \div 0.212$.
4. $\sqrt{51.7} \div 103$.
5. $0.763 \div \sqrt{0.0296}$.
6. $\frac{5.66 \times (7.48)^2}{79}$.
7. $\frac{2.56 \times 4.86}{(1.365)^2}$.
8. $\frac{(2.38)^2 \times 19.7}{18.14}$.
9. $\frac{6.76}{2.17\,(2.7)^2}$.
10. $\frac{\sqrt{277}}{5.34 \times \sqrt{7.02}}$.
11. $\frac{645}{5.34\sqrt{13.6}}$.
12. $14.3 \times 47.5\sqrt{0.344}$.
13. $20.6 \times \sqrt{7.89} \times \sqrt{0.571}$.
14. $\frac{7.92\sqrt{7.89}}{\sqrt{0.571}}$.

20. Combined operations involving square roots and squares. The principle of Example 2 §16 may be applied to evaluate a fraction containing indicated square roots as well as numbers and reciprocals of numbers. If the learner will recall that when the hairline is set to a number on the CI scale it is automatically set to the reciprocal of the number on the C scale and when set to a number on the B scale it is automatically set to the square root of the number on the C scale, he will easily understand that the method used in this article is essentially the same as that used in §16. The principle of determining whether B left or B right should be used is the same whether we are merely extracting the square root of a number or whether the square root is involved with other numbers.

Example 1. Evaluate $\dfrac{\sqrt{365} \times 915}{804}$.

Solution. To evaluate this expression, we may think "divide $\sqrt{365}$ by 804 and multiply the result by 915." To set $\sqrt{365}$ on D, set the hairline to 365 on A left. Hence

push hairline to 365 on A left,
draw 804 of C under the hairline,
push hairline to 915 on C,
at the hairline read **21.7** on D.

The following diagram indicates the setting:

A	opposite 365 (left)	
C	set 804	opposite 915
D		read **21.7**

Example 2. Evaluate $\dfrac{\sqrt{832} \times \sqrt{365} \times 1863}{(1/736) \times 89{,}400}$.

Solution. Before making the setting indicated in this solution, the learner should read the italicized statement in §16, p. 24.

Push hairline to 832 on A left,
draw 736 of CI under the hairline,
push hairline to 365 on B left,
draw 894 of C under the hairline,
push hairline to 1863 on CF,
at the hairline read **8450** on DF.

Example 3. Evaluate $\dfrac{0.286 \times 652 \times \sqrt{2350} \times \sqrt{5.53}}{785\sqrt{1288}}$

Solution. Write the expression in the form

$$\frac{0.286 \times \sqrt{2350} \times \sqrt{5.53}}{(1/652) \times 785 \times \sqrt{1288}} \qquad \text{and}$$

push hairline to 286 on D,
draw 652 of CI under the hairline,

push hairline to 235 on B right,
draw 785 of C under the hairline,
push hairline to 553 on B left,
draw 1288 of B right under hairline,
opposite the index of C read **0.755** on D.

Example 4. Evaluate $\dfrac{\pi^2 \times 875 \times 278}{(72.2)^2\ (0.317)^2}$.

Solution. Using the A and B scales as fundamental scales,
push hairline to 3.142 on D,
draw 722 of C under the hairline,
push hairline to 875 on B,
draw 317 of C under the hairline,
push hairline to 278 on B,
at the hairline read **4580** on A.

EXERCISES

1. $\dfrac{7.87 \times \sqrt{377}}{2.38}$.

$\dfrac{86 \times \sqrt{734} \times \pi}{775 \times \sqrt{0.685}}$.

3. $\dfrac{4.25 \times \sqrt{63.5} \times \sqrt{7.75}}{0.275 \times \pi}$.

4. $\dfrac{(2.60)^2}{2.17 \times 7.28}$.

5. $\dfrac{20.6 \times 7.89^2 \times 6.79^2}{4.67^2 \times 281}$.

6. $\dfrac{189.7 \times \sqrt{0.00296} \times \sqrt{347} \times 0.274}{\sqrt{2.85} \times 165 \times \pi}$.

21. Cubes. The cube of a number is the result of using the number three times as a factor. Thus the cube of 3 (written 3^3) is $3 \times 3 \times 3 = 27$.

The K scale is so constructed that *when the hairline is set to a number on the D scale, the cube of the number is at the hairline on the K scale.* To convince himself of this the operator should set the hairline to 2 on D, read 8 at the hairline on K, set the hairline to 3 on D, read 27 at the hairline on K, etc. To find 21.7^3, set the hairline to 217 on D and read 102 on K. Since $20^3 = 8000$, the answer is near 8000. Hence we write 10,200 as the answer. To obtain this answer otherwise, write

$$21.7^3 = \frac{21.7 \times 21.7}{(1/21.7)}$$

and use the general method of combined operations. This latter method is more accurate as it is carried out on the full length scales.

EXERCISES

1. Cube each of the following numbers by using the K scale and also by using the method of combined operations: 2.1, 3.2, 62, 75, 89, 733, 0.452, 3.08, 1.753, 0.0334, 0.943, 5270, 3.85×10^6.

2. How many gallons will a cubical tank hold that measures 26 inches in depth? (1 gal. = 231 cu. in.)

22. Cube roots. There are three parts to the K scale, each the same as the others except in position. We shall refer to the left hand part, the middle part, and the right hand part as K left, K middle, and K right respectively.

The cube root of a given number is a second number whose cube is the given number.

Rule. *To find the cube root of a number between* 1 *and* 10 *set the hairline to the number on K left, read its cube root at the hairline on D. To find the cube root of a number between* 10 *and* 100, *set the hairline to the number on K middle, and read its cube root at the hairline on D. The cube root of a number between* 100 *and* 1000 *is found on the D scale opposite the number on K right.* In each of the three cases the decimal point is placed after the first digit. To see how this rule is used, set the hairline to 8 on K left, read 2 at the hairline on D; set the hairline to 27 on K middle, read 3 at the hairline on D; set the hairline to 343 on K right, read 7 at the hairline on D.

To obtain the cube root of any number, move the decimal point over three places (or digits) *at a time until a number between* 1 *and* 1000 *is obtained, then apply the rule written above in italics; finally move the decimal point one third as many places as it was moved in the original number but in the opposite direction.* The learner may also place the decimal point in accordance with information derived from a rough approximation.

For example, to find the cube root of 23,400,000, move the decimal point 6 places to the left thus obtaining 23.4. Since this is between 10 and 100, set the hairline to 234 on K middle, read 2.86 at the hairline on D. Move the decimal point $\frac{1}{3}(6) = 2$ places to the right to obtain the answer 286. The decimal point could have been placed after observing that $\sqrt[3]{27{,}000{,}000} = 300$.

To obtain $\sqrt[3]{0.000585}$, move the decimal point 6 places to the right to obtain $\sqrt[3]{585}$, set the hairline to 585 on K right, and read $\sqrt[3]{585} = 8.36$. Then move the decimal point $\frac{1}{3}(6) = 2$ places to the left to obtain the answer 0.0836.

EXERCISES

1. Find the cube root of each of the following numbers: 8.72, 30, 729, 850, 7630, 0.00763, 0.0763, 0.763, 89,600, 0.625, 75×10^7, 10, 100, 100,000.

23. Combined Operations. By setting the hairline to numbers on various scales we may set square roots, cube roots and reciprocals of numbers on the *D* scale and on the *C* scale. Hence we can use the slide rule to evaluate expressions involving such quantities, and we can solve proportions involving them. The position of the decimal point is determined by a rough calculation.

Example 1. Find the value of $\frac{\sqrt[3]{385}}{2.36}$.

Solution. We may think of this as a division or write the proportion $\frac{x}{1} = \frac{\sqrt[3]{385}}{2.36}$, and then make the setting indicated in the following diagram:

C	set 2.36	opposite 1
D		read x = **3.08**
K	opposite 385 (right)	

Example 2. Find the value of $\frac{5.37\sqrt[3]{0.0835}}{\sqrt{52.5}}$.

Solution. Equating the given expression to x and applying Rule *B* §12, we write

$$\frac{x}{5.37} = \frac{\sqrt[3]{0.0835}}{\sqrt{52.5}}.$$

This proportion suggests the setting indicated in the following diagram:

C		opposite 537
D		read x = **0.324**
K	to 835 (middle)	
B	set 52.5 (right)	

Example 3. Evaluate

$$\frac{(1.736)\ (6.45)\ \sqrt{8590}\ \sqrt[3]{581}}{\sqrt{27.8}}.$$

Solution. By using rule (C) of §15, write the given expression in the form

$$\frac{\sqrt[3]{581}\ (6.45)\ \sqrt{8590}}{(1/1.736)\ \sqrt{27.8}},$$

and

set the hairline to 581 on K right,
draw 1736 of CI under the hairline,
push hairline to 645 on C,
draw 278 of B right under the hairline,
change indices (see § 6),
push hairline to 8590 on B right,
at the hairline read **1643** on D.

Note that Examples 1 and 2 were attacked by the proportion principle whereas Example 3 was considered as a series of multiplications and divisions. When no confusion results, the student should always think of an exercise as a series of multiplications and divisions. The proportion principle should be used in case of doubt.

EXERCISES

1. $\sqrt[3]{73.2}\ (0.523)$.

2. $24.3\ \sqrt[3]{0.0661\pi}$.

3. $489 \div \sqrt[3]{732}$.

4. $27\pi \div \sqrt[3]{661{,}000}$.

5. $\sqrt[3]{531} \div \sqrt{28.4}$.

6. $\sqrt{9.80} \div \sqrt[3]{160{,}000}$.

7. $(72.3)^2 \times 8.25$.

8. $\dfrac{\pi(0.213)^2}{0.0817}$.

9. $\dfrac{\sqrt[3]{19.2^2}}{(7.13)^2 \times 0.122}$.

10. $\dfrac{\pi\sqrt{740}}{4.46 \times \sqrt{28.5}}$.

11. $3.83 \times 6.26 \times \sqrt[3]{54.2}$.

12. $0.437 \times \sqrt{564} \times \sqrt[3]{1.86}$.

24. The L scale. The problems of this chapter could well be solved by means of logarithms. The following statements indicate how the L scale is used to find the logarithms of numbers to the base 10.

(A) When the hairline is set to a number on the D scale it is at the same time set to the mantissa (fractional part) of the common logarithm of the number on the L scale, and conversely, when the hairline is set to a number on the L scale it is set on the D scale to the antilogarithm of that number.

(B) The characteristic (integral part) of the common logarithm of a number greater than 1 is positive and is one less than the number of digits to the left of the decimal point; the characteristic of a number less than 1 is negative and is numerically one greater than the number of zeros immediately following the decimal point.

Example. Find the logarithm of (a) 50; (b) 1.6; (c) 0.35; (d) 0.00905.

Solution. (a) To find the mantissa of log 50,
push hairline to 50 on D,
at hairline on L read .699

Since 50 has two digits to the left of the decimal point, its characteristic is **1.**

Therefore $\log 50 = \mathbf{1.699.}$

(b) Push hairline to 16 on D,
at hairline on L read .204

Supplying the characteristic in accordance with (B), we have

$$\log 1.6 = \mathbf{0.204.}$$

(c) Push hairline to 35 on D,
at hairline on L read .544

Hence, in accordance with (B), we have

$$\log 0.35 = \mathbf{9.544 - 10.}$$

(d) Push hairline to 905 on D,
at hairline on L read .956

Hence, in accordance with (B), we have

$$\log 0.00905 = \mathbf{7.956 - 10.}$$

EXERCISE

Find the logarithms of the following numbers: 32.7, 6.51, 980,000, 0.676, 0.01052, 0.000412, 72.6, 0.267, 0.00802, 432.

CHAPTER IV

PLANE TRIGONOMETRY

25. Some important formulas from plane trigonometry. The following formulas from plane trigonometry, given for the convenience of the student, will be employed in the slide rule solution of trigonometric problems considered in this chapter.

In the right triangle ABC of Fig. 1, the side opposite the angle A is designated by a, the side opposite B by b, and the hypotenuse by c. Referring to this figure, we write the following definitions and relations.

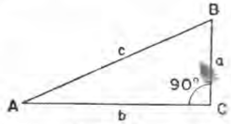


FIG. 1.

Definitions of the sine, cosine, and tangent:

$$\text{sine } A \text{ (written } \sin A) = \frac{a}{c} = \frac{\text{opposite side}}{\text{hypotenuse}},$$

$$\text{cosine } A \text{ (written } \cos A) = \frac{b}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}},$$

$$\text{tangent } A \text{ (written } \tan A) = \frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}}.$$

Reciprocal relations:

$$\text{cosecant } A \text{ (written } \csc A) = \frac{c}{a} = \frac{1}{\sin A},$$

$$\text{secant } A \text{ (written } \sec A) = \frac{c}{b} = \frac{1}{\cos A},$$

$$\text{cotangent } A \text{ (written } \cot A) = \frac{b}{a} = \frac{1}{\tan A}.$$

Relations between complementary angles:

$$\cos A = \sin (90° - A),$$
$$\tan A = \cot (90° - A),$$
$$\cot A = \tan (90° - A).$$

Relations between supplementary angles:

$$\sin (180° - A) = \sin A,$$
$$\cos (180° - A) = - \cos A,$$
$$\tan (180° - A) = - \tan A.$$

Relation between angles in a right triangle:

$$A + B = 90°.$$

If in any triangle such as ABC of Fig. 2, A, B, and C represent the angles and a, b, and c, represent, respectively, the lengths of the sides opposite these angles, the following relations hold true:

Law of sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos A$.

$$A + B + C = 180°.$$

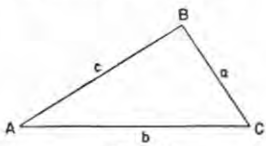


FIG. 2.

26. The S (Sine) and ST (Sine Tangent) scales. The numbers on the sine scales S and ST* represent angles. In order to set the indicator to an angle on the sine scales it is necessary to determine the value of the angles represented by the subdivisions. Thus since there are six primary intervals between 4° and 5°, each represents 10′; since each of the primary intervals is subdivided into five secondary intervals, each of the latter represents 2′. Again since there are five primary intervals between 20° and 25°, each represents 1°; since each primary interval here is subdivided into 2 secondary intervals, each of the latter represent 30′; as each secondary interval is subdivided into three parts, these smallest intervals represent 10′. These illustrations indicate the manner in which the learner should analyze the part of the scale involved to find the value of the smallest interval to be considered. In general when setting the hairline to an angle the student should always have in mind the value of the smallest interval on the part of the slide rule under consideration.

When the indicator is set to a number (angle) on scale S or ST, the sine of the angle is on scale C at the hairline and hence on scale D when the indices on scales C and D coincide.

When scale C is used to read sines of angles on ST, the left index of C is taken as 0.01, the right index as 0.1. When reading sines of angles on S, the left index of C is taken as 0.1, the right index as 1.

*The ST scale is a sine scale, but since it is also used as a tangent scale it is designated ST.

Thus to find sin 36° 26′, opposite 36° 26′ on scale *S*, read 0.594 on scale *C*; to find sin 3°24′, opposite 3°24′ on scale *ST*, read 0.0593 on scale *C*. Fig. 3 shows scales *ST*, *S*, and *C* on which certain angles and their sines are indicated. As an exercise, read from your slide rule the sines of the angles shown in the figure and compare your results with those given.

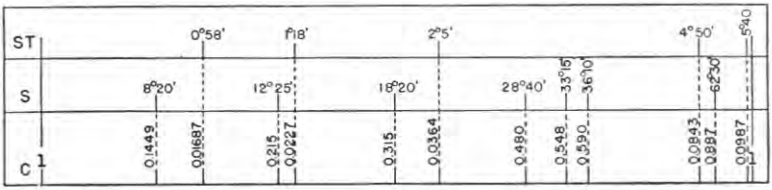


FIG. 3.

EXERCISE

1. Find the sines of each of the following angles:

(*a*) 30°.	(*b*) 38°.	(*c*) 3°20′.	(*d*) 90°.	(*e*) 87°45′
(*f*) 1°35′.	(*g*) 14°38′.	(*h*) 22°25′.	(*i*) 11°48′.	(*j*) 51°30′.

27. The T (Tangent) scale. *When the indicator is set to an angle on the T scale, the tangent of the angle is on the D scale at the hairline.* For this reading the left index is taken as 0.1 and the right index as 1. Thus to find tan 36° push the hairline to 36° on T, at the hairline read **0.727** on D. To find the co-tangent of 36°, find the reciprocal of 0.727 to obtain $\cot 36° = 1/0.727 =$ **1.376.** Also close the rule, push hairline to 36° on T, at the hairline read $\cot 36° =$ **1.376** on CI.

In finding the tangent of an angle less than 5° 43′ the ST scale is used. It is shown in trigonometry that the sine and the tangent of an angle less than 5° 43′ are approximately equal. Hence, as far as the slide rule is concerned, the tangent of an angle less than 5° 43′ may be replaced by the sine of the angle. Thus to find tan 2° 15′, push the hairline to 2° 15′ on ST*, at the hairline read **0.0393** on C.

To find the tangent of an angle greater than 45°, use the relation $\tan \theta = \cot (90° - \theta) = 1/\tan (90° - \theta)$. Thus to find tan 56°, close the rule, push the hairline to $34° = (90° - 56°)$ on T, and read $\tan 56° =$ **1.483** on CI.

To find angles when their tangents are given, use in reverse the processes just described.

EXERCISES

1. Fill out the following table:

φ	8° 6′	27° 15′	62° 19′	1° 7′	74° 15′	87°	47° 28′
tan φ							
cot φ							

2. The following numbers are tangents of angles. Find the angles.

(*a*) 0.24.	(*b*) 0.785.	(*c*) 0.92.	(*d*) 0.54.	(*e*) 0.059.
(*f*) 0.082.	(*g*) 0.432.	(*h*) 0.043.	(*i*) 0.0149.	(*j*) 0.374.
(*k*) 3.72.	(*l*) 4.67.	(*m*) 17.01.	(*n*) 1.03.	(*o*) 1.232.

3. The numbers in Exercise 2 are contangents of angles. Find the angles.

* The ST scale is a sine scale, but since it is used as a tangent scale it is designated ST.

28. Evaluation of trigonometric functions. The values of various trigonometric functions of given angles are found by expressing them in terms of sines and tangents and then using the scales S, ST and T. For this purpose use

$$\cos\theta = \sin(90^\circ - \theta),\ \cot\theta = \tan(90^\circ - \theta),$$
$$\sec\theta = 1/\cos\theta = 1/\sin(90^\circ - \theta),\ \csc\theta = 1/\sin\theta.$$

In the following exercises the notation

$$\sin^{-1} \text{m},\ \cos^{-1} \text{m},\ \sec^{-1} \text{m},\ \text{etc.},$$

will be used to mean: *the angle whose sine is m, the angle whose cosine is m, the angle whose secant is m, etc.*

EXERCISES

1. Find the cosine of each of the following angles by using the relation $\cos\varphi = \sin(90^\circ - \varphi)$:

(*a*) 30°. (*b*) 38°. (*c*) 3°20′. (*d*) 90°. (*e*) 87°45′.
(*f*) 1°35′. (*g*) 14°38′. (*h*) 22°25′. (*i*) 11°48′. (*j*) 51°30′.

2. For each of the following values of x,

(*a*) 0.5. (*b*) 0.875. (*c*) 0.375. (*d*) 0.1. (*e*) 0.015.
(*f*) 0.62. (*g*) 0.062. (*h*) 0.031. (*i*) 0.92. (*j*) 0.885.

find the value of φ less than 90°, (A) if $\varphi = \sin^{-1}x$; (B) if $\varphi = \cos^{-1}x$.

3. Find the cosecant of each of the angles in Exercise 1, by using the relation $\csc\varphi = \dfrac{1}{\sin\varphi}$.

Hint. Set the angle on S, read the cosecant on CI.

4. Find the secant of each of the angles in Exercise 1 by using the relation $\sec\varphi = \dfrac{1}{\cos\varphi}$.

5. For each of the following values of x,

(*a*) 2. (*b*) 2.4. (*c*) 1.7. (*d*) 6.12. (*e*) 80.2. (*f*) 4.72.

find the value of φ less than 90°, (A) if $\varphi = \csc^{-1}x$; (B) if $\varphi = \sec^{-1}x$.

29. Combined operations. The method for evaluating expressions involving combined operations as stated in §§16 and 23 applies without change when some of the numbers are trigonometric functions. This is illustrated in the following examples.

Example 1. Evaluate $\dfrac{6.1\sqrt{17}\sin 72^\circ \tan 20^\circ}{2.2}$.

Solution. Use rule C §15 to write $\dfrac{\sqrt{17}\sin 72^\circ \tan 20^\circ}{2.2\left(\dfrac{1}{6.1}\right)}$.

Push hairline to 20° on T,
draw 2.2 of C under the hairline,
push hairline to 17 on B right,
draw 6.1 of CI under the hairline,
push hairline to left index of C,
draw right index of C under the hairline,
push hairline to 72° on S,
at the hairline read **3.96** on D.

Example 2. Evaluate $\dfrac{7.9 \csc 17^\circ \cos 41^\circ}{18 \tan 48^\circ \sqrt{38}}$.

Solution. Replacing csc 17° by $\dfrac{1}{\sin 17^\circ}$, cos 41° by sin 49°, and tan 48° by $\dfrac{1}{\tan 42^\circ}$ and using rule C §15, we obtain

$$\frac{\tan 42^\circ \times 7.9 \sin 49^\circ}{18\sqrt{38}\sin 17^\circ}.$$

Hence:

push hairline to 42° on T,
draw 18 of C under the hairline,
interchange indices (see §6),
push hairline to 79 on C,
Draw 38 of B right under the hairline,
push hairline to 49° on S,
draw 17° of S under the hairline,
at the index of C read **0.1655** on D.

The *CF* scale may often be used to avoid shifting the slide. In the process of evaluating a fraction consisting of a number of factors in the numerator over a number of factors in the denominator, the hairline may be pushed to a number of the numerator on the *CF* scale provided that a number of the denominator on the *CF* scale is drawn under the hairline later in the process, and conversely. In other words the *CF* scale may be used at any time for a multiplication (or division) if it is later used for a division (or multiplication).

EXERCISES

Evaluate the following:

1. $\dfrac{18.6 \sin 36°}{\sin 21°}$.

2. $\dfrac{32 \sin 18°}{27.5}$.

3. $\dfrac{4.2 \tan 38°}{\sin 45° 30'}$.

4. $\dfrac{34.3 \sin 17°}{\tan 22° 30'}$.

5. $\dfrac{17.2 \cos 35°}{\cot 50°}$.

6. $\dfrac{7.8 \csc 35° 30'}{\cot 21° 25'}$.

7. $3.14 \sin 13° 10' \csc 32°$.

8. $7.1\pi \sin 47° 35'$.

9. $\dfrac{0.61 \csc 12° 15'}{\cot 35° 16'}$.

10. $\dfrac{3.1 \sin 61° 35' \csc 15° 18'}{\cos 27° 40' \cot 20°}$.

11. $\dfrac{\sin 51° 30'}{(39.1)\ (6.28)}$.

12. $\dfrac{\csc 49° 30'}{(19.1)\ (7.61)\ \sqrt{69.4}}$.

13. $(48.1)\ (1.68) \sin 39°$.

14. $\dfrac{1.01 \cos 71° 10' \sin 15°}{\sqrt{4.81} \cos 27° 10'}$.

30. The use of the trigonometric scales in solving proportions. Scales S and D can be used together in the same way as other scales have been used in previous chapters for making a setting indicated by a proportion involving sines and numbers. For example let us find the values of x and φ in the proportion

$$\frac{\sin 36^\circ}{270} = \frac{\sin \varphi}{320} = \frac{\sin 10^\circ 2'}{x}.$$

Since in the ratio $\frac{\sin 36^\circ}{270}$ both numerator and denominator are known, opposite 270 on scale D, set 36° on scale S, push hairline to 320 on D, at the hairline read $\varphi = $ **44°10′** on S; push hairline to 10°2′ on S, and at the hairline read $x =$ **80** on D. The following form gives a diagramatic setting for the proportion:

S	set 36°	read $\varphi =$ **44°10′**	opposite 10°2′
D	opposite 270	opposite 320	read $x =$ **80.0**

The learner may well write such a form for the first few exercises but he should, as soon as possible, make the setting directly from the proportion. In any case the decimal point can be placed by means of a rough calculation. For this purpose the student may sometimes find it necessary to replace the symbols in the proportion by rough values found from the slide rule.

To find the value of a product involving sines, we may use the S and D scales together to perform the multiplication in the usual way, or we may write the given expression in the form of a proportion. Thus treating 8 sin 40° as a product, to 8 on D, set the right index of S, push the hairline to 40° on S, and at the hairline read the product **5.14** on D.

To find this product by means of a proportion we write

$$x = 8 \sin 40^\circ,$$

use Rule B §12 to write

$$\frac{x}{\sin 40^\circ} = \frac{8}{1\ (= \sin 90^\circ)},$$

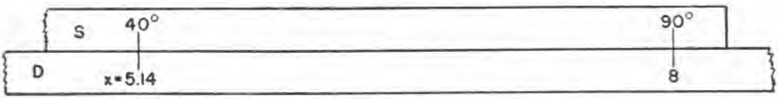


FIG. 4.

opposite 8 on D set 90° on S, and opposite 40° on S read x = **5.14** on D (see Fig. 4).

Example 1. Find θ if $\sin\theta = \frac{3}{5}$.

Solution. We write the given equation in the form

$$\frac{\sin\theta}{3} = \frac{1\ (=\sin 90°)}{5},$$

to 5 on D set right index of scale S, push the hairline to 3 on D, and at the hairline read θ = **36°52′** on S.

Example 2. Find θ if $\cos\theta = 2/3$.

Solution. Since $\cos\theta = \sin(90° - \theta)$, write the given equation in the form
$$\frac{\sin(90° - \theta)}{2} = \frac{1\ (=\sin 90°)}{3},$$
to 3 on D set right index of S, opposite 2 on D, read $90° - \theta = 41°49'$ on S. Hence θ = **48°11′**.

Scales T and C can be used together in the usual way to make the setting indicated by a proportion. Thus to find

$$y = \frac{16 \tan 37°}{0.017}\text{, apply Rule } B, \text{§12 to obtain } \frac{y}{16} = \frac{\tan 37°}{0.017},$$

opposite 37° on T, set 17 of C, and opposite 16 on C read y = **710** on D. This setting is also explained by the following diagram:

T	opposite 37°	
C	set 17	opposite 16
D		read y = **710**

Again to find $\theta = \tan^{-1}\frac{4.23}{6.72}$*, write $\frac{\tan\theta}{1} = \frac{4.23}{6.72}$ and from this proportion obtain θ = **32°11′**.

It is worthy of attention that the CF and DF scales may be used in place of the C and D scales in this process of solving proportions. For example find the values of x and φ in the proportion

$$\frac{11}{\sin 50°36'} = \frac{x}{\sin 43°} = \frac{13}{\sin\varphi},$$

by using the setting explained by the following diagram:

DF	opposite 11	read **9**	opposite 13
S	set 56°36′	opposite 43°	read **80°24′**

*The symbol $\tan^{-1}y$ is read "*the angle whose tangent is y.*"

EXERCISES

1. In each of the following proportions find the unknowns:

(a) $\dfrac{\sin 50^\circ 25'}{7} = \dfrac{\sin 42^\circ 10'}{x} = \dfrac{\sin\theta}{8}$.

(b) $\dfrac{\sin\theta}{30.5} = \dfrac{\sin 35^\circ}{x} = \dfrac{\sin 60^\circ 30'}{32.8}$.

(c) $\dfrac{\sin 25^\circ}{20} = \dfrac{\sin 40^\circ}{x} = \dfrac{\sin 70^\circ}{y}$.

(d) $\dfrac{\sin\theta}{15.6} = \dfrac{\sin\varphi}{25.6} = \dfrac{\sin 12^\circ 55'}{40.7}$.

2. Find the value of each of the following:

(a) $5 \sin 30^\circ$.
(b) $12 \sin 60^\circ$.
(c) $22/\sin 30^\circ$.
(d) $15/\sin 20^\circ$.
(e) $28 \cos 25^\circ$.
(f) $35 \csc 52^\circ 20'$.
(g) $17 \sec 16^\circ$.
(h) $55 \sin 32^\circ \sin 18^\circ$.

3. Find the value of θ in the following:

(a) $\sin\theta = \dfrac{307 \sin 42^\circ 30'}{2030}$.

(b) $\sin\theta = \dfrac{413 \sin 77^\circ 43'}{488}$.

4. Find the value of each of the following:

(a) $\dfrac{179.5 \sin 6^\circ 25'}{\sin 34^\circ 30'}$.

(b) $\dfrac{3.27 \sin 73^\circ}{\sin 2^\circ 13'}$.

5. Find the value of each of the following:

(a) $\dfrac{4 \sin 35^\circ - 5.4 \sin 17^\circ}{\sin 47^\circ}$

(b) $\dfrac{8 - 6 \sin 70^\circ}{\sin 37^\circ - 0.21}$.

6. Solve for the unknowns in the following equations:

(a) $\dfrac{\tan\theta}{27} = \dfrac{\tan\alpha}{49} = \dfrac{\tan 33^\circ 15'}{38}$.

(b) $y = \dfrac{\tan 24^\circ 10'}{6.15} = \dfrac{\tan\theta}{1.07}$.

(c) $y = (407 \cot 82^\circ 53')^2$.

(d) $y = \dfrac{17.2}{\tan 34^\circ 10'}$.

(e) $y = \dfrac{84.1 \tan 75^\circ}{27.4}$.

(f) $y = \dfrac{9.32 \tan 17^\circ}{32.2}$.

(g) $y = \dfrac{10.7}{15.1 \cot 42^\circ}$.

(h) $\tan\theta = \dfrac{4.77 \tan 21^\circ 15'}{25.7}$.

(i) $\tan\theta = \dfrac{472 \tan 11^\circ 45'}{333}$.

31. Law of sines applied to right triangles. We have just seen how a proportion may be solved with the slide rule. A method of writing a proportion from a triangle is given by the law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c},$$

which applies to all triangles.

Consider its application to a right triangle in which $C = 90°$ and write

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin 90°}{c}.$$

By inspection we note that if any two parts, other than the two legs a and b, are known, one of the ratios in the proportion is known. Hence we can find all the unknowns by the principle explained in the preceding article.

Example 1. Given a right triangle in which $A = 36°$ and $a = 520$, find b, c, and B (see Fig. 5).

Solution. Since $A + B = 90°$, $B = 90° - A = 90° - 36° = \mathbf{54°}$.

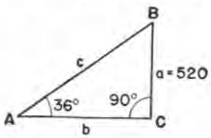


FIG. 5.

Application of the law of sines to the right triangle of Fig. 5 gives

$$\frac{\sin 36°}{520} = \frac{\sin 54°}{b} = \frac{\sin 90°}{c}.$$

The solution of this proportion for b and c is accomplished by using the setting (see Fig. 6) explained by the following diagram:

S	set 36°	opposite 54°	opposite 90°
D	opposite 520	read $b =$ **716**	read $c =$ **885**

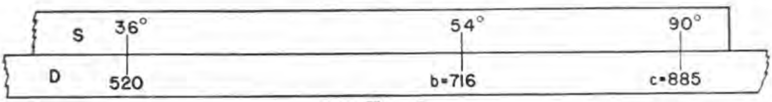


FIG. 6.

Example 2. Given a right triangle in which $A = 44°$, $c = 51$, find a, B, b.

Solution. $B = 90° - A = 90° - 44° = \mathbf{46°}$. Application of the law of sines to the triangle of Fig. 7, gives

$$\frac{\sin 44°}{a} = \frac{\sin 46°}{b} = \frac{\sin 90°}{51}.$$

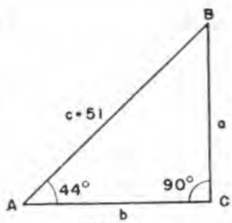


FIG. 7.

The solution of this proportion for a and b is accomplished by using the setting explained by the following diagram:

S	set 90°	opposite 44°	opposite 46°
D	opposite 51	read $a =$ **35.4**	read $b =$ **36.7**

Example 3. Given a right triangle in which $c = 49$ and $a = 22$, find A, B, and b (see Fig. 8).

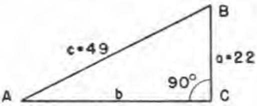


FIG. 8.

Solution. Application of the law of sines to the triangle of Fig. 8, gives

$$\frac{\sin A}{22} = \frac{\sin 90°}{49} = \frac{\sin B}{b}.$$

The solution of this proportion for b and c is accomplished by using the setting explained by the following diagram:

S	set 90°	read A = **26°40′**	opposite B = **63°20′** (= 90° − A)
D	opposite 49	opposite 22	read b = **43.8**

Whenever an angle less than 5°44′, that is, an angle whose sine or tangent is less than 0.1, is involved in the solution of a triangle, it is necessary to use the ST scale, instead of the S or T scale, as illustrated in the following example.

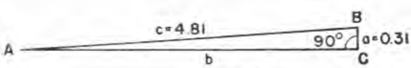


FIG. 9.

Example 4. Given the right triangle in which $c = 4.81$ and $a = 0.31$, find A, B, and b (see Fig. 9).

Solution. Application of the law of sines to the right triangle of Fig. 9, gives

$$\frac{\sin A}{0.31} = \frac{\sin B}{b} = \frac{\sin 90°}{4.81}.$$

The solution of this proportion for b and A is accomplished by using the setting explained by the following diagram:

ST		read A = **3°42′**	
S	set 90°		opposite **86°18′** (= 90° − 3°42′)
D	opposite 481	opposite 31	read b = **4.80**

Note that the DF scale may be used in place of the D scale in making a setting based on the law of sines.

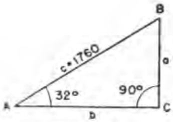


FIG. 10.

Example 5. Given the right triangle of Fig. 10 in which $c = 1760$ and $A = 32°$, find a, b, and B.

Solution. Application of the law of sines to the right triangle of Fig. 10 gives

$$\frac{1760}{\sin 90°} = \frac{a}{\sin 32°} = \frac{b}{\sin B}.$$

The solution of this proportion for a and b is accomplished by using the following setting explained by the following diagram:

DF	opposite 1760	read a = **933**	read b = **1490**
S	set 90°	opposite 32°	opposite 58° (= 90° − 32°)

In the foregoing examples the law of sines was written in each case. After the student has solved a few exercises, he should, to save time, make the setting on the slide rule directly from the figure. To do this *select from the figure* (see Fig. 11) *a known side* **a** *opposite a known angle* **A**, *push the hairline to* **a** *on scale D, draw* **A** *of scale S under the hairline, push the hairline to a known part, and at the hairline read the opposite unknown part.*

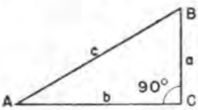


FIG. 11.

In other words it is unnecessary to write a proportion. *As soon as a side and an angle opposite are known, set them opposite on the slide rule,* thus making the setting necessary for solving the triangle.

EXERCISES

Solve the following right triangles. In each case draw a figure and write a proportion from the law of sines.

1. $a = 60$, $c = 100$.	**4.** $b = 200$, $A = 64°$.	**7.** $b = 47.7$, $B = 62°56'$.
2. $a = 50.6$, $A = 38°40'$.	**5.** $c = 37.2$, $B = 6°12'$.	**8.** $a = 0.624$, $c = 0.910$.
3. $a = 729$, $B = 68°50'$.	**6.** $c = 11.2$, $A = 43°30'$.	**9.** $a = 83.4$, $A = 72°7'$.

Solve the following right triangles. In each case draw a figure, from it make the setting directly, and write the results without first writing the law of sines.

10. $b = 4247$, $A = 52°41'$.	**13.** $c = 35.7$, $A = 58°39'$.	**16.** $a = 52$, $c = 60$.
11. $b = 2.89$, $c = 5.11$.	**14.** $c = 0.726$, $B = 10°51'$.	**17.** $a = 1875$, $B = 2°20'$.
12. $b = 512$, $c = 900$.	**15.** $a = 0.821$, $B = 21°34'$.	**18.** $b = 9$, $A = 88°26'$.

19. The length of a kite string is 250 yds., and the angle of elevation of the kite is 40°. If the line of the kite string is straight, find the height of the kite.

20. A vector is directed due N.E. and its magnitude is 10. Find the component in the direction of north.

21. Find the angle made by the diagonal of a cube with the diagonal of a face of the cube drawn from the same vertex.

32. The law of sines applied to right triangles with two legs given. When the two legs of a right triangle are the given parts, we may first find the *smaller* acute angle from its tangent and then apply the law of sines to complete the solution.

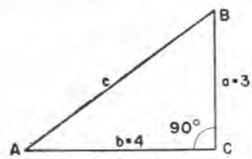


FIG. 12.

Example. Given the right triangle of Fig. 12 in which $a = 3$, $b = 4$; solve the triangle.

Solution. From the triangle we read $\tan A = \frac{3}{4}$. This equation when written in the form

$$\frac{\tan A}{3} = \frac{1}{4}$$

indicates the setting explained by the following diagram:

C	set 4	opposite 3
T	opposite index	read $A = \mathbf{36°52'}$, $B = \mathbf{90°} - A = \mathbf{53°8'}$

Since an angle and its opposite side are now known, we may apply the law of sines to the triangle to obtain

$$\frac{\sin 36°52'}{3} = \frac{\sin 90° \text{ (either index)}}{c}$$

and make the setting explained in the following diagram:

S	set $A = 36°52'$	opposite 90°
D	opposite 3	read $c = \mathbf{5}$

To solve a right triangle having two known legs, first find the angle opposite the small leg and then use the law of sines to find the hypotenuse.

EXERCISES

Solve the following right triangles:

1. $a = 12.3$, $b = 20.2$.	**4.** $a = 273$, $b = 418$.	**7.** $a = 13.2$, $b = 13.2$.
2. $a = 101$, $b = 116$.	**5.** $a = 28$, $b = 34$.	**8.** $a = 42$, $b = 71$.
3. $a = 50$, $b = 23.3$.	**6.** $a = 12$, $b = 5$.	**9.** $a = 0.31$, $b = 4.8$.

10. The length of the shadow cast by a 10-ft. vertical stick on a horizontal plane is 8.39 ft. Find the angle of elevation of the sun.

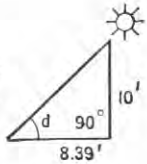


FIG. 13.

33. Law of sines applied to oblique triangles when two opposite parts are known. The same process used to solve right triangles may be used to solve any triangle when a side and angle opposite are given, since the law of sines which we have been using applies to any triangle.

Example 1. Given an oblique triangle (see Fig. 14) in which $a = 50$, $A = 65°$, and $B = 40°$. Find b, c, and C.

Solution. Since $A + B + C = 180°$, $C = 180° - (A + B) = \mathbf{75°}$.

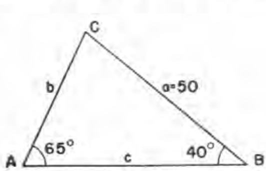


FIG. 14.

Application of the law of sines to the triangle gives

$$\frac{\sin 65^\circ}{50} = \frac{\sin 40^\circ}{b} = \frac{\sin 75^\circ}{c}.$$

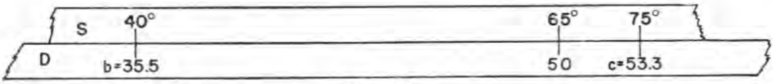


FIG. 15.

The solution of this proportion for b and c is accomplished by using the setting (see Fig. 15) explained in the following diagram:

S	set 65°	opposite 40°	opposite 75°
D	opposite 50	read b = **35.5**	read c = **53.3**

Example 2. Given an oblique triangle in which $A = 75°$, $a = 40$, and $b = 30$, find c, B, and C (see Fig. 16).

Solution. From the figure we observe that $a = 40$ and $A = 75°$ are the known parts which are opposite. Hence push hairline to 40 on D, draw 75° on S under the hairline, push hairline to 30 of D, and at the hairline read B = **46°25′** on S, push hairline to C = **58°35′** (= 180° − A − B) on S and at the hairline read c = **35.3** on D.

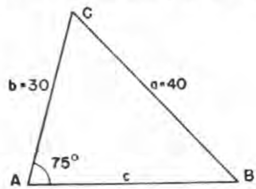


FIG. 16.

Example 3. Given the oblique triangle of Fig. 17 in which $A = 75°$, $a = 40$, $b = 3$, find c, B, and C.

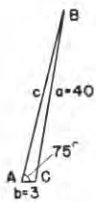


FIG. 17.

Solution. From the figure we observe that $a = 40$ and $A = 75°$ are the known parts which are opposite. Hence by pushing the hairline to 40 on D and drawing 75° of S under the hairline, we make the setting used in solving the triangle. The solution is explained in the following diagram:

ST		read B = **4°9′16″**	
S	set 75°		opposite 79°9′ (= $A + B$)
D	opposite 40	opposite 3	read c = **40.7.**

To obtain C = **100°50′44″** we use the relation $C = 180° - (A + B)$.

EXERCISES

Solve the following oblique triangles.

1. $a = 50$, $A = 65°$, $B = 40°$.	**5.** $a = 120$, $b = 80$, $A = 60°$.	**9.** $b = 91.1$, $c = 77$, $B = 51°7'$
2. $c = 60$, $A = 50°$, $B = 75°$.	**6.** $b = 0.234$, $c = 0.198$, $B = 109°$.	**10.** $a = 50$, $c = 66$, $A = 123°11'$
3. $a = 550$, $A = 10°12'$ $B = 46°36'$	**7.** $a = 795$, $A = 79°59'$ $B = 44°41'$	**11.** $a = 8.66$, $c = 10$, $A = 59°57'$
4.* $a = 222$, $b = 4570$, $C = 90°$.	**8.*** $a = 21$, $A = 4°10'$ $B = 75°$.	**12.*** $b = 8$, $a = 120$, $A = 60°$.

13. A ship at point S can be seen from each of two points, A and B, on the shore. If $AB = 800$ ft., angle $SAB = 67°43'$, and angle $SBA = 74°21'$, find the distance of the ship from A.

14. To determine the distance of an inaccessible tower A from a point B, a line BC and the angles ABC and BCA were measured and found to be 1000 yd., 44°, and 70°, respectively. Find the distance AB.

34. Law of sines applied to oblique triangles, continued. The ambiguous case. When the given parts of a triangle are two sides and an angle opposite one of them, and when the side opposite the given angle is less than the other given side, there may be two triangles which have the given parts. We have already solved triangles in which the side opposite the given angle is greater than the other side. In this case there is always only one solution. Let us now consider a case where there are two solutions.

Example. Given $a = 175$, $b = 215$, and $A = 35°30'$; solve the triangle.

Solution. Fig. 18 shows the two possible triangles having the given parts. Application of the law of sines to triangle ABC, gives

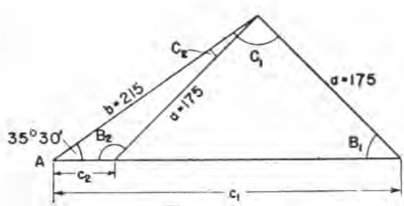


FIG. 18.

$$\frac{\sin 35°30'}{175} = \frac{\sin B_1}{215} = \frac{\sin C_1}{c_1}.$$

*The ST scale must be used in the solution.

The setting and results are shown in the following diagram.

S	set $35°30'$	read $B_1 =$ **$45°30'$**	opposite **$81°$** $(= A + B)$
D	opposite 175	opposite 215	read $c_1 =$ **298.**

It appears from the Fig. 18 that $B_2 = 180° - B_1$. Hence to solve triangle $A\ B_2\ C_2$ we write

$$B_2 = 180° - 45°30' = \mathbf{134°30'},$$
$$C_2 = 180° - (A + B_2) = 180° - 170° = \mathbf{10°}$$

and
$$\frac{\sin 35°30'}{175} = \frac{\sin 10°}{c_2}.$$

The setting and results are shown in the following diagram.

S	set $35°30'$	opposite $10°$
D	opposite 175	read $c_2 =$ **52.3.**

The student should notice that the solution of both triangles was made with a single setting of the slide rule, since each of the two proportions used contained the same known ratio

$$\frac{175}{\sin 35°30'}.$$

When the known ratio $\left(\frac{\sin A}{a}\right)$ in the ambiguous case is set on the slide rule, we find opposite the index of scale S on scale D a number equal to or greater than the largest value that leg b may have consistent with the proportion $\frac{\sin A}{a} = \frac{\sin B}{b}$. Therefore the following rule applies:

Set angle A on S opposite a on D and read the value on D opposite the index of S. If this value is greater than b, *there are two solutions; if it is equal to* b, *there is one solution, a right triangle; if it is less than* b, *there is no solution.*

EXERCISES

Solve the following oblique triangles.

1. $a = 18$,
$b = 20$,
$A = 55°24'$.

2. $b = 19$,
$c = 18$,
$C = 15°49'$.

3. $a = 32.2$,
$c = 27.1$,
$C = 52°\,24'$

4. $b = 5.16$,
$c = 6.84$,
$B = 44°3'$

5. $a = 177$,
$b = 216$,
$A = 35°36'$

6. $a = 17{,}060$,
$b = 14{,}050$,
$B = 40°$.

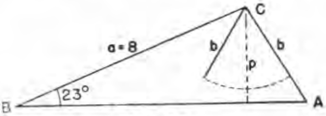


FIG. 19.

7. Find the length of the perpendicular p for the triangle of Fig. 19. How many solutions will there be for triangle ABC if (a) $b = 3$? (b) $b = 4$? (c) $b = p$?

35. Law of sines applied to an oblique triangle in which two sides and the included angle are given. To solve an oblique triangle in which two sides and the included angle are given, it is convenient to divide the triangle into two right triangles. The method is illustrated in the following example.

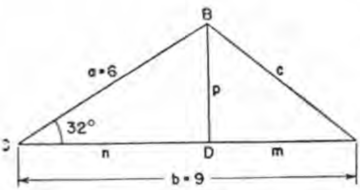


FIG. 20.

Example. Given an oblique triangle in which $a = 6$, $b = 9$, and $C = 32°$, solve the triangle.

Solution. From B of Fig. 20, drop the perpendicular p to side b. Applying the law of sines to the right triangle CBD, we obtain

$$\frac{\sin 90°}{6} = \frac{\sin 32°}{p} = \frac{\sin 58°}{n}.$$

Solving this proportion, we find $p = 3.18$ and $n = 5.09$. From the figure $m = 9 = 5.09 = 3.91$. Hence, in triangle ABD, we have

$$\tan A = \frac{p}{m} = \frac{3.18}{3.91},$$

or

$$\frac{\tan A}{3.18} = \frac{1}{3.91}.$$

Solving this proportion, we get $A = 39°8'$. Now applying the law of sines to triangle ABD, we obtain

$$\frac{\sin 39°8'}{3.18} = \frac{\sin 90°}{c}.$$

Solving this proportion, we find $c =$ **5.04.** Finally, using the relation, $A + B + C = 180°$, we obtain $B =$ **108°52'**.

If the given angle is obtuse, the perpendicular falls outside the triangle, but the method of solution is essentially the same as that used in the above example.

EXERCISES

Solve the following triangles:

1. $a = 94$, $b = 56$, $C = 29°$.

2. $a = 100$, $c = 130$, $B = 51°49'$

3. $a = 235$, $b = 185$, $C = 84°36'$

4. $b = 2.30$, $c = 3.57$, $A = 62°$.

5. $a = 27$, $c = 15$, $B = 46°$.

6. $a = 6.75$, $c = 1.04$, $B = 127°9'$

7. $a = 0.085$, $c = 0.0042$, $B = 56°30'$

8. $a = 17$, $b = 12$, $C = 59°18'$

9. $b = 2580$, $c = 5290$, $A = 138°21'$

10. The two diagonals of a parallelogram are 10 and 12 and they form an angle of 49.3° Find the length of each side.

11. Two ships start from the same point at the same instant. One sails due north at the rate of 10.44 mi. per hr., and the other due northeast at the rate of 7.71 mi. per hr. How far apart are they at the end of 40 minutes?

12. In a land survey find the latitude and departure of a course whose length is 525 ft. and bearing N 65°30′ E. See Fig. 21.

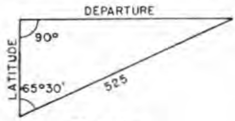


FIG. 21.

13. Solve Ex. 1 by means of the law of tangents

$$\frac{a-b}{\tan\frac{1}{2}(A-B)} = \frac{a+b}{\tan\frac{1}{2}(A+B)}.$$

36. Law of cosines applied to oblique triangles in which three sides are given. When the three sides are the given parts of an oblique triangle, we may find one angle by means of the law of cosines $a^2 = b^2 + c^2 - 2bc \cos A$ and then complete the solution by using the law of sines.

Example. Given the oblique triangle of Fig. 22, in which $a = 15$, $b = 18$, and $c = 20$, find A, B, and C.

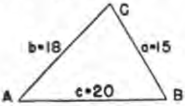


FIG. 22.

Solution. From the law of cosines we write $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

or $$\cos A = \sin(90° - A) = \frac{18^2 + 20^2 - 15^2}{2 \times 18 \times 20} = \frac{499}{720}.$$

The ratio indicates the setting from which we find $90° - A = 43°52'$ or $A = 46°8'$. Since we now know an angle and the side opposite, we may apply the law of sines to obtain

$$\frac{\sin 46°8'}{15} = \frac{\sin B}{18} = \frac{\sin C}{20}.$$

Making the setting indicated by the proportion, we find

$$B = \mathbf{59°52'},\ C = \mathbf{74°}.$$

We may use the relation $A + B + C = 180°$ as a check. Thus: $46°8' + 59°52' + 74° = 180°$ *check.*

EXERCISES

Solve the following triangles:

1. $a = 3.41$, $b = 2.60$, $c = 1.58$.

2. $a = 111$, $b = 145$, $c = 40$.

3. $a = 35$, $b = 38$, $c = 41$.

4. $a = 61.0$, $b = 49.2$, $c = 80.5$.

5. $a = 7.93$, $b = 5.08$, $c = 4.83$.

6. $a = 21$, $b = 24$, $c = 27$.

7. $a = 97.9$, $b = 106$, $c = 139$.

8. $a = 57.9$, $b = 50.1$, $c = 35.0$.

9. $a = 13$, $b = 14$, $c = 15$.

10. The sides of a triangular field measure 224 ft., 245 ft., and 265 ft. Find the angles at the vertices.

11. Find the largest angle of the triangle whose sides are 13, 14, 16.

37. MISCELLANEOUS EXERCISES

Solve the following triangles:

1. $c = 80$, $A = 20°$, $C = 90°$.
2. $b = 30$, $A = 10°$, $C = 90°$.
3. $a = 80$, $A = 75°$, $C = 90°$.
4. $a = 10.11$, $b = 17.3$, $C = 90°$.
5. $a = 2$, $b = 3$, $c = 4$.
6. $a = 5.6$, $b = 4.3$, $c = 4.9$.
7. $c = 1$, $A = 36°$, $C = 90°$.
8. $a = 795$, $A = 79°59'$, $B = 44°41'$.
9. $a = 500$, $A = 10°12'$, $B = 46°36'$.
10. $b = 29.0$, $A = 87°40'$. $C = 33°15'$.
11. $a = 55.6$, $c = 66.7$, $C = 77°42'$.
12. $a = 51.38$, $c = 67.94$, $B = 79°13'$.
13. $a = 0.321$, $b = 0.361$, $c = 0.402$.
14. $a = 4$, $b = 7$, $c = 6$.
15. $a = 78$, $b = 83.4$, $B = 56°30'$.
16. $b = 8000$, $A = 24°30'$, $B = 86°30'$.
17. $a = 42{,}930$, $c = 73{,}480$, $C = 127°38'$.
18. $a = 61.3$, $b = 84.7$, $c = 47.6$.

19. If the sides of a triangular field are 70 ft., 110 ft., and 96 ft. long, find the angle opposite the longest side.

20. The diagonals of a parallelogram are 5 ft. and 6 ft. in length. If the angle they form is 49°18′, find the sides of the parallelogram.

21. A car is traveling at a rate of 44 ft. per second up a grade which makes an angle of 10° with the horizontal. Find how long it takes for the car to rise 200 ft.

22. A lighthouse is 16 mi. in the direction 29°30′ east of north from a cliff. Another lighthouse is 12 mi. in the direction 72°45′ west of south from the cliff. What is the direction of the first lighthouse from the second?

23. A 52-ft. ladder is placed 20 ft. from the foot of an inclined buttress, and reaches 46 ft. up its face. What is the inclination of the buttress?

24. If in a circle a chord of 41.36 ft. subtends an arc of 145°37′, find the radius of the circle.

38. To change radians to degrees or degrees to radians. In the next article we shall find it convenient to use the angular unit called the radian. To change radians to degrees or degrees to radians we use the proportion

$$\frac{\pi}{180} = \frac{r \text{ (number of radians)}}{d \text{ (number of degrees)}}.$$

The setting is as follows:

Opposite π on DF set 180 on CF,

Opposite radians on D (or DF) read degrees on C (or CF),

or

Opposite degrees on C (or CF) read radians on D (or DF).

Thus to convert 1.5 radians to degrees, we make the following setting:
Opposite π on DF set 180 on CF,
Opposite 1.5 on DF read **85.9°** on CF.
To convert 45° to radians, we make the following setting:
Opposite π on DF set 180 on CF,
Opposite 45 on C (or CF) read **0.785** on D (or DF).

EXERCISES

1. Express the following angles in radians:

(a) 45°.	(d) 180°.	(g) 22°30′.
(b) 60°.	(e) 120°.	(h) 200°.
(c) 90°.	(f) 135°.	(i) 3000°.

2. Express the following angles in degrees:

(a) $\pi/3$ radians.	(c) $\pi/72$ radian.	(e) $20\pi/3$ radians.
(b) $3\pi/4$ radians.	(d) $7\pi/6$ radians.	(f) 0.98π radians.

3. Express in radians the following angles:

(a) 1°.	(c) 1″.	(e) 180°34′20″.
(b) 1′.	(d) 10°11′.	(f) 300°25′43″.

4. Find the following angles in degrees and minutes:

(a) $\frac{1}{10}$ radian; (b) $2\frac{1}{2}$ radians; (c) 1.6 radians; (d) 6 radians.

39. Sines and tangents of small angles. To find $\sin\theta$ or $\tan\theta$ for an angle smaller than those given on the ST scale, we may use the approximate relation

$$\sin\theta = \tan\theta = \theta \text{ (in radians)}, \qquad \text{(approximately)}.$$

Since $1° = \frac{\pi}{180}$ radians, set 180 on CF to π on DF; opposite the angles expressed in degrees on C or CF, read the same angles expressed in radians on D or DF respectively. Thus $\sin 0.3° = 0.3\,\frac{\pi}{180} = 0.00524$.

Since $1' = \frac{\pi}{180 \times 60}$ radian, and since $1'' = \frac{\pi}{180 \times 60 \times 60}$ radian, we can find the sine or tangent of any small angle expressed in minutes (or seconds) by multiplying it by the value of $1'$ (or $1''$) in radians. Thus $\sin 18' = 18\,\frac{\pi}{180 \times 60} = 0.00524$; $\sin 35'' = 35\,\frac{\pi}{180 \times 60 \times 60} = 0.0001697$.

For convenience the value of $\frac{180 \times 60}{\pi}$ has been marked by a "minute" gauge point half way between the ST and B scales near the 2° division, and the value of $\frac{180 \times 60 \times 60}{\pi}$ has been marked by a "second" gauge point between ST and B near the 1°10′ division

To approximate an answer for the purpose of placing the decimal point, it is convenient to remember that **sin 0.1° = .002 (2 zeros, 2)** nearly, that **sin 1′ = .0003 (3 zeros, 3)** nearly, and that **sin 1″ = .000005 (5 zeros, 5)** nearly.

To find sin 35″ or tan 35″ set the slide rule as indicated in the following diagram:

ST	set "second" gauge point	opposite left index
D	opposite 35	read **0.0001697**

To find 540 sin 28′ set the rule as indicated in the following diagram:

ST	set "minute" gauge point	
C		opposite 540
D	opposite 28	read 4.40

To find 540 sin 0.467°, set the rule as indicated in the following diagram:

CF	set 180	
C		opposite 540
D	opposite 0.467	read 4.40

EXERCISES

Find the values of the following:

1. sin 5′.
2. sin 5″.
3. sin 21′.
4. sin 32″.
5. tan 7′.
6. tan 52″.
7. cos 89° 45′.
8. cos 89° 59′ 19″.
9. sec 89° 40′.
10. csc 16″.
11. csc 2′.
12. cot 30′.
13. 250 sin 23′.
14. 42 tan 19′.
15. 150 cos 89° 40′.
16. 38 sin 52″.
17. 500 tan 35″.
18. 432 sin 33′.
19. $\dfrac{\tan 12'}{0.0001745}$.
20. $\dfrac{\sin 18'}{0.131}$.

40. Applications. The solutions of many practical problems are obtained by dealing with rectilinear figures. In finding the length of a specified line segment of a rectilinear figure, the beginner is likely to read a number of lengths which are not needed. This may be well at first, but the efficient operator reads and tabulates only useful numbers. The following examples and solutions indicate efficient methods of finding desired parts of rectilinear figures.

Example 1. Find the line segment marked x in Fig. 23.

Solution. By using the law of sines, we write

$$\frac{368}{\sin 39^\circ} = \frac{y}{\sin 65^\circ}, \quad \frac{y}{\sin 50^\circ} = \frac{x}{\sin 28^\circ}$$

and then find x by making the following settings:

push hairline to 368 on D,
draw 39° of S under the hairline,
push hairline to 65° on S,
draw 50° of S under the hairline,
push hairline to 28° on S,
at the hairline read x = **325** on D.

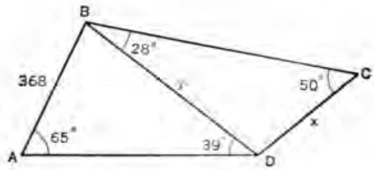


FIG. 23.

The value of y was not tabulated, but it could have been read at the hairline on scale D when the hairline was set to 65° of scale S. Also it was not necessary to write the ratios; for, when one remembers that each ratio is that of a side of a triangle to the sine of the opposite angle, he has no difficulty in perceiving, from an inspection of the figure, the settings to be made.

Generally it is necessary to compute the magnitudes of a number of angles before the slide rule computation can be carried out. This process is illustrated in Example 2.

Example 2. Find the length of the side marked z in Fig. 24(a).

Solution. To find the length of the side marked z in Fig. 24(a), first draw Fig. 24(b), compute the angles shown in the figure, and push the hairline to 289 on D,
draw 77° (= 180° − 103°) of S under the hairline,
push hairline to 32° on S,
draw 38° of S under the hairline,
push hairline to 65° on S,

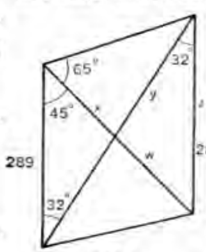


FIG. 24(a).

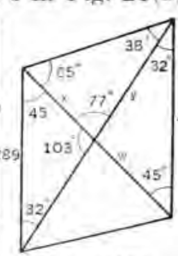


FIG. 24(b).

draw 45° of S under the hairline
push hairline to 77° on S,
at the hairline read z = 319 on D.

In some problems it is necessary to perform some of the earlier settings in a chain of settings, compute some parts on the basis of the results, make some more settings, compute more parts, etc.

EXERCISES

1. Find the length of the line segment BC in Fig. 23.

2. Find the length of the line segment marked w in Fig. 24a.

3. In Fig. 25 find the length of the line segment marked x.

4. Line segment AB in Fig. 26 is horizontal and CD is vertical. Find the length of CD.

5. In the statement of Ex. 4, replace "Fig. 26" by "Fig. 27" and solve the resulting problem.

FIG. 25.

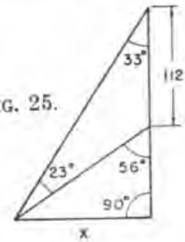


FIG. 26.

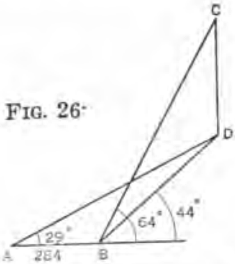
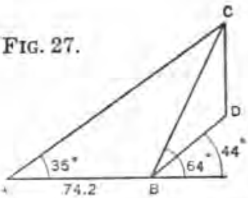


FIG. 27.



6. A tower and a monument stand on a level plane. (See Fig. 28). The angles of depression of the top and bottom of the monument viewed from the top of the tower are 13° and 31° respectively; the height of the tower is 145 ft. Find the height of the monument.

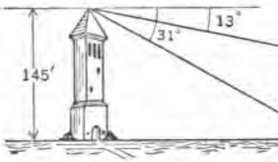


FIG. 28.

7. The captive balloon shown in Fig. 29 is connected to a ground station A by a cable of length 842 ft. inclined 65° to the horizontal. In a vertical plane with the balloon and its station and on the opposite side of the balloon from A a target B was sighted from the balloon on a level with A. If the angle of depression of the target from the balloon is 4° find the distancef rom the target to a point C directly under the balloon.

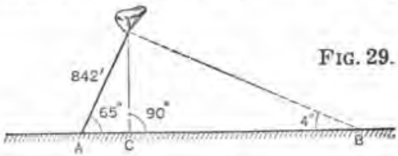


FIG. 29.

8. Fig. 30 represents a 600 ft. radio tower. AC and AD are two cables in the same vertical plane anchored at two points C and D on a level with the base of the tower. The angles made by the cables with the horizontal are 44° and 58° as indicated. Find the lengths of the cables and the distance between their anchor points.

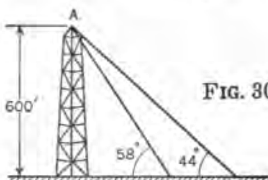


FIG. 30.

ANSWERS

§5. Page 8

1. 6
2. 7
3. 10
4. 9.1
5. 6.75
6. 9.62
7. 49.8
8. 340
9. 47.0
10. 0.0826
11. 3220
12. 0.836
13. 9.86
14. 3.08

§6. Page 9

1. 15
2. 15.5
3. 3530
4. 42.1
5. 0.001322
6. 1737
7. 9.98
8. 1,340

§7. Page 10

1. 2.32
2. 165.2
3. 0.0767
4. 106.1
5. 0.000713
6. 77.5
7. 1861
8. 26.3
9. 1.154
10. 0.0419

§8. Pages 11, 12

1. (*a*) 1576
   (*b*) 2.60
   (*c*) 5.25
   (*d*) 4.59
2. (*a*) 26.1%
   (*b*) 64.4%
   (*c*) 220%
   (*d*) 2.73%
3. (*a*) 178.9 mi.
   (*b*) 121.1 mi.
   (*c*) 2140 mi.
4. (*a*) 9.22 yds./sec.
   (*b*) 15.02 ft./sec.
   (*c*) 186,000 mi./sec.
5. (*a*) 10.13 sec.
   (*b*) 29.5 hrs.
   (*c*) 322 hrs.

§9. Page 13

1. 36.7
2. 8.35
3. 0.0000632
4. 3400
5. 0.00357
6. 13,970
7. 1586
8. 0.0223
9. 0.01311
10. 2.36
11. 0.0414
12. 2460
13. 249
14. 0.275
15. 0.1604
16. 0.0977

§11. Pages 17, 18

1. $x = 5.22$
2. $x = 2.30, \quad y = 31.8$
3. $x = 51.7, \quad y = 3370$
4. $\begin{cases} x = 3.97 \\ y = 0.984 \\ z = 0.272 \end{cases}$
5. $\begin{cases} x = 0.1013 \\ z = 0.0769 \end{cases}$
6. $\begin{cases} x = 1.586 \\ y = 41.4 \end{cases}$
7. $\begin{cases} x = 106.2 \\ y = 30.4 \end{cases}$
8. $\begin{cases} x = 0.1170 \\ y = 0.927 \end{cases}$
9. $\begin{cases} x = 186 \\ y = 13.42 \\ z = 50.3 \end{cases}$

§12. Page 19

1. 48.7
2. 0.396
3. 9.46
4. 42.0
5. 3.14
6. 3.20
7. 4.07
8. 0.1264
9. 104.6
10. 9.69

§13. Page 20

1. 167.8 cm.
2. 274 m.
3. 720 lb.
4. 235 sq. cm.
5. (a) 25,700 watts
(b) 3,940,000 watts
(c) 621 watts
6. (a) 1.122 gal.
(b) 0.00255 gal.
(c) 0.1504 gal.

§14. Page 21

1. 0.0625, 0.00385, 1.389, 15.38
0.0575, 0.0541, 0.01490
2. 2,162
3. 74.0, 10.97
4. 200, 8.55

§15. Page 23

1. $x = 16.98$, $y = 12.74$
2. $x = 0.0640$, $y = 1.417$
3. $x = 154.9$, $y = 6940$
4. $x = 0.00247$, $y = 645$
5. $\begin{cases} x = 0.0481 \\ y = 0.0435 \\ z = 44.95 \end{cases}$
6. $\begin{cases} x = 11.07 \\ y = 0.0484 \\ z = 0.465 \end{cases}$

§16. Page 25

1. 0.001156
2. 1.512
3. 1.015
4. 17.2
5. 96.1
6. 0.1111
7. 150,800
8. 15.32
9. 9.76
10. 0.00288
11. 144,700
12. 0.0267
13. 0.279
14. 41.3
15. 111.1
16. 3430

§17. Page 27

1. 625, 1024, 3720, 5620, 7920, 537,000, 204,000, 4.33, 3.07, 0.1116, 0.00001267
0.908, 27,800,000, 2.24×10^{13}
2. (a) 5.94 ft.^{2} (b) 3500 ft.^{2} (c) 0.445 ft.^{2} (d) 2.76 ft.^{2}
3. (a) 37.6 ft.^{2} (b) 0.00597 ft.^{2} (c) 966 ft.^{2} (d) 2.35×10^8 ft.^{2}

§18. Page 28

1. 2.83, 3.46, 4.12, 9.43, 2.98, 29.8, 0.943, 85.3, 0.252, 0.00797, 252, 316
2. (a) 231 ft. (b) 0.279 ft. (c) 5720 ft.
3. (a) 18.05 ft. (b) 0.992 ft. (c) 49.8 ft.

§19. Page 30

1. 24.2
2. 0.416
3. 8.54
4. 0.0698
5. 4.43
6. 4.01
7. 6.69
8. 6.14
9. 0.428
10. 1.176
11. 32.8
12. 398
13. 43.7
14. 29.4

§20. Page 32

1. 64.2
2. 11.41
3. 109.2
4. 0.428
5. 9.65
6. 0.0602

§21. Page 33

1. 9.260, 32.8, 238,000, 422,000, 705,000, 3.94×10^8, 0.0923, 29.2, 5.39, 0.0000373, 0.839, 1.46×10^{11}, 5.71×10^{19}. 2. 76.2

§22. Page 34

1. 2.06, 3.11, 9.00, 9.47, 19.69, 0.1969, 0.424, 0.914, 44.8, 0.855, 909, 2.15, 4.64, 46.4

§23. Page 35

1. 2.19	**7.** 43,100
2. 30.9	**8.** 1.745
3. 54.2	**9.** 1.156
4. 0.974	**10.** 1.192
5. 1.522	**11.** 90.7
6. 0.0577	**12.** 12.77

§24. Page 36

1.515, 0.814, 5.991, 9.830–10, 8.022–10, 6.615–10.
1.861, 9.427–10, 7.904–10, 2.636

§26. Page 39

1.	(*a*) 0.5	(*b*) 0.616	(*c*) 0.0581	(*d*) 1	(*e*) 0.999
	(*f*) 0.0276	(*g*) 0.253	(*h*) 0.381	(*i*) 0.204	(*j*) 0.783

§27. Page 40

1. 0.1423, 0.515, 1.906, 0.01949, 3.55, 19.08, 1.09
7.03, 1.942, 0.525, 51.3, 0.282, 0.0524, 0.917

2.	(*a*) 13°30′	(*b*) 38°8′	(*c*) 42°37′	(*d*) 28°22′	(*e*) 3°23′
	(*f*) 4°42′	(*g*) 23°22′	(*h*) 2°28′	(*i*) 51′13″	(*j*) 20°30′
	(*k*) 74°57′	(*l*) 77°55′	(*m*) 86°38′	(*n*) 45°51′	(*o*) 50°56′
3.	(*a*) 76°30′	(*b*) 51°52′	(*c*) 47°23′	(*d*) 61°38′	(*e*) 86°37′
	(*f*) 85°18′	(*g*) 66°38′	(*h*) 87°32′	(*i*) 89°8′47″	(*j*) 69°30′
	(*k*) 15°3′	(*l*) 12°5′	(*m*) 3°22′	(*n*) 44°9′	(*o*) 39°4′

§28. Page 41

1.	(*a*) 0.866	(*b*) 0.788	(*c*) 0.998	(*d*) 0	(*e*) 0.0393
	(*f*) 1.00	(*g*) 0.968	(*h*) 0.924	(*i*) 0.979	(*j*) 0.623
2. *A.*	(*a*) 30°	(*b*) 61°6′	(*c*) 22°2′	(*d*) 5°44′	(*e*) 51′34″
	(*f*) 38°19′	(*g*) 3°33′	(*h*) 1°46′34″	(*i*) 66°56′	(*j*) 62°15′
B.	(*a*) 60°	(*b*) 28°54′	(*c*) 67°58′	(*d*) 84°16′	(*e*) 89°8′26″
	(*f*) 51°41′	(*g*) 86°27′	(*h*) 88°13′26″	(*i*) 23°4′	(*j*) 27°45′
3.	(*a*) 2	(*b*) 1.623	(*c*) 17.21	(*d*) 1	(*e*) 1.001
	(*f*) 36.2	(*g*) 3.95	(*h*) 2.63	(*i*) 4.90	(*j*) 1.277
4.	(*a*) 1.155	(*b*) 1.27	(*c*) 1.002	(*d*) ∞	(*e*) 25.5
	(*f*) 1	(*g*) 1.033	(*h*) 1.082	(*i*) 1.021	(*j*) 1.605
5. *A.*	(*a*) 30°	(*b*) 24°38′	(*c*) 36°	(*d*) 9°24′	(*e*) 0°43′
	(*f*) 12°14′				
B.	(*a*) 60°	(*b*) 65°22′	(*c*) 54°	(*d*) 80°36′	(*e*) 89°17′
	(*f*) 77°46′				

§29. Page 43

1. 30.5	**6.** 5.27	**11.** 0.00319
2. 0.360	**7.** 1.349	**12.** 0.001086
3. 4.61	**8.** 16.47	**13.** 50.8
4. 24.2	**9.** 2.033	**14.** 0.0433
5. 16.79	**10.** 4.24	

§30. Page 46

1. (a) $x = 6.09$, $\theta = 61°45'$ (b) $\theta = 54°1'$, $x = 21.6$ (c) $x = 30.4$, $y = 44.5$ (d) $\theta = 4°55'$, $\varphi = 8°5'$
2. (a) 2.5 (b) 10.39 (c) 44 (d) 43.9 (e) 25.4 (f) 44.1 (g) 17.68 (h) 9.01
3. (a) $5°52'$ (b) $55°48'$
4. (a) 35.4 (b) 81.0
5. (a) 0.977 (b) 6.02
6. (a) $\theta = 24°58'$, $\alpha = 40°13'$ (b) $y = 0.0729$, $\theta = 4°28'$ (c) $y = 2568$ (d) $y = 25.3$ (e) $y = 11.45$ (f) $y = 0.0885$ (g) $y = 0.638$ (h) $\theta = 4°8'$ (i) $16°25'$

§31. Pages 49, 50

1. $A = 36°52'$, $B = 53°8'$, $b = 80$
2. $B = 51°20'$, $c = 80.9$, $b = 63.2$
3. $A = 21°10'$, $b = 1884$, $c = 2020$
4. $B = 26°$, $a = 410$, $c = 457$
5. $A = 83°48'$, $a = 36.98$, $b = 4.02$
6. $B = 46°30'$, $a = 7.71$, $b = 8.12$
7. $A = 27°4'$, $a = 24.37$, $c = 53.56$
8. $A = 43°18'$, $B = 46°42'$, $b = 0.662$
9. $B = 17°53'$, $b = 26.91$, $c = 87.6$
10. $B = 37°19'$, $a = 5570$, $c = 7007$
11. $B = 34°24'$, $A = 55°36'$, $a = 4.22$
12. $A = 55°19'$, $B = 34°41'$, $a = 740$
13. $B = 31°21'$, $a = 30.5$, $b = 18.59$
14. $A = 79°9'$, $a = 0.713$, $b = 0.1367$
15. $A = 68°26'$, $b = 0.3245$, $c = 0.883$
16. $A = 60°4'$, $B = 29°56'$, $b = 29.94$
17. $A = 87°40'$, $b = 76.1$, $c = 1877$
18. $B = 1°34'$, $a = 329$, $c = 329$
19. 160.7 yd.
20. 7.07
21. $35°16'$

§32. Page 51

1. $A = 31°20'$, $B = 58°40'$, $c = 23.7$
2. $A = 41°2'$, $B = 48°58'$, $c = 153.8$
3. $A = 65°$, $B = 25°$, $c = 55.2$
4. $A = 33°9'$, $B = 56°51'$, $c = 499$
5. $A = 39°30'$, $B = 50°30'$, $c = 44$
6. $A = 67°23'$, $B = 22°37'$, $c = 13$
7. $A = 45°$, $B = 45°$, $c = 18.67$
8. $A = 30°37'$, $B = 59°23'$, $c = 82.5$
9. $A = 3°42'$, $B = 86°18'$, $c = 4.8$
10. $50°$

§33. Page 53

1. $C = 75^\circ$
$b = 35.46$
$c = 53.3$

2. $C = 55^\circ$
$b = 70.7$
$a = 56.1$

3. $C = 123^\circ\,12'$
$b = 2257$
$c = 2599$

4. $A = 2^\circ\,47'$
$B = 87^\circ\,13'$
$c = 4570$

5. $B = 35^\circ\,16'$
$C = 84^\circ\,44'$
$c = 138$

6. $A = 17^\circ\,41'$
$C = 53^\circ\,19'$
$a = 0.0751$

7. $C = 55^\circ\,20'$
$b = 568$
$c = 664$

8. $b = 279$
$c = 284$
$C = 100^\circ\,50'$

9. $A = 87^\circ\,41'$
$C = 41^\circ\,12'$
$a = 116.9$

10. Impossible.

11. $B = 30^\circ\,3'$
$C = 90^\circ$
$b = 5.01$

12. $c = 123.8$
$B = 3^\circ\,18'\,35''$
$C = 116^\circ\,41'\,25''$

13. 1254 ft.

14. 1029 yds.

§34. Pages 54, 55

1. $B_1 = 66^\circ\,10'$
$C_1 = 58^\circ\,26'$
$c_1 = 18.6$
$B_2 = 113^\circ\,50'$
$C_2 = 10^\circ\,46'$
$c_2 = 4.08$

2. $B_1 = 16^\circ\,43'$
$A_1 = 147^\circ\,28'$
$a_1 = 35.5$
$B_2 = 163^\circ\,17'$
$A_2 = 0^\circ\,54'$
$a_2 = 1.04$

3. $A_1 = 70^\circ\,12'$
$B_1 = 57^\circ\,24'$
$b_1 = 28.79$
$A_2 = 109^\circ\,48'$
$B_2 = 17^\circ\,48'$
$b_2 = 10.45$

4. $A_1 = 68^\circ\,47'$
$C_1 = 67^\circ\,10'$
$a_1 = 6.92$
$A_2 = 23^\circ\,7'$
$C_2 = 112^\circ\,50'$
$a_2 = 2.91$

5. $B_1 = 45^\circ\,16'$
$C_1 = 99^\circ\,8'$
$c_1 = 300$
$B_2 = 134^\circ\,44'$
$C_2 = 9^\circ\,40'$
$c_2 = 51.1$

6. $A_1 = 51^\circ\,19'$
$C_1 = 88^\circ\,41'$
$c_1 = 21,850$
$A_2 = 128^\circ\,41'$
$C_2 = 11^\circ\,19'$
$c_2 = 4290$

7. $p = 3.13$; (*a*) none, (b) 2, (*c*) 1

§35. Page 56

1. $A = 119^\circ\,54'$
$B = 31^\circ\,6'$
$c = 52.6$

2. $A = 49^\circ\,4'$
$C = 79^\circ\,7'$
$b = 104.1$

3. $A = 55^\circ\,2'$
$B = 40^\circ\,21'$
$c = 285$

4. $B = 39^\circ\,16'$
$C = 78^\circ\,44'$
$a = 3.21$

5. $A = 100^\circ\,57'$
$C = 33^\circ\,3'$
$b = 19.8$

6. $A = 46^\circ\,26'$
$C = 6^\circ\,24'$
$b = 7.43$

7. $A = 121^\circ\,4'$
$C = 2^\circ\,26'$
$b = 0.0828$

8. $A = 77^\circ\,12'$
$B = 43^\circ\,30'$
$c = 15$

9. $B = 13^\circ\,22'$
$C = 28^\circ\,17'$
$a = 7420$

10. 10 and 4.68

11. 4.93 mi.

12. Lat. = 218 ft.
Dep. = 478 ft.

§36. Page 57

1. $A = 106^\circ\,47'$
$B = 46^\circ\,53'$
$C = 26^\circ\,20'$

2. $A = 27^\circ\,21'$
$B = 143^\circ\,8'$
$C = 9^\circ\,32'$

3. $A = 52^\circ\,26'$
$B = 59^\circ\,23'$
$C = 68^\circ\,12'$

4. $A = 49^\circ\,12'$
$B = 37^\circ\,36'$
$C = 93^\circ\,12'$

5. $A = 106^\circ\,18'$
$B = 37^\circ\,55'$
$C = 35^\circ\,47'$

6. $A = 48^\circ\,11'$
$B = 58^\circ\,25'$
$C = 73^\circ\,24'$

7. $A = 44^\circ\,42'$
$B = 49^\circ\,37'$
$C = 85^\circ\,40'$

$A = 83^\circ\,42'$
$B = 59^\circ\,22'$
$C = 36^\circ\,56'$

9. $A = 53^\circ\,8'$
$B = 59^\circ\,30'$
$C = 67^\circ\,22'$

10. $51^\circ\,53'$
$59^\circ\,32'$
$68^\circ\,35'$

11. $72^\circ\,36'$

§37. Page 58

1. $B = 70°$, $a = 27.4$, $b = 75.2$
2. $B = 80°$, $a = 5.29$, $x = 30.5$
3. $B = 15°$, $b = 21.4$, $c = 82.8$
4. $A = 30° 18'$, $B = 59° 42'$, $c = 20.04$
5. $A = 29°$, $B = 46° 36'$, $C = 104° 24'$
6. $A = 74° 36'$, $B = 47° 48'$, $C = 57° 36'$
7. $B = 54°$, $a = 0.589$, $b = 0.809$
8. $C = 55° 20'$, $b = 568$, $c = 664$
9. $C = 123° 12'$, $b = 2050$, $c = 2360$
10. $B = 59° 5'$, $a = 33.78$, $c = 18.54$
11. $A = 54° 30'$, $B = 47° 48'$, $b = 50.5$
12. $A = 40° 53'$, $C = 59° 54'$, $b = 77.1$
13. $A = 49° 24'$, $B = 58° 36'$, $C = 72°$
14. $A = 34° 48'$, $B = 86° 24'$, $C = 58° 48'$
15. $C = 72° 18'$, $A = 51° 12'$, $c = 95.2$
16. $A = 69°$, $a = 3320$, $c = 7480$
17. $B = 24° 47'$, $b = 38{,}900$, $A = 27° 35'$
18. $A = 45° 12'$, $B = 101° 24'$, $C = 33° 24'$
19. 81°
20. 5 ft., 2.34 ft.
21. 26.17 sec.
22. 47° 54' east of no.
23. 84°
24. 21.7 ft.

§38. Page 59

1. (*a*) 0.785 (*b*) 1.047 (*c*) 1.571 (*d*) 3.14 (*e*) 2.09 (*f*) 2.36 (*g*) 0.393 (*h*) 3.49 (*i*) 52.4
2. (*a*) 60° (*b*) 135° (*c*) 2.5° (*d*) 210° (*e*) 1200° (*f*) 176.4°
3. (*a*) 0.01745 (*b*) 0.0002909 (*c*) 0.00000485 (*d*) 0.1778 (*e*) 3.152 (*f*) 5.24
4. (*a*) 5° 44' (*b*) 143° 12' (*c*) 91° 42' (*d*) 343° 48'

§39. Page 60

1. 0.001454
2. 0.0000242
3. 0.00611
4. 0.0001551
5. 0.00204
6. 0.000252
7. 0.00436
8. 0.0001988
9. 171.8
10. 12,890
11. 1718
12. 114.5
13. 1.67
14. 0.232
15. 0.873
16. 0.00958
17. 0.0848
18. 4.15
19. 20
20. 0.04

§40. Page 62

1. 677
2. 173.4
3. 129.4
4. 415
5. 41.7
6. 89.3 ft.
7. 10,910 ft.
8. 864 ft., 708 ft., 246 ft.

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