SLIDE RULE MANUAL

LOG LOG DUPLEX DECITRIG®

No. 4081

By

LYMAN M. KELLS,

WILLIS F. KERN

and

JAMES R. BLAND

KEUFFEL & ESSER CO.

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DECITRIG®
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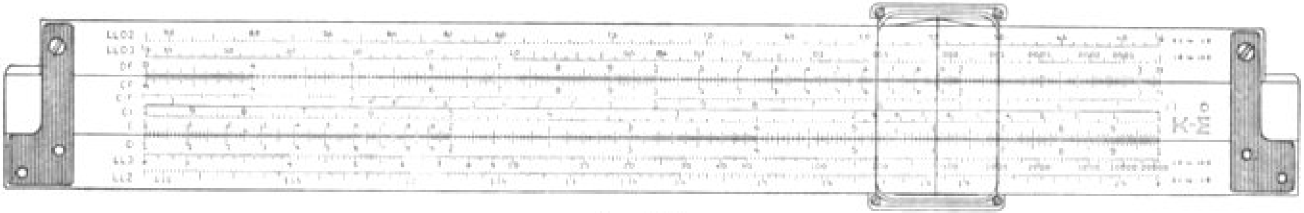

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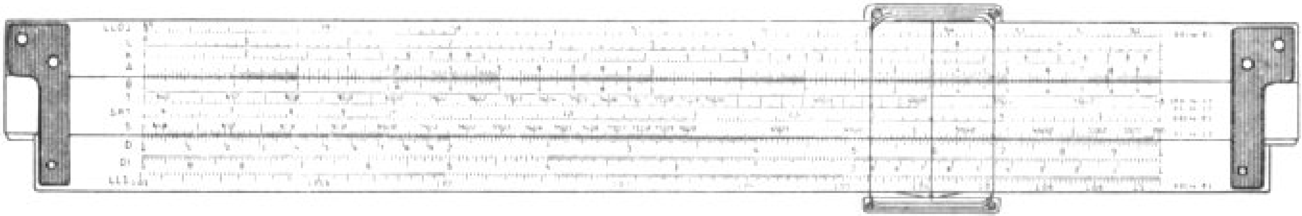

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PREFACE

This slide rule manual has been written for study without the aid of a teacher. For this reason one might suspect that the treatment is superficial. On the contrary, the subject matter is so presented that the beginner uses two general principles while he is learning to read the scales and perform the simpler operations. The mastery of these two principles gives the power to devise the best settings for any particular purpose, and to recall settings which have been forgotten.

These principles are so simple and so carefully explained and illustrated both by diagram and by example that they are easily mastered. In Chapters I and II, they are applied to simple problems in multiplication and division; in Chapters III, IV, and V they are used to solve problems involving multiplication, division, square and cube root, trigonometry, logarithms, and powers of numbers.

Chapter VI explains the slide rule from the logarithmic standpoint. Those who desire a theoretical treatment are likely to be surprised to find that the principles of the slide rule are so easily understood in terms of logarithms.

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CHAPTER I

MULTIPLICATION AND DIVISION

1. Introduction. This manual is designed to enable any interested person to learn to use the slide rule efficiently. The beginner should keep his slide rule before him while reading the manual, should make all settings indicated in the illustrative examples, and should compute answers for a large number of the exercises. The principles involved are easily understood but a certain amount of practice is required to enable one to use the slide rule efficiently and with a minimum of error.

2. Reading the scales.* Everyone has read a ruler in measuring a length. The number of inches is shown by a number appearing on the ruler, then small divisions are counted to get the number of 16th's of an inch in the fractional part of the inch, and finally in close measurement, a fraction of a 16th of an inch may be estimated. We first read a primary length, then a secondary length, and finally estimate a tertiary length. Exactly the same method is used in reading the slide rule. The divisions on the slide rule are not uniform in length, but the same principle applies.

Figure 1 represents, in skeleton form, the fundamental scale of the slide rule, namely the *D* scale. An examination of this actual

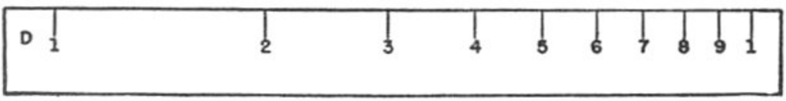


FIG. 1.

scale on the slide rule will show that it is divided into 9 parts by primary marks which are numbered 1, 2, 3, . . . , 9, 1. The space between any two primary marks is divided into ten parts by nine secondary marks. These are not numbered on the actual scale except

*The description here given has reference to the 10″ slide rule. However anyone having a rule of different length will be able to understand his rule in the light of the explanation given.

between the primary marks numbered 1 and 2. Fig. 2 shows the secondary marks lying between the primary marks of the *D* scale. On this scale each italicized number gives the reading to be associated

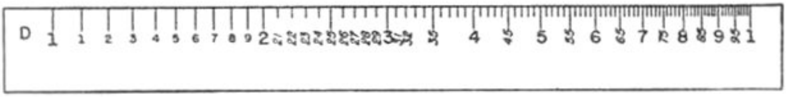


Fig. 2.

with its corresponding secondary mark. Thus, the first secondary mark after 2 is numbered 21, the second 22, the third 23, etc.; the first secondary mark after 3 is numbered 31, the second 32, etc. Between the primary marks numbered 1 and 2 the secondary marks are numbered 1, 2, . . . , 9. Evidently the readings associated with these marks are 11, 12, 13, . . . , 19. Finally between the secondary marks, see Fig. 3, appear smaller or tertiary marks which aid in

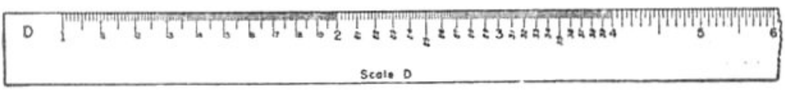


Fig. 3.

obtaining the third digit of a reading. Thus between the secondary marks numbered 22 and 23 there are 4 tertiary marks. If we think of the end marks as representing 220 and 230, the four tertiary marks divide the interval into five parts each representing 2 units. Hence with these marks we associate the numbers 222, 224, 226, and 228; similarly the tertiary marks between the secondary marks numbered 32 and 33 are read 322, 324, 326, and 328, and the tertiary marks between the primary marks numbered 3 and the first succeeding secondary mark are read 302, 304, 306, and 308. Between any pair of secondary marks to the right of the primary mark numbered 4, there is only one tertiary mark. Hence, each smallest space represents five units. Thus the tertiary mark between the secondary marks representing 41 and 42 is read 415, that between the secondary marks representing 55 and 56 is read 555, and the first tertiary mark to the right of the primary mark numbered 4 is read 405.

The reading of any position between a pair of successive tertiary marks must be based on an estimate. Thus a position half way between the tertiary marks associated with 222 and 224 is read 223 and a position two fifths of the way from the tertiary mark numbered 415 to the next mark is read 417. The principle illustrated by these readings applies in all cases.

Consider the process of finding on the *D* scale the position representing 246. The first figure on the left, namely 2, tells us that the position lies between the primary marks numbered 2 and 3. This region is indicated by the brace in Fig. (*a*). The second figure from the left, namely 4, tells us that the position lies between

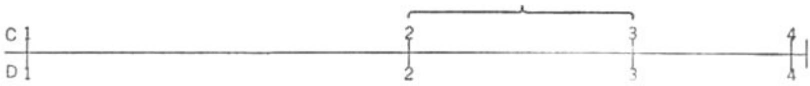


Fig. *a*.

the secondary marks associated with 24 and 25. This region is indicated by the brace in Fig. (*b*). Now there are 4 marks between

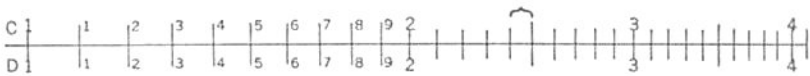


Fig. *b*.

the secondary marks associated with 24 and 25. With these are associated the numbers 242, 244, 246, and 248 respectively. Thus

Fig. *c*.

the position representing 246 is indicated by the arrow in Fig. (*c*). Fig. (*abc*) gives a condensed summary of the process.

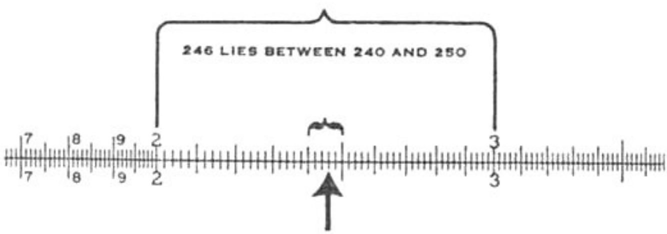


Fig. *abc*.

It is important to note that the decimal point has no bearing upon the position associated with a number on the *C* and *D* scales. Consequently, the arrow in Fig. (*abc*) may represent 246, 2.46, 0.000246, 24,600, or any other number whose principal digits are 2, 4, 6. The placing of the decimal point will be explained later in this chapter.

For a position between the primary marks numbered 1 and 2, four gits should be read; the first three will be exact and the last one

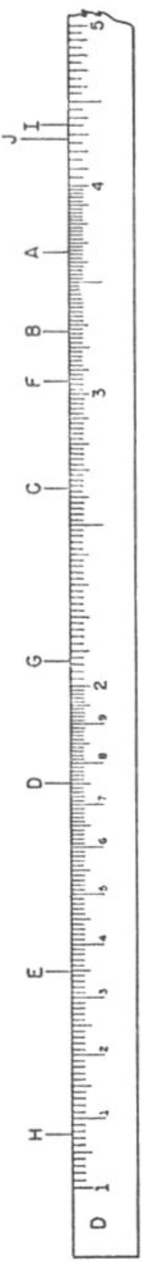


FIG. 4.

estimated. No attempt should be made to read more than three digits for positions to the right of the primary mark numbered 2.

While making a reading, the learner should have definitely in mind the number associated with the smallest space under consideration. Thus between 1 and 2, the smallest division is associated with 10 in the fourth place; between 2 and 3, the smallest division has a value 2 in the third place; while to the right of 4, the smallest division has a value 5 in the third place.

The learner should read from Fig. 4 the numbers associated with the marks lettered A, B, C, . . . and compare his readings with the following numbers: A 365, B 327, C 263, D 1745, E 1347, F 305, G 207, H 1078, I 435, J 427.

3. Accuracy of the slide rule. From the discussion of § 2 it appears that we read four figures of a result on one part of the scale and three figures on the remaining part. Assuming that the error of a reading is one tenth of the smallest interval following the left-hand index of D, we conclude that the error is roughly 1 in 1000 or one tenth of one per cent. The effect of the assumed error in judging a distance is inversely proportional to the length of the rule. Hence we associate with a 10-inch slide rule an error of one tenth of one per cent, with a 20-inch slide rule an error of one twentieth of one per cent or 1 part in 2000, and with the Thacher Cylindrical slide rule an error of a hundredth of one per cent or one part in 10,000. The accuracy obtainable with the 10-inch slide rule is sufficient for many practical purposes; in any case the slide rule result serves as a check.

4. Definitions. The central sliding part of the rule is called the *slide*, the other part the *body*. The glass runner is called the *indicator* and the line on the indicator is referred to as the *hairline*.

The mark associated with the primary number 1 on any scale is called the *index* of the scale. An examination of the D scale shows that it has two indices, one at the left end and the other at the right end.

Two positions on different scales are said to be *opposite* if, without moving the slide, the hairline may be brought to cover both positions at the same time.

5. Multiplication. The process of multiplication may be performed by using scales C and D. The C scale is on the slide, but in other respects it is like the D scale and is read in the same manner.

To multiply 2 by 4,

to 2 on D set index of C,
push hairline to 4 on C,
at the hairline read **8** on D.

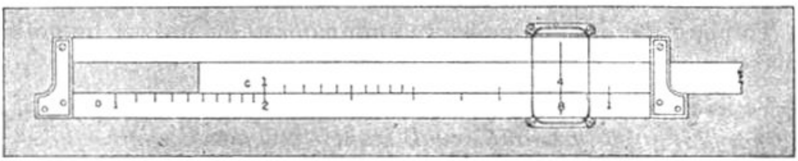


Fig. 5.

Fig. 5 shows the rule in skeleton form set for multiplying 2 by 4.

To multiply 3×3,

to 3 on D set index of C,
push hairline to 3 on C,
at the hairline read **9** on D.

See Fig. 6 for the setting in skeleton form.

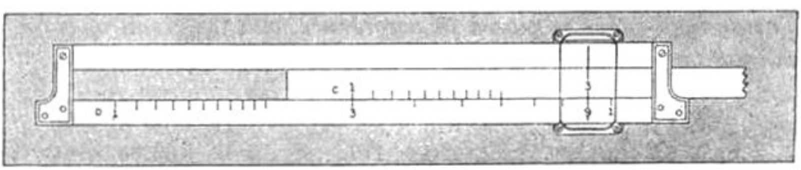


Fig. 6.

To Multiply 1.5×3.5, disregard the decimal point and
to 15 on D set index of C,
push hairline to 35 on C,
at the hairline read **525** on D.

By inspection we know that the answer is near to 5 and is therefore **5.25.**

To find the value of 16.75×2.83 (see Fig. 7).

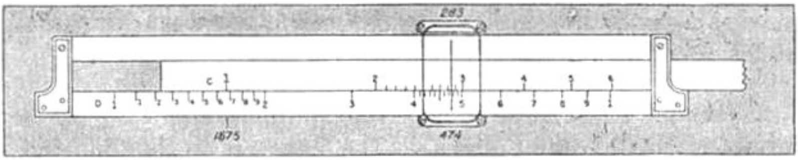


Fig. 7.

disregard the decimal point and
to 1675 on D set index of C,
push hairline to 283 on C,
at the hairline read **474** on D.

To place the decimal point we approximate the answer by noting that it is near to $3 \times 16 = 48$. Hence the answer is **47.4.**

To find the value of 0.001753×12.17,
to 1753 on D set left index of C,
push hairline to 1217 on C,
at the hairline read **213** on D.

To place the decimal point, approximate the answer by writing $.002 \times 10 = .02$. Hence the answer is **0.0213.**

These examples illustrate the use of the following rule.

Rule. *To find the product of two numbers, disregard the decimal points, opposite either of the numbers on the D scale set the index of the C scale, push the hairline of the indicator to the second number on the C scale, and read the answer under the hairline of the D scale. The decimal point is placed in accordance with the result of a rough calculation.*

EXERCISES

1. 3×2.
2. 3.5×2.
3. 5×2.
4. 2×4.55.
5. 4.5×1.5
6. 1.75×5.5.
7. 4.33×11.5.
8. 2.03×167.3.
9. 1.536×30.6.
10. 0.0756×1.093.
11. 1.047×3080.
12. 0.00205×408.
13. $(3.142)^2$.
14. $(1.756)^2$.

6. Either index may be used. It may happen that a product cannot be read when the left index of the C scale is used in the rule of §5. This will be due to the fact that the second number of the product is on the part of the slide projecting beyond the body. In this case reset the slide using the right index of the C scale in place of the left, or use the following rule:

Rule. *When a number is to be read on the D scale opposite a number of the C scale and cannot be read, push the hairline to the index of the C scale inside the body and draw the other index of the C scale under the hairline. Then make the desired reading.*

This rule, slightly modified to apply to the scales being used, is generally applicable when an operation calls for setting the hairline to a position on the part of the slide extending beyond the body.

If, to find the product of 2 and 6, we set the left index of the C scale opposite 2 on the D scale, we cannot read the answer on the D scale opposite 6 on the C scale. Hence, we set the right index of C opposite 2 on D; opposite 6 on C read the answer, **12**, on D.

Again, to find 0.0314×564,

to 314 on D set the right index of C,
push hairline to 564 on C,
at the hairline read **1771** on D.

A rough approximation is obtained by finding $0.03 \times 600 = 18$. Hence the product is **17.71**.

EXERCISES

Perform the indicated multiplications:

1. 3×5.
2. 3.05×5.17.
3. 5.56×634.
4. 743×0.0567.
5. 0.0495×0.0267.
6. 1.876×926.
7. 1.876×5.32.
8. 42.3×31.7.
9. 912×0.267.
10. 48.7×1.173.
11. 0.298×0.544.
12. 0.0456×4.40.
13. 8640×0.01973.
14. $(75.0)^2$.
15. $(83.0)^2$.
16. 4.98×576.

7. Division. The process of division is performed by using the C and D scales.

To divide 8 by 4 (see Fig. 8),

push hairline to 8 on D,
draw 4 of C under the hairline,
opposite index of C read **2** on D.

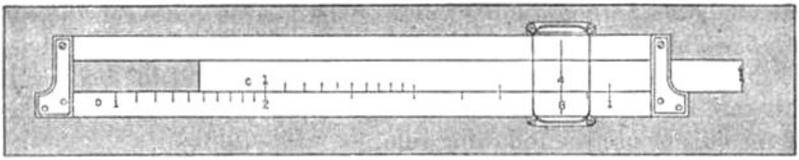


FIG. 8.

To divide 876 by 20.4,

push hairline to 876 on D,
draw 204 of C under the hairline,
opposite index of C read **429** on D.

The rough calculation $800 \div 20 = 40$ shows that the decimal point must be placed after the 2. Hence the answer is **42.9.**

These examples illustrate the use of the following rule.

Rule. *To find the quotient of two numbers, disregard the decimal points, opposite the numerator on the D scale set the denominator on the C scale, opposite the index of the C scale read the quotient on the D scale. The position of the decimal point is determined from information gained by making a rough calculation.*

EXERCISES

Perform the indicated operations:

1. $87.5 \div 37.7$.
2. $3.75 \div 0.0227$.
3. $0.685 \div 8.93$.
4. $1029 \div 9.70$.
5. $0.00377 \div 5.29$.
6. $2875 \div 37.1$.
7. $871 \div 0.468$.
8. $0.0385 \div 0.001462$.
9. $3.14 \div 2.72$.
10. $3.42 \div 81.7$.
11. $529 \div 565$.
12. $0.0456 \div 0.0297$.
13. $396 \div 0.643$.
14. $0.0592 \div 1.983$.
15. $0.378 \div 0.0762$.
16. $10.05 \div 30.3$.

8. Simple applications, percentage, rates. Many problems involving percentage and rates are easily solved by means of the slide rule.

One per cent (1%) of a number N is $N \times 1/100$; hence 5% of N is $N \times 5/100$, and, in general, p% of N is $pN/100$. Hence to find 83% of 1872

to 1872 on D set right index of C,
push hairline to 83 on C,
at the hairline read **1554** on D.

Since $(83/100) \times 1872$ is approximately $\frac{80}{100} \times 2000 = 1600$, the answer is **1554**.

To find the answer to the question "M is what per cent of N?" we must find $100\ M \div N$. Thus, to find the answer to the question "87 is what per cent of 184.7?" we must divide $87 \times 100 = 8700$ by 184.7. Hence

push hairline to 87 on D,
draw 1847 of C under the hairline,
opposite index of C read **471** on D.

The rough calculation $\frac{9000}{200} = 45$ shows that the decimal point should be placed after the 7. Hence the answer is **47.1%**.

For a body moving with a constant velocity, distance = rate times time. Hence if we write d for distance, r for rate, and t for time, we have

$$d = rt, \quad \text{or} \quad r = \frac{d}{t}, \quad \text{or} \quad t = \frac{d}{r}.$$

To find the distance traveled by a car going 33.7 miles per hour for 7.75 hours, write $d = 33.7 \times 7.75$, and

to 337 on D set right index of C,
push hairline to 775 on C,
at hairline read **261** on D.

Since the answer is near to $8 \times 30 = 240$ miles, we have $d =$ **261** miles.

To find the average rate at which a driver must travel to cover 287 miles in 8.75 hours, write $r = 287 \div 8.75$, and

push hairline to 287 on D,
draw 875 of C under the hairline,
opposite the index of C read **328** on D.

Since the rate is near $280 \div 10 = 28$, we have $r =$ **32.8** miles per hour.

EXERCISES

1. Find (*a*) 86.3 per cent of 1826.
   (*b*) 75.2 per cent of 3.46.
   (*c*) 18.3 per cent of 28.7.
   (*d*) 0.95 per cent of 483.

2. What per cent of
   (a) 69 is 18?
   (b) 132 is 85?
   (c) 87.6 is 192.8?
   (d) 1027 is 28?
3. Find the distance covered by a body moving
   (a) 23.7 miles per hour for 7.55 hours.
   (b) 68.3 miles per hour for 1.773 hours.
   (c) 128.7 miles per hour for 16.65 hours.
4. At what rate must a body move to cover
   (a) 100 yards in 10.85 seconds?
   (b) 386 feet in 25.7 seconds?
   (c) 93,000,000 miles in 8 minutes and 20 seconds?
5. Find the time required to move
   (a) 100 yards at 9.87 yards per second.
   (b) 3800 miles at 128.7 miles per hour.
   (c) 25,000 miles at 77.5 miles per hour.

9. Use of the scales *DF* and *CF* (folded scales). The DF and the CF scales are the same as the D and the C scales respectively except in the position of their indices. The fundamental fact concerning the folded scales may be stated as follows: *if for any setting of the slide, a number M of the C scale is opposite a number N on the D scale, then the number M of the CF scale is opposite the number N on the DF scale.* Thus, if the learner will draw 1 of the CF scale opposite 1.5 on the DF scale, he will find the following opposites on the CF and DF scales

CF	1	2	4	5	6	6.67	—,
DF	1.5	3	6	7.5	9	1	

and the same opposites will appear on the C and D scales.

The following statement relating to the folded scales is basic. *The process of* **setting the hairline** *to a number N on scale C to find its opposite M on scale D may be replaced by* **setting the hairline** *to n on scale CF to find its opposite m on scale DF.* The statement holds true if letters C and D are interchanged.

In accordance with the principle stated above, if the operator wishes to read a number on the D scale opposite a number N on the C scale but cannot do so, he can generally read the required number on the DF scale opposite N on the CF scale. For example to find 2×6,

to 2 on D set left index of C,
push hairline to 6 on CF,
at the hairline read **12** on DF.

By using the CF and DF scales we saved the trouble of moving the slide as well as the attendant source of error. This saving, entering as it does in many ways, is a main reason for using the folded scales.

The folded scales may be used to perform multiplications and divisions just as the C and D scales are used. Thus to find 6.17×7.34,

to 617 on DF set index of CF,
push hairline to 734 on CF,
at the hairline read **45.3** on DF;

or

to 617 on DF set index of CF,
push hairline to 734 on C,
at the hairline read **45.3** on D.

Again to find the quotient $7.68/8.43$,

push hairline to 768 on DF,
draw 843 of CF under the hairline,
opposite the index of CF read **0.911** on DF;

or

push hairline to 768 on DF,
draw 843 of CF under the hairline,
opposite the index of C read **0.911** on D.

It now appears that we may perform a multiplication or a division in several ways by using two or more of the scales C, D, CF, and DF. The sentence written in italics near the beginning of the article sets forth the guiding principle.

A convenient method of multiplying or dividing a number by π ($= 3.14$ approx.) is based on the statement: *any number on DF is π times its opposite on D, and any number on D is $1/\pi$ times its opposite on DF.* For example, to find the value of 4π,

push hairline to 4 on D,
at hairline read on DF, **12.57** $= 4\pi$,

to find the value of $\frac{3}{\pi}$

push hairline to 3 on DF,
at hairline read on D, **0.955** $= 3/\pi$.

EXERCISES

Perform each of the operations indicated in the following exercises. Whenever possible without resetting, read the answer on D and also on DF:

1. 5.78×6.35.
2. 7.84×1.065.
3. $0.00465 \div 73.6$.
4. $0.0634 \times 53{,}600$.
5. $1.769 \div 496$.
6. $946 \div 0.0677$.
7. 813×1.951.
8. $0.00755 \div 0.338$.
9. $0.0948 \div 7.23$.
10. $149.0 \div 63.3$.
11. $2.718 \div 65.7$.
12. $783\,\pi$.
13. $783 \div \pi$.
14. $0.0876\,\pi$.
15. $0.504 \div \pi$.
16. $1.072 \div 10.97$.

17. The circumference of a circle measures 8.43 inches. Find its diameter.

18. A cylindrical tube is 13 inches long and has an outside diameter of $2\frac{1}{8}$ inches. Find its outside surface area.

CHAPTER II

THE PROPORTION PRINCIPLE AND COMBINED OPERATIONS

10. Introduction. The *ratio* of two numbers a and b is the quotient of a divided by b or a/b. A statement of equality between two ratios is called a proportion. Thus

$$\frac{2}{3} = \frac{6}{9}, \qquad \frac{x}{5} = \frac{7}{11}, \qquad \frac{a}{b} = \frac{c}{d}$$

are proportions. We shall at times refer to equations having such forms as

$$\frac{2}{3} = \frac{x}{5} = \frac{9}{y} = \frac{10}{z}, \text{ and } \frac{a}{b} = \frac{c}{d} = \frac{e}{f}$$

as proportions.

An important setting like the one for multiplication, the one for division, and any other one that the operator will use frequently should be practiced until it is made without thought. But, in the process of devising the best settings to obtain a particular result, of making a setting used infrequently, or of recalling a forgotten setting, the application of proportions as explained in the next article is very useful.

11. Use of Proportions. If the slide is drawn to any position, the ratio of any number on the D scale to its opposite on the C scale is, in accordance with the setting for division, equal to the number on the D scale opposite the index on the C scale. In other words, *when the slide is set in any position, the ratio of any number on the D scale to its opposite on the C scale is the same as the ratio of any other number on the D scale to its opposite on the C scale.* For example

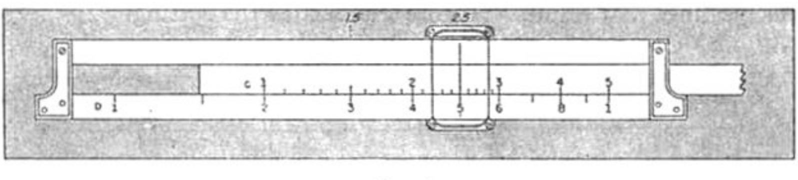


FIG. 1.

draw 1 of C opposite 2 on D (see Fig. 1) and find the opposites indicated in the following table:

C (or CF)	1	1.5	2.5	3	4	5	,
D (or DF)	2	3	5	6	8	10	

and draw 2 of C over 1 on D and read the same opposites. The same statement is true if in it we replace C scale by CF scale and D scale by DF scale. Hence, *if both numerator n and denominator d of a ratio in a given proportion are known, we can set n of the C scale opposite d on the D scale and then read, for an equal ratio having one part known, its unknown part opposite the known part.* We could also begin by setting d on the C scale opposite n on the D scale. It is important to observe that *all the numerators of a series of equal ratios must appear on one scale and the denominators on the other.* For example, let it be required to find the value of x satisfying

$$\frac{x}{56} = \frac{9}{7}.$$

Here the known ratio is 9/7. Hence

push hairline to 7 on D,
draw 9 of C under the hairline,
push hairline to 56 on D,
at the hairline read **72** on C;

or

push hairline to 9 on D,
draw 7 of C under the hairline,
push hairline to 56 on C,
at the hairline read **72** on D.

The CF and DF scales could have been used to obtain exactly the same settings and results. Figure 2 indicates the setting.

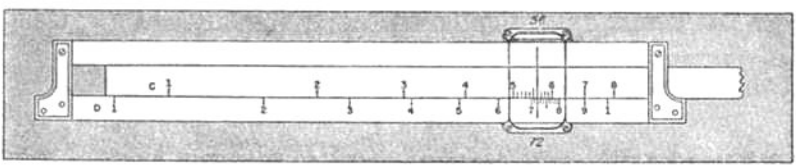


FIG. 2.

To find the values of x, y, and z defined by the equations

$$\frac{C}{D}: \qquad \frac{3.15}{5.29} = \frac{x}{4.35} = \frac{57.6}{y} = \frac{z}{183.4},$$

note that C and D indicate the respective scales for the numerators and the denominators, observe that 3.15/5.29 is the known ratio, and

push hairline to 529 on D,
draw 315 of C under the hairline,
opposite 435 on D, read x = **2.59** on C,
opposite 576 on C, read y = **96.7** on D,
opposite 1834 on D, read z = **109.2** on C.

The positions of the decimal points were determined by noticing that each denominator had to be somewhat less than twice its associated numerator because 5.29 is somewhat less than twice 3.15.

When an answer cannot be read, apply the italicized rule of §6. Thus to find the values of x and y satisfying

$$\frac{C}{D}: \qquad \frac{x}{587} = \frac{14.56}{97.6} = \frac{5.78}{y},$$

to 976 on D set 1456 of C; then, since the answers cannot be read, push the hairline to the index on C, draw the right index of C under the hairline and

opposite 587 on D, read x = **87.6** on C,
opposite 578 on C, read y = **38.7** on D.

Here the positions of the decimal points were determined by observing that each denominator had to be about six times the associated numerator.

When a result cannot be read on the C scale nor on the D scale it may be possible to read it on the CF scale or on the DF scale. Thus, to find x and y satisfying the equations

$$\frac{C \text{ (or } CF)}{D \text{ (or } DF)}: \qquad \frac{4.92}{x} = \frac{1}{3.23} = \frac{y}{13.08},$$

to 323 on D set left index of C,
opposite 492 on CF, read x = **15.89** on DF,
opposite 1308 on DF, read y = **4.05** on CF.

A slight inspection of the scales will show the value of the statement: If the difference of the first digits of the two numbers of the known ratio is small use the C and D scales for the initial setting; if the difference is large use the CF and DF scales. Since in the next to the last example, the difference between the first digits was great, the CF and DF scales should have been used for the initial setting This would have eliminated the necessity for shifting the slide.

EXERCISES

Find, in each of the following equations, the values of the unknowns:

1. $\dfrac{x}{5} = \dfrac{78}{9}$.

2. $\dfrac{x}{120} = \dfrac{240}{170}$.

3. $\dfrac{7}{8} = \dfrac{249}{x}$.

4. $\dfrac{2}{3} = \dfrac{x}{7.83}$.

5. $\dfrac{x}{1.804} = \dfrac{y}{25} = \dfrac{1}{0.785}$;

6. $\dfrac{x}{709} = \dfrac{246}{y} = \dfrac{28}{384}$.

7. $\dfrac{17}{x} = \dfrac{1.365}{8.53} = \dfrac{4.86}{y}$.

8. $\dfrac{8.51}{1.5} = \dfrac{9}{x} = \dfrac{235}{y}$.

9. $\dfrac{x}{2.07} = \dfrac{3}{61.3} = \dfrac{z}{1.571}$.

10. $\dfrac{x}{0.204} = \dfrac{y}{0.0506} = \dfrac{5.28}{z} = \dfrac{2.01}{0.1034}$;

11. $\dfrac{0.813}{2.85} = \dfrac{x}{4.61} = \dfrac{0.435}{y}$.

12. $\dfrac{x}{0.429} = \dfrac{y}{0.789} = \dfrac{2.43}{0.0276}$.

13. $\dfrac{x}{0.00560} = \dfrac{0.743}{1} = \dfrac{0.0615}{y}$;

14. $\dfrac{x}{y} = \dfrac{y}{7.34} = \dfrac{3.75}{29.7}$.

15. $\dfrac{x}{49.6} = \dfrac{z}{y} = \dfrac{y}{3.58} = \dfrac{1.076}{0.287}$;

12. Forming proportions from equations. Since proportions are algebraic equations, they may be rearranged in accordance with the laws of algebra. For example, if

$$x = \frac{ab}{c}, \tag{1}$$

we may write the proportion

$$\frac{x}{1} = \frac{ab}{c}, \tag{2}$$

or we may divide both sides by a to get

$$\frac{x}{a} = \frac{ab}{ac}, \text{ or } \frac{x}{a} = \frac{b}{c}, \tag{3}$$

or we may multiply both sides by c/x to obtain

$$\frac{cx}{x} = \frac{cab}{xc}, \text{ or } \frac{c}{1} = \frac{ab}{x}. \tag{4}$$

Rule (*A*). *A number may be divided by 1 to form a ratio.* This was done in obtaining proportion (2).

Rule (*B*). *A factor of the numerator of either ratio of a proportion may be replaced by 1 and written as a factor of the denominator of the other ratio, and a factor of the denominator of either ratio may be replaced by 1 and written as a factor of the numerator of the other ratio.* Thus (3) could have been obtained from (1) by transferring a from the numerator of the right hand ratio to the denominator of the left hand ratio.

For example, to find $\frac{16 \times 28}{35}$, write $x = \frac{16 \times 28}{35}$, apply Rule (*B*) to obtain $\frac{C}{D}$: $\quad \frac{x}{16} = \frac{28}{35}$,

and push hairline to 35 on D,
draw 28 of C under the hairline,
opposite 16 on D, read x = **12.8** on ***C***.

Figure 3 indicates the setting.

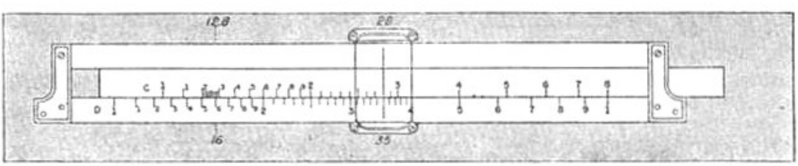


FIG. 3.

To recall the rule for dividing a given number M by a second given number N, write $x = \frac{M}{N}$, apply Rule (A) to obtain $\frac{D}{C}$: $\quad \frac{x}{1} = \frac{M}{N}$,

and push hairline to M on D,
draw N of C under the hairline,
opposite index of C, read x on D.

To recall the rule for multiplication, set $x = \frac{MN}{1}$, apply Rule(B) to obtain $\frac{D}{C}$: $\quad \frac{x}{M} = \frac{N}{1}$,

and to N on D set index of C,
opposite M on C, read x on D.

To find x if $\frac{1}{x} = \frac{864}{(7.48)\ (25.5)}$, use Rule (B) to get $\frac{7.48}{x} = \frac{864}{25.5}$, make the corresponding setting and read $x =$ **0.221**. The position of the decimal point was determined by observing that x must be about $\frac{1}{40}$ of 8, or 0.2.

EXERCISES

Find in each case the value of the unknown quantity:

1. $y = \frac{8 \times 12}{7}$.
2. $7.4 = \frac{9y}{28}$.
3. $8y = 75.6 \times 9$.
4. $y = \frac{86 \times 70.8}{125}$.
5. $y = \frac{147.5 \times 8.76}{3260}$.
6. $y = \frac{0.797 \times 5.96}{0.502}$.
7. $\frac{37 \times 86}{y} = 75.7$.
8. $498 = \frac{89.3x}{0.563}$.
9. $0.874 = \frac{3.95 \times 0.707}{x}$.
10. $0.695 = \frac{0.0879}{x}$.
11. $\frac{1}{386} = \frac{0.772}{2.85y}$.
12. $2580y = 17.9 \times 587$.
13. $3.14y = 0.785 \times 38.7$.
14. $\frac{0.876y}{5.49} = 7.59$.

13. Equivalent expressions of quantity.* When the value of a quantity is known in terms of one unit, it is a simple matter to find its value in terms of a second unit. Thus to find the number of square feet in 3210 sq. in., write

$$\frac{1}{144} = \frac{\text{no. of sq. ft.}}{\text{no. of sq. in.}} = \frac{x}{3210},$$

since there are 144 sq. in. in a square foot; hence

to 144 on D, set index of C,
opposite 3210 on D, read x = **22.3** on C;

that is, there are 22.3 sq. ft. in 3210 sq. in.

Again consider the problem of finding the number of nautical miles in 28.5 ordinary miles. Since there are 5280 ft. in an ordinary mile and 6080 ft. in a nautical mile, write

$$\frac{5280}{6080} = \frac{\text{no. of naut. mi.}}{\text{no. of ord. mi.}} = \frac{x}{28.5},$$

make the corresponding setting and read x = **24.8** naut. mi.

EXERCISES

1. An inch is equivalent to 2.54 cm. Find the respective length in cm. of rods 66 in. long, 98 in. long, and 386 in. long. Note the proportion:

$$\frac{\text{in.}}{\text{cm.}} : \frac{1}{2.54} = \frac{66}{x} = \frac{98}{y} = \frac{386}{z}.$$

2. One yd. is equivalent to 0.9144 meters. Find the number of meters in a distance of (*a*) 300 yd. (*b*) 875 yd. (*c*) 2.78 yd.

$$\textit{Hint.}\ \frac{\text{yd.}}{\text{m}} : \frac{1}{0.914} = \frac{300}{x} = \frac{875}{y} = \frac{2.78}{z}.$$

3. If 7.5 gal. water weighs 62.4 lbs., find the weight of (*a*) 86.5 gal. water. (*b*) 247 gal. water. (*c*) 3.78 gal. water.

4. 31 sq. in. is approximately 200 sq. cm. How many square centimeters in (*a*) 36.5 sq. in.? (*b*) 144 sq. in.? (*c*) 65.3 sq. in.?

5. If one horse-power is equivalent to 746 watts, how many watts are equivalent to (*a*) 34.5 horsepower? (*b*) 5280 horsepower? (*c*) 0.832 horsepower?

6. If one gallon is equivalent to 3790 cu. cm., find the number of gallons of water in a bottle which contains (*a*) 4250 cu. cm. (*b*) 9.68 cu. cm. (*c*) 570 cu. cm. of the liquid.

*A table of equivalents is included with each K & E slide rule.

7. The intensity of pressure due to a column of mercury 1 inch high (1 inch of mercury) is 0.49 lb. per sq. in. If atmospheric pressure is 14.2 lb. per sq. in., what is atmospheric pressure in inches of mercury? What is a pressure of 286 lb. per sq. in. in inches of mercury? What is a pressure of 128 inches of mercury in lb. per sq. in.?

8. If P_1 represents the pressure per square unit on a given quantity of a perfect gas and V_1 the corresponding volume, then for two states of the gas at the same temperature

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}.$$

The volume of a gas at constant temperature and pressure 14.7 lb. per sq. in. is 125 cu. in. (*a*) Find the respective pressures at which the volumes of the gas are 300 cu. in., 250 cu. in., 75.0 cu. in. (*b*) Find the respective volumes of the gas under the pressures: 85 lb. per sq. in., 55 lb. per sq. in., 23 lb. per sq. in , 10 lb. per sq. in.

14. The *DI, CI,* and *CIF* (reciprocal) scales. The reciprocal of a number is obtained by dividing 1 by the number. Thus, $\frac{1}{2}$ is the reciprocal of 2, $\frac{2}{3}$ $(= 1 \div \frac{3}{2})$ is the reciprocal of $\frac{3}{2}$, and $\frac{1}{a}$ is the reciprocal of a.

The reciprocal scales *DI*, *CI*, and *CIF* are marked and numbered like the *D*, *C*, and *CF* scales respectively but in the reverse (or inverted) order; that is, the numbers represented by the marks on these scales increase from right to left. The red numbers associated with the reciprocal scales enable the operator to recognize these scales. A very important consideration may be stated as follows: *When the hairline is set to a number on the C scale, the reciprocal (or Inverse) of the number is at the hairline on the CI scale; conversely, when the hairline is set to a number on the CI scale, its reciprocal is at the hairline on the C scale.*

The same relation exists between the *D* and *DI* scales and between the *CF* and *CIF* scales.

To fix this relation in mind push the hairline in succession to the

D	1	0.5 (=1/2)	0.25 (=1/4)	0.2 (=1/5)	0.125 (=1/8)	0.1111 (=1/9)
DI	1	2	4	5	8	9

numbers on *DI* in the second row of the diagram and read on *D* the respective opposites written in the first row. Also opposite the numbers 1, 2, 4, 5, 8, and 9 on CI read their respective reciprocals on C. Again find the same opposites on CIF and CF.

By using the facts just mentioned, we can multiply a number or divide it by the reciprocal of another number. Thus to find $\frac{28}{7}$, we may think of it as $28 \times \frac{1}{7}$ and

to 28 on D set index of C,
opposite 7 on CI read **4** on D.

Again to find 12×3, we may think of it as $12 \div \frac{1}{3}$ and

push hairline to 12 on D,
draw 3 of CI under the hairline,
opposite index of C, read **36** on D.

When the CI scale is used in multiplication and division, the position of the decimal point is determined in the usual way.

The DF and CIF scales may be used to perform multiplications and divisions in the same manner as the D and CI scales; thus to multiply 40.3 by 1/9.04,

to 403 on DF set index of CF,
opposite 904 on CIF, read **4.46** on DF.

Again to multiply 40.3 by 1/0.207,

to 403 on D set left index of C,
opposite 207 on CIF read **194.7** on DF.

EXERCISES

1. Use the DI scale to find the reciprocals of 16, 260, 0.72, 0.065, 17.4, 18.5, 67.1

2. Find 18.2×21.7 in the usual way and then read 1/(18.2 x 21.7) on DI opposite the first answer on D. Similarly find the values of $1/(2.87 \times 623)$, and $1/(0.324 \times 0.497)$.

3. Using the D scale and the CI scale, multiply 18 by 1/9 and divide 18 by 1/9.

4. Using the D scale and the CI scale multiply 28.5 by 1/0.385 and divide 28.5 by 1/0.385. Also find 28.5/0.385 and 28.5×0.385 by using the C scale and the D scale.

5. Using the D scale and the CI scale multiply 41.3 by 1/0.207 and divide 41.3 by 1/0.207.

6. Perform the operations of Exercises 2, 3, and 4 by using the CIF scale and the DF scale.

7. Set the hairline to 8.62 on DF, and read at the hairline $8.62/\pi$ on D and $\pi/8.62$ on DI. Also find the values of $1.23/\pi$, $\pi/1.23$, $39.4/\pi$, and $\pi/39.4$.

15. Proportions involving the reciprocal scales. The reciprocal scales may be used in connection with proportions containing reciprocals. Since any number $a = 1 \div \frac{1}{a}$ and since $\frac{1}{a} = \frac{1}{a} \div 1$, we have

Rule (C). *The value of any ratio is not changed if any factor of its numerator be replaced by 1 and its reciprocal be written in the denominator, or if any factor of its denominator be replaced by 1 and its reciprocal be written in the numerator.* Thus $\frac{a}{b} = a\left(\frac{1}{b}\right) = \frac{1}{b\,(1/a)}$. Hence if $\frac{x}{a} = bc$, we may write $\frac{x}{a} = \frac{b}{(1/c)} = \frac{c}{(1/b)}$; if $ax = bc$, we may write $\frac{x}{(1/a)} = \frac{b}{(1/c)} = \frac{c}{(1/b)}$. A few examples will indicate the method of applying these ideas in computations.

To find the value of y which satisfies $\frac{y}{4.27} = 0.785 \times 3.76$, apply Rule (C) to get $\frac{D}{C}$: $\quad \frac{y}{4.27} = \frac{0.785}{(1/3.76)}$.

Since when 3.76 of CI is under the hairline, $1/3.76$ of C is also under the hairline

push hairline to 785 on D,
draw 376 of CI under the hairline,
opposite 427 on CF, read $y =$ **12.60** on DF.

The position of the decimal point was determined by observing that y was near to $4 \times 1 \times 4 = 16$.

To find the value of y which satisfies $1/(7.89y) = 0.381/0.0645$, use Rule (C) to obtain $\frac{D}{C}$: $\quad \frac{(1/y)}{7.89} = \frac{0.381}{0.0645}$, and

push hairline to 381 on D,
draw 645 of C under the hairline,
opposite 7.89 on C read $y =$ **0.0215** on DI.

The position of the decimal point was obtained by observing that 0.381 is about 6×0.06 and therefore that $1/y$ is about 6×7.89, or $y = 0.02$ approximately.

To find the values of x and y which satisfy $57.6x = 0.846y = 7$, use Rule (C) to obtain

$$\frac{D}{CI}: \qquad \frac{x}{(1/57.6)} = \frac{y}{(1/0.846)} = \frac{7}{1}, \tag{a}$$

and to 7 on D set index of CI,
opposite 576 on CI, read $x =$ **0.1215** on D,
opposite 846 on CIF, read $y =$ **8.27** on DF.

The folded scales may also be used in this connection. Thus to solve equation (a),

to 7 on DF set index of CIF,
opposite 576 on CIF, read $x =$ **0.1215** on DF,
opposite 846 on CIF, read $y =$ **8.27** on DF.

EXERCISES

In each of the following equations find the values of the unknown numbers:

1. $3.3x = 4.4y = \dfrac{75.2}{1.342}\cdot$
2. $76.1x = 3.44y = \dfrac{111}{22.8}\cdot$
3. $1.83x = \dfrac{y}{24.5} = (162)\ (1.75).$
4. $\dfrac{0.342}{x} = \dfrac{y}{4.65} = (189)\ (0.734).$
5. $5.83x = 6.44y = \dfrac{12.6}{z} = 0.2804.$
6. $3.42x = \dfrac{1.83}{y} = \dfrac{17.6}{z} = (2.78)\ (13.62).$

16. Combined multiplication and division.

Example 1. Find the value of $\dfrac{7.36 \times 8.44}{92}$.

Solution. Reason as follows: first divide 7.36 by 92 and then multiply the result by 844. This would suggest that we

push hairline to 736 on D,
draw 92 of C under the hairline,
opposite 844 on C, read **0.675** on D.

Example 2. Find the value of $\dfrac{18 \times 45 \times 37}{23 \times 29}$.

Solution. Reason as follows: (a) divide 18 by 23, (b) multiply the result by 45, (c) divide this second result by 29, (d) multiply this third result by 37. This argument suggests that we

push hairline to 18 on D,
draw 23 of C under the hairline,
push hairline to 45 on C,
draw 29 of C under the hairline,
push hairline to 37 on C,
at the hairline read 449 on D.

To determine the position of the decimal point write $\dfrac{20 \times 40 \times 40}{20 \times 30} =$ about 50. Hence the answer is **44.9.**

A little reflection on the procedure of Example 2 will enable the operator to evaluate by the shortest method expressions similar to the one just considered. He should observe that: the D scale was used only twice, once at the beginning of the process and once at its end; *the process for each number of the denominator consisted in drawing that number, located on the C scale, under the hairline; the process for each number of the numerator consisted in pushing the hairline to that number located on the C scale.*

If at any time the indicator cannot be placed because of the projection of the slide, apply the rule of §6, or carry on the operations using the folded scales.

It is interesting to observe that when an answer is read on D its reciprocal can be read at once on DI opposite this answer.

Example 3. Find the value of $1.843 \times 92 \times 2.45 \times 0.584 \times 365$, and of 1 divided by this product.

Solution. By using Rule (C) of §15, write the given expression in the form

$$\frac{1.843 \times 2.45 \times 365}{(1/92)\ (1/0.584)}$$

and reason as follows: (a) divide 1.843 by (1/92), (b) multiply the result by 2.45, (c) divide this second result by (1/0.584), (d) multiply the third result by 365. This argument suggests that we

push hairline to 1843 on D,
draw 92 of CI under the hairline,
push hairline to 245 on C,
draw 584 of CI under the hairline,
push hairline to 365 on C,
at the hairline read 886 on D, and 1129 on DI.

To approximate the first answer we write $\dfrac{2 \times 2 \times 400}{0.01 \times 2} = 80{,}000$. Hence the answers are **88,600** and **0.00001129**.

Example 4. Find the value of

$$1/(352 \times 621 \times 0.0154 \times 0.00392).$$

Solution. This computation could be made by computing the denominator in the usual manner and then reading the reciprocal of the denominator on the DI scale. However the regular process is effective. Hence write the given expression in the form

$$\frac{(1/352)\ (1/0.0154)}{621 \times 0.00392}$$

and

push hairline to 352 on DI,
draw 621 of C under the hairline,
push hairline to 154 on CI,
draw 392 of C under the hairline,
opposite index of C read 758 on D.

To approximate the answer write $1/(300 \times 600 \times .02 \times .004) = 1/14$(nearly) $= .07$(nearly). Therefore the answer is **0.0758**.

The following rule summarizes the process.

Rule. *To compute a number defined by a series of multiplications and divisions:*

(*a*) *arrange the expression in fractional form with one more factor in the numerator than in the denominator,* (1 *may be used if necessary.*)

(*b*) *push the hairline to the first number in the numerator on the DI scale,*

(*c*) *using the C or CI scale take the other numbers alternately, drawing each number of the denominator under the hairline, and pushing the hairline to each number of the numerator,*

(*d*) *read the answer on the D scale,*

(*e*) *to get a rough approximation, compute the value of the expression obtained by replacing each number of the given expression by a convenient approximate number involving one, or at most two, significant figures.*

When necessary use the rule of §6 to make a setting possible. Also the folded scales may be used to avoid shifting the slide. At any time the hairline may be pushed to a number on C or on CF; it is a good plan in combined-operation problems always to follow the operation of pushing the hairline to a mark on C or CF by drawing a mark of the same scale under the hairline.*

When a problem involving combined operations contains π as a factor the statement dealing with π at the end of §9 can be used in the solution.

*In the combined-operation computation considered above, the scale of operation may be changed at will from the C scale to the CF scale or vice versa. In general, however, if the answer is read on the D scale, the number of times the hairline has been pushed to a mark on CF must be the same as the number of times a mark on CF has been drawn under the hairline. If the answer is read on DF the process of pushing the hairline to a number on CF must have been used exactly one more time than the process of drawing a mark of CF under the hairline.

EXERCISES

1. $\dfrac{7 \times 8}{5}$.

2. $\dfrac{11 \times 12 \times 1}{7 \times 8}$.

3. $\dfrac{9 \times 7 \times 1}{8 \times (1/5)}$.

4. $\dfrac{1375 \times 0.0642}{76{,}400}$.

5. $\dfrac{45.2 \times 11.24}{336}$.

6. $\dfrac{218}{4.23 \times 50.8}$.

7. $\dfrac{235}{3.86 \times 3.54}$.

8. $2.84 \times 6.52 \times 5.19$.

9. $9.21 \times 0.1795 \times 0.0672$.

10. $37.7 \times 4.82 \times 830$.

11. $\dfrac{65.7 \times 0.835}{3.58}$.

12. $\dfrac{362}{3.86 \times 9.61}$.

13. $\dfrac{24.1}{261 \times 32.1}$.

14. $\dfrac{75.5 \times 63.4 \times 95}{3.14}$.

15. $\dfrac{3.97}{51.2 \times 0.925 \times 3.14}$:

16. $\dfrac{47.3 \times 3.14}{32.5 \times 16.4}$.

17. $\dfrac{3.82 \times 6.95 \times 7.85 \times 436}{79.8 \times 0.0317 \times 870}$.

18. $187 \times 0.00236 \times 0.0768 \times 1047 \times 3.14$.

19. $\dfrac{0.917 \times 8.65 \times 1076 \times 3152}{7840}$.

20. $\dfrac{45.2 \times 11.24\pi}{336}$.

21. $\dfrac{45.2 \times 11.24}{336\pi}$.

In evaluating the expressions numbered 22–25, compute the denominators in the usual manner and read the reciprocals of the denominators on the *DI* scale.

22. $\dfrac{1}{421 \times 632}$.

23. $\dfrac{1}{827 \times 6.28 \times 273}$.

24. $\dfrac{1}{0.153 \times 0.646 \times 5.72 \times 0.628}$.

25. $\dfrac{1}{3.14 \times 2.72 \times 1.414 \times 1.572}$.

26. Solve problems 22–25 by using the method of Example 4.

Expressions having the form

$$l\,m\,n - \frac{1}{l\,m\,p} \tag{5}$$

occur frequently in electrical engineering practice. To evaluate expressions having the form (5)

push hairline to l on D,
draw m of CI under the hairline,
opposite n on C read $l\ m\ n$ on D,
opposite p on C read $1/lmp$ on DI.
Subtract the second result from the first.

Evaluate expression (5) for the sets of values numbered 27–32:

27. $l = 5.41$, $m = 3.14$, $n = 0.226$, $p = 0.0635$,

28. $l = 0.759$, $m = 60.1$, $n = 0.154$, $p = 0.00632$.

29. $l = 6.28$, $m = 54.2$, $n = 0.0246$, $p = 0.00542$.

30. $l = 6.28$, $m = 63.2$, $n = 0.562$, $p = 0.0000653$.

31. $l = 6.28$, $m = 60.0$, $n = 0.247$, $p = 15.00 \times 10^{-5}$.

32. $l = 6.28$, $m = 63.5$, $n = 0.152$, $p = 5.16 \times 10^{-5}$.

Another form of practical importance in electricity is

$$\frac{1}{l\,m}\left[\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \ldots \frac{1}{n_k}\right]. \tag{6}$$

To evaluate expressions having form (6), use the method of Example 4 for each term and

push hairline to l on DI,
draw m of C under the hairline,
at $n_1, n_2, \ldots, n_k$, of CI

in succession, read the corresponding terms on D and add these results.

Evaluate expression (6) for the sets of values numbered 33–38:

33. $l = 2.50$, $m = 40.1$, $n_1 = 0.641$, $n_2\ (=n_k) = 1.03$.

34. $l = 3.65$, $m = 30.6$, $n_1 = 0.00347$, $n_2 = 0.00297$, $n_3\ (=n_k) = 0.00472$.

35. $l = 6.28$, $m = 60.0$, $n_1 = 0.000346$, $n_2 = 0.000463$, $n_3\ (=n_k) = 0.000645$.

36. $l = 6.28$, $m = 61.3$, $n_1 = 3.42 \times 10^{-6}$, $n_2 = 2.71 \times 10^{-6}$, $n_3\ (=n_k) = 5.62 \times 10^{-6}$.

37. $l = 6.28$, $m = 62.4$, $n_1 = 3.01 \times 10^{-6}$, $n_2 = 5.62 \times 10^{-6}$, $n_3\ (=n_k) = 5.81 \times 10^{-6}$.

38. $l = 6.28$, $m = 40.3$, $n_1 = 5.21 \times 10^{-6}$, $n_2 = 8.21 \times 10^{-6}$, $n_3\ (=n_k) = 7.51 \times 10^{-6}$.

CHAPTER III

SQUARES AND SQUARE ROOTS, CUBES AND CUBE ROOTS

17. Squares. The square of a number is the result of multiplying the number by itself. Thus $2^2 = 2 \times 2 = 4$.

The A scale is so designed that *when the hairline is set to a number on the D scale, the square of the number is found under the hairline on the A scale.* Similarly, if the hairline be set to a number on the C scale its square may be read at the hairline on the B scale. Note that the rule can be turned at will to enable the user to refer from one face to the other. For example, if one hairline of the indicator is set to 2 on C, the number $4 = 2^2$ will be found at the other hairline on scale B.

To gain familiarity with the relations between these scales the operator should set the hairline to 3 on the D scale, and read 9 at the hairline on the A scale; set the hairline to 4 on D, read 16 at the hairline on A; etc. To find 278^2, set the hairline to 278 on D, read 773 at the hairline on A. Since $300^2 = 90{,}000$, we write 77,300 as the answer. Actually $278^2 = 77{,}284$. The answer obtained on the slide rule is accurate to three figures.

EXERCISE

Use the slide rule to find, accurate to three figures, the square of each of the following numbers: 25, 32, 61, 75, 89, 733, 452, 2.08, 1.753, 0.334, 0.00356, 0.953, 5270, 4.73×10^6.

18. Square roots. The square root of a given number is a second number whose square is the given number. Thus the square root of 4 is 2 and the square root of 9 is 3, or, using the symbol for square root, $\sqrt{4} = 2$, and $\sqrt{9} = 3$.

The A scale consists of two parts which differ only in slight details. We shall refer to the left hand part as *A left* and to the right hand part as *A right*. Similar reference will be made to the B scale.

Rule. *To find the square root of a number between 1 and 10, set the hairline to the number on scale A left, and read its square root at* *the hairline on the D scale. To find the square root of a number between* 10 *and* 100, *set the hairline to the number on scale A right, and read its square root at the hairline on the D scale.* In either case place the decimal point after the first digit. A similar statement relating to the B scale and the C scale holds true. For example, set the hairline to 9 on scale A left, read 3 ($=\sqrt{9}$) at the hairline on D, set the hairline to 25 on scale B right, read 5 ($=\sqrt{25}$) at the hairline on C.

To obtain the square root of any number, *move the decimal point an even number of places to obtain a number between* 1 *and* 100; *then apply the rule written above in italics; finally move the decimal point one half as many places as it was moved in the original number but in the opposite direction.** The learner may also place the decimal point in accordance with information derived from a rough approximation.

For example, to find the square root of 23,400, move the decimal point 4 places to the left thus getting 2.34 (a number between 1 and 10), set the hairline to 2.34 on scale A left, read 1.530 at the hairline on the D scale, finally move the decimal point ½ of 4 or 2 places to the right to obtain the answer 153.0. The decimal point could have been placed after observing that $\sqrt{10{,}000} = 100$ or that $\sqrt{40{,}000} = 200$. Also the left B scale and the C scale could have been used instead of the left A scale and the D scale.

To find $\sqrt{3850}$, move the decimal point 2 places to the left to obtain $\sqrt{38.50}$, set the hairline to 38.50 on scale A right, read 6.20 at the hairline on the D scale, move the decimal point one place to the right to obtain the answer **62.0.** The decimal point could have been placed by observing that $\sqrt{3600} = 60$.

To find $\sqrt{0.000585}$, move the decimal point 4 places to the right to obtain $\sqrt{5.85}$, find $\sqrt{5.85} = 2.42$, move the decimal point two places to the left to obtain the answer **0.0242.**

*The following rule may also be used: If the square root of a number greater than unity is desired, use A left when it contains an odd number of digits to the left of the decimal point, otherwise use A right. For a number less than unity use A left if the number of zeros immediately following the decimal point is odd; otherwise, use A right.

EXERCISES

1. Find the square root of each of the following numbers: 8, 12, 17, 89, 8.90, 890, 0.89, 7280, 0.0635, 0.0000635, 63,500, 100,000.

2. Find the length of the side of a square whose area is (*a*) 53,500 ft.^{2} (*b*) 0.0776 ft.^{2} (*c*) 3.27×10^7 ft.^{2}

3. Find the diameter of a circle having area (*a*) 256 ft.^{2} (*b*) 0.773 ft.^{2} (*c*) 1950 ft.^{2}

19. Evaluation of simple expressions containing square roots and squares. When the hairline is set to a number on the proper one of the two *B* scales, its square root is automatically set to the hairline on the *C* scale. Consequently we may multiply and divide numbers by square roots of other numbers or we may find the value of the unknown in a proportion involving square roots.

For example to find $3\sqrt{3.24}$ set the left index of *C* to 3 on *D*, then set the hairline to 3.24 on *B* left and therefore to $\sqrt{3.24}$ on *C*, and at the hairline read 540 on *D*. Since $3\sqrt{3.24}$ is nearly equal to $3\sqrt{4}=6$, we have $3\sqrt{3.24} = \mathbf{5.40}$. Observe that the process is that for multiplication by means of the *C* and *D* scales, the *B* scale being used as a means of setting $\sqrt{3.24}$ on the *C* scale.

To find the value of $x = \dfrac{28\sqrt{375}}{369}$, in accordance with rule (B) of §12, write

$$\frac{D}{C}: \quad \frac{x}{\sqrt{375}} = \frac{28}{369}$$

and push the hairline to 28 on *D*,
draw 369 of *C* under the hairline,
push hairline to 375 on *B* left,
at hairline read $x = \mathbf{1.469}$ on *D*.

The approximate answer $30\sqrt{400} \div 400 = 1.5$ indicated the position of the decimal point. Note also that $\sqrt{375}$ of the *C* scale was drawn under the hairline indirectly by drawing its opposite 375 on *B* left under the hairline.

To find the value of $x = \dfrac{347}{4.92\sqrt{0.465}}$, use rule (B) of §12 to get $4.92x = 347/\sqrt{0.465}$ and then rule (C) of §15 to obtain

$$\frac{D}{C}: \quad \frac{x}{1/4.92} = \frac{347}{\sqrt{0.465}},$$

and

set the hairline to 347 on D,
draw 465 of B right under the hairline,
push hairline to 492 on CI,
at the hairline read x = **103.4** on D.

The decimal point was placed in accordance with the approximate value $350 \div (5 \times \sqrt{.49}) = 350 \div 3.5 = 100$.

When the hairline is set to a number on the D scale it is automatically set to the square of the number on the A scale, and when set to a number on the C scale it is automatically set to the square of the number on the B scale. Hence by using the A and B scales as fundamental scales, many expressions involving squares can be evaluated conveniently. Thus to find $x = \dfrac{(24.6)^2 \times 0.785}{4.39}$, write

$$\frac{A}{B}: \quad \frac{x}{0.785} = \frac{(24.6)^2}{4.39},$$

and

push hairline to 246 on D,
draw 439 of B (either left or right) under the hairline,
push hairline to 785 on B (left or right),
at the hairline read **108.2** on A.

The decimal point was placed in the usual manner. Of course this computation could have been carried out on the C and D scales, but one will find it convenient at times to use the setting just indicated.

The area of a circle may be conveniently found when its radius is known by using the A, B, C, and D scales. If π represents a mathematical constant whose value is approximately 3.14, and r represents the radius of a circle, then the area A equals πr^2. Similarly if d represents the diameter of a circle then its area is given by the formula $A = (\pi/4)\, d^2 = 0.785\, d^2$ nearly. Hence to find the area of a circle,

to index of A set $\pi/4$ (= 0.785 approx.) on B,
opposite any diameter on D, read area on B.

Note that a special mark toward the right end of the A and B scales gives the exact position of $\pi/4$. Thus to find the area of a circle of diameter 17.5,

to index on A set $\pi/4$ on B right,
opposite 175 on D, read **241** on B.

Therefore the area is **241 sq. ft.**

EXERCISES

1. $42.2\sqrt{0.328}$.
2. $1.83\sqrt{0.0517}$.
3. $\sqrt{3.28} \div 0.212$.
4. $\sqrt{51.7} \div 103$.
5. $0.763 \div \sqrt{0.0296}$.
6. $\dfrac{5.66 \times (7.48)^2}{79}$.
7. $\dfrac{2.56 \times 4.86}{(1.365)^2}$.
8. $\dfrac{(2.38)^2 \times 19.7}{18.14}$.
9. $\dfrac{6.76}{2.17\ (2.7)^2}$.
10. $\dfrac{\sqrt{277}}{5.34 \times \sqrt{7.02}}$.
11. $\dfrac{645}{5.34\sqrt{13.6}}$.
12. $14.3 \times 47.5\sqrt{0.344}$.
13. $20.6 \times \sqrt{7.89} \times \sqrt{0.571}$.
14. $\dfrac{7.92\sqrt{7.89}}{\sqrt{0.571}}$.

15. Find the area of a circle having diameter (*a*) 2.75 ft. (*b*) 66.8 ft. (*c*) 0.753 ft. (*d*) 1.876 ft.

16. Find the area of a circle having radius (*a*) 3.46 ft. (*b*) 0.0436 ft. (*c*) 17.53 ft. (*d*) 8650 ft.

20. Combined operations involving square roots and squares. The principle of Example 2 §16 may be applied to evaluate a fraction containing indicated square roots as well as numbers and reciprocals of numbers. If the learner will recall that when the hairline is set to a number on the *CI* scale it is automatically set to the reciprocal of the number on the *C* scale and when set to a number on the *B* scale it is automatically set to the square root of the number on the *C* scale, he will easily understand that the method used in this article is essentially the same as that used in §16. The principle of determining whether *B* left or *B* right should be used is the same whether we are merely extracting the square root of a number or whether the square root is involved with other numbers.

Example 1. Evaluate $\dfrac{915 \times \sqrt{36.5}}{804}$.

Solution. Remembering that the hairline is automatically set to $\sqrt{36.5}$ on the *C* scale when it is set to 365 on *B* right, use the rule of §16 and

push the hairline to 915 on *D*,
draw 804 of the *C* scale under the hairline,
push hairline to 365 on *B* right,
at the hairline read **6.88** on *D*.

Example 2. Evaluate $\dfrac{\sqrt{832} \times \sqrt{365} \times 1863}{(1/736) \times 89{,}400}$.

Solution. Before making the setting indicated in this solution, the learner should read the italicized rule in §16.

Push hairline to 832 on A left,
draw 736 of CI under the hairline,
push hairline to 365 on B left,
draw 894 of C under the hairline,
push hairline to 1863 on CF,
at the hairline read **8450** on DF.

To get an approximate value write $\dfrac{(30)\ (18)\ (2000)\ (700)}{90{,}000} = 8400.$*

Example 3. Evaluate $\dfrac{0.286 \times 652 \times \sqrt{2350} \times \sqrt{5.53}}{785\sqrt{1288}}$.

Solution. Write the expression in the form

$$\frac{0.286 \times \sqrt{2350} \times \sqrt{5.53} \times 1}{(1/652) \times 785 \times \sqrt{1288}} \text{ and}$$

push hairline to 286 on D,
draw 652 of CI under the hairline,
push hairline to 235 on B right,
draw 785 of C under the hairline,
push hairline to 553 on B left,
draw 1288 of B right under hairline,
opposite the index of C read **0.755** on D.

As an approximate value use $\dfrac{.3(700)\ (50)\ (2)}{800(30)} = 0.9.$

Example 4. Evaluate $\dfrac{\pi^2 \times 875 \times 278}{(72.2)^2\ (0.317)^2}$.

Solution. Using the A and B scales as fundamental scales
push hairline to 3.142 on D,
draw 722 of C under the hairline,
push hairline to 875 on B,
draw 317 of C under the hairline,
push hairline to 278 on B,
at the hairline read **4580** on A.

* In problems of this type involving very large numbers, it is often desirable to use the powers-of-ten method as an aid in placing the decimal point of the answer. For a discussion of this subject see §53.

EXERCISES

1. $\dfrac{7.87 \times \sqrt{377}}{2.38}$.

2. $\dfrac{86 \times \sqrt{734} \times \pi}{775 \times \sqrt{0.685}}$.

3. $\dfrac{4.25 \times \sqrt{63.5} \times \sqrt{7.75}}{0.275 \times \pi}$.

4. $\dfrac{(2.60)^2}{2.17 \times 7.28}$.

5. $\dfrac{20.6 \times 7.89^2 \times 6.79^2}{4.67^2 \times 281}$.

6. $\dfrac{189.7 \times \sqrt{0.00296} \times \sqrt{347} \times 0.274}{\sqrt{2.85} \times 165 \times \pi}$.

7. $\sqrt{285} \times 667 \times \sqrt{6.65} \times 78.4 \times \sqrt{0.00449}$.

8. $\dfrac{239 \times \sqrt{0.677} \times 374 \times 9.45 \times \pi}{84.3 \times \sqrt{9350} \times \sqrt{28400}}$.

21. Cubes. The cube of a number is the result of using the number three times as a factor. Thus the cube of 3 (written 3^3) is $3 \times 3 \times 3 = 27$.

The K scale is so constructed that *when the hairline is set to a number on the D scale, the cube of the number is at the hairline on the K scale.* To convince himself of this the operator should set the hairline to 2 on D, read 8 at the hairline on K, set the hairline to 3 on D, read 27 at the hairline on K, etc. To find 21.7^3, set the hairline to 217 on D and read 102 on K. Since $20^3 = 8000$, the answer is near 8000. Hence we write 10,200 as the answer. To obtain this answer otherwise, write

$$21.7^3 = \frac{21.7 \times 21.7}{(1/21.7)}$$

and use the general method of combined operations. This latter method is more accurate as it is carried out on the full length scales.

EXERCISES

1. Cube each of the following numbers by using the K scale and also by using the method of combined operations: 2.1, 3.2, 62, 75, 89, 733, 0.152, 3.08, 1.753, 0.0334, 0.943, 5270, 3.85×10^6.

2. How many gallons will a cubical tank hold that measures 26 inches in depth? (1 gal. = 231 cu. in.)

22. Cube roots. There are three parts to the K scale, each the same as the others except in position. We shall refer to the left hand part, the middle part, and the right hand part as K left, K middle, and K right respectively.

The cube root of a given number is a second number whose cube is the given number.

Rule. *To find the cube root of a number between* 1 *and* 10 *set the hairline to the number on K left, read its cube root at the hairline on D. To find the cube root of a number between* 10 *and* 100, *set the hairline to the number on K middle, and read its cube root at the hairline on D. The cube root of a number between* 100 *and* 1000 *is found on the D scale opposite the number on K right.* In each of the three cases the decimal point is placed after the first digit. To see how this rule is used, set the hairline to 8 on K left, read 2 at the hairline on D; set the hairline to 27 on K middle, read 3 at the hairline on D; set the hairline to 343 on K right, read 7 at the hairline on D.

To obtain the cube root of any number, move the decimal point over three places (or digits) at a time until a number between 1 *and* 1000 *is obtained, then apply the rule written above in italics; finally move the decimal point one third as many places as it was moved in the original number but in the opposite direction.* The learner may also place the decimal point in accordance with information derived from a rough approximation.

For example, to find the cube root of 23,400,000, move the decimal point 6 places to the left, thus obtaining 23.4. Since this is between 10 and 100, set the hairline to 234 on K middle, read 2.86 at the hairline on D. Move the decimal point $\frac{1}{3}$ (6) = 2 places to the right to obtain the answer **286.** The decimal point could have been placed after observing that $\sqrt[3]{27{,}000{,}000} = 300$.

To obtain $\sqrt[3]{0.000585}$, move the decimal point 6 places to the right to obtain $\sqrt[3]{585}$, set the hairline to 585 on K right, and read $\sqrt[3]{585} = 8.36$. Then move the decimal point $\frac{1}{3}$ (6) = 2 places to the left to obtain the answer **0.0836.**

EXERCISE

Find the cube root of each of the following numbers: 8.72, 30, 729, 850, 7630, 0.00763, 0.0763, 0.763, 89,600, 0.625, 75×10^7, 10, 100, 100,000.

23. Combined Operations. By setting the hairline to numbers on various scales we may set square roots, cube roots and reciprocals of numbers on the D scale and on the C scale. Hence we can use the slide rule to evaluate expressions involving such quantities, and we can solve proportions involving them. The position of the decimal point is determined by a rough calculation.

Example 1. Find the value of $\dfrac{\sqrt[3]{385}}{2.36}$.

Solution. We may think of this as a division or write the proportion $\frac{x}{1} = \frac{\sqrt[3]{385}}{2.36}$, and then

push the hairline to 385 on K right,
draw 2.36 of C under the hairline,
opposite index of C read **3.08** on D.

Example 2. Find the value of $\frac{5.37\sqrt[3]{0.0835}}{\sqrt{52.5}}$.

Solution. Equating the given expression to x and applying Rule (B) §12, we write

$$\frac{x}{5.37} = \frac{\sqrt[3]{0.0835}}{\sqrt{52.5}}.$$

This proportion suggests the following setting:

push hairline to 835 on K middle,
draw 52.5 of B right under the hairline,
push the hairline to 537 on C,
at hairline read x = **0.324** on D.

Example 3. Evaluate

$$\frac{(1.736)\ (6.45)\ \sqrt{8590}\ \sqrt[3]{581}}{\sqrt{27.8}}.$$

Solution. By using Rule (C) of §15, write the given expression in the form

$$\frac{\sqrt[3]{581}\ (6.45)\ \sqrt{8590}}{(1/1.736)\ \sqrt{27.8}},$$

and

set the hairline to 581 on K right,
draw 1736 of CI under the hairline,
push hairline to 645 on C,
draw 278 of B right under the hairline,
push hairline to 8590 on B right,
at the hairline read **1643** on D.

Note that Examples 1 and 2 were attacked by the proportion principle whereas Example 3 was considered as a series of multiplications and divisions. When no confusion results, the student should always think of an exercise as a series of multiplications and divisions. The proportion principle should be used in case of doubt.

EXERCISES

1. $\sqrt[3]{73.2}\ (0.523)$.
2. $24.3\sqrt[3]{0.0661}\pi$.
3. $489 \div \sqrt[3]{732}$.
4. $27\pi \div \sqrt[3]{661{,}000}$.
5. $\sqrt[3]{531} \div \sqrt{28.4}$.
6. $\sqrt{9.80} \div \sqrt[3]{160{,}000}$.
7. $(72.3)^2 \times 8.25$.
8. $\dfrac{\pi(0.213)^2}{0.0817}$.
9. $\dfrac{\sqrt[3]{19.2^2}}{(7.13)^2 \times 0.122}$.
10. $\dfrac{\pi\sqrt[3]{740}}{4.46 \times \sqrt{28.5}}$.
11. $3.83 \times 6.26 \times \sqrt[3]{54.2}$.
12. $0.437 \times \sqrt{564} \times \sqrt[3]{1.86}$.
13. $675 \times \sqrt{0.346} \times \sqrt[3]{0.00711}$.
14. $\dfrac{\sqrt[3]{32.1}\ (0.0585)\pi}{(1/3.63)}$.
15. $\dfrac{3.57 \times \sqrt{643} \times 4250}{0.0346\sqrt{0.00753}}$.
16. $\dfrac{\sqrt[3]{0.00335}\sqrt{273}}{787\sqrt{0.723}}$.
17. $\dfrac{0.0872 \times 36.8 \times \sqrt{2.85}}{0.343\pi}$.
18. $76.2\sqrt[3]{56.1}\sqrt{877}\ (1/3.78)$.
19. $\dfrac{\sqrt{1.735}}{0.0276\sqrt{58{,}300} \times 7.63 \times 0.476}$.
20. $\dfrac{68.7\sqrt[3]{3160}\sqrt{0.0317} \times 89.3}{17.6 \times 277}$.
21. $\dfrac{\sqrt[3]{0.0645} \times 1834 \times \sqrt{21.6}}{89.6 \times 748 \times \sqrt{3460}}$.
22. $\sqrt{(27.5)^2 - (3.483)^2}$.

23. The maximum time E in hours that an airplane will remain aloft may be approximated by

$$E = \frac{750\ N\sqrt{w}\ (L/D)}{C\,V_c}\left(\frac{1}{\sqrt{w_1}} - \frac{1}{\sqrt{w_0}}\right),$$

where the letters represent certain quantities for an airplane. Compute E if $w = 21500$, $V_c = 190$, $N = 0.85$, $C = 0.465$, $L/D = 20.0$, $w_1 = 19100$, $w_0 = 24000$.

Hint. Push hairline to 750 on D,
draw 190 of C under the hairline,
push hairline to 215 on B left,
draw 465 of C under the hairline,
push hairline to 0.85 on CF,
draw 20 of CIF under the hairline,
push hairline to 1 on C,
draw 191 of B left under hairline, read $F_1 = 153.1$ on D at index of C,
draw 24 of B left under hairline, read $F_2 = 136.6$ on D at index of C,

and subtract F_2 from F_1 to get **16.5** hours.*

* In dealing with combinations of very large numbers or very small numbers it is advisable to use the powers-of-ten notations in placing the decimal point. Article 53 indicates the method to be used.

24. The L scale.* The problems of this chapter could well be solved by means of logarithms. The following statements indicate how the L scale is used to find the logarithms of numbers to the base 10.

(A) When the hairline is set to a number on the D scale it is at the same time set to the mantissa (fractional part) of the common logarithm of the number on the L scale, and conversely, when the hairline is set to a number on the L scale it is set on the D scale to the antilogarithm of that number.

(B) The characteristic (integral part) of the common logarithm of a number greater than 1 is positive and is one less than the number of digits to the left of the decimal point; the characteristic of a number less than 1 is negative and is numerically one greater than the number of zeros immediately following the decimal point.

Example. Find the logarithm of (a) 50. (b) 1.6. (c) 0.35. (d) 0.00905.

Solution. (a) To find the mantissa of log 50,
push hairline to 50 on D,
at hairline on L read 699.

Hence the mantissa is .699. Since 50 has two digits to the left of the decimal point, its characteristic is 1.

Therefore $\log 50 = \mathbf{1.699.}$

Solution. (b) Push hairline to 16 on D,
at hairline on L read 204.

Supplying the characteristic in accordance with (B), we have

$$\log 1.6 = \mathbf{0.204.}$$

Solution. (c) Push hairline to 35 on D,
at hairline on L read 544.

Hence, in accordance with (B), we have

$$\log 0.35 = \mathbf{9.544 - 10.}$$

Solution (d) Push hairline to 905 on D,
at hairline on L read 956.

Hence, in accordance with (B), we have

$$\log 0.00905 = \mathbf{7.956 - 10.}$$

EXERCISE

Find the logarithms of the following numbers: 32.7, 6.51, 980,000, 0.676, 0.01052, 0.000412, 72.6, 0.267, 0.00802, 432.

*On rules, such as Log Log Duplex Vector, No. 4083, which are without the L scale, logarithms can be found on the LL scales as explained on page 88.

CHAPTER IV

TRIGONOMETRY*

25. Some important formulas from plane trigonometry. The following formulas from plane trigonometry, given for the convenience of the student, will be employed in the slide rule solution of trigonometric problems considered in this chapter.

In the right triangle ABC of Fig. 1, the side opposite the angle A is designated by a, the side opposite B by b, and the hypotenuse by c. Referring to this figure, we write the following definitions and relations.

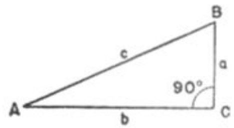


FIG. 1.

Definitions of the sine, cosine, and tangent:

$$\text{sine } A \text{ (written sin } A) = \frac{a}{c} = \frac{\text{opposite side}}{\text{hypotenuse}}, \tag{1}$$

$$\text{cosine } A \text{ (written cos } A) = \frac{b}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}}, \tag{2}$$

$$\text{tangent } A \text{ (written tan } A) = \frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}} \tag{3}$$

Reciprocal relations:

$$\text{cosecant } A \text{ (written csc } A) = \frac{c}{a} = \frac{1}{\sin A}, \tag{4}$$

$$\text{secant } A \text{ (written sec } A) = \frac{c}{b} = \frac{1}{\cos A}, \tag{5}$$

$$\text{cotangent } A \text{ (written cot } A) = \frac{b}{a} = \frac{1}{\tan A}. \tag{6}$$

Relations between complementary angles:

$$\sin A = \cos (90° - A), \tag{7}$$

$$\cos A = \sin (90° - A), \tag{8}$$

$$\tan A = \cot (90° - A), \tag{9}$$

$$\cot A = \tan (90° - A). \tag{10}$$

*See the authors' "Plane and Spherical Trigonometry," McGraw-Hill Book Co., New York, N.Y., for a thorough treatment of the solution of triangles both by logarithmic computation and by means of the slide rule.

Relations between supplementary angles:

$$\sin(180^\circ - A) = \sin A, \tag{11}$$
$$\cos(180^\circ - A) = -\cos A, \tag{12}$$
$$\tan(180^\circ - A) = -\tan A. \tag{13}$$

Relation between angles in a right triangle:

$$A + B = 90^\circ. \tag{14}$$

If in any triangle such as ABC of Fig. 2, A, B, and C represent the angles and a, b, and c, represent, respectively, the lengths of the sides opposite these angles, the following relations hold true:

Law of sines: $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}. \tag{15}$$

Law of cosines: $$a^2 = b^2 + c^2 - 2bc \cos A. \tag{16}$$

$$A + B + C = 180^\circ. \tag{17}$$

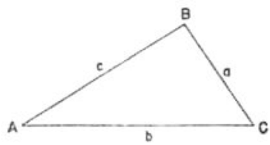


Fig. 2.

26. The *S* (Sine) and *SRT* (Sine, Radian, Tangent) scales. The graduations on the sine scales S and SRT represent *angles*. Accordingly, for convenience, we shall speak *of setting the hairline to an angle* or *drawing an angle under the hairline*.

There are really two S scales, one called S or the sine scale specified by the black numbers on S; the other S *red* or the cosine *scale* specified by the red numbers. Note that the graduations on the sine scale represent angles increasing from left to right, and that the graduations on the cosine scale represent angles increasing from right to left. The sine scale is the predominant scale. In what follows any reference to an angle on a trigonemetric scale will be to the angle in black unless otherwise indicated.

The scale designated SRT is, as its name indicates, used to find sines, radian equivalents, and tangents of angles, ranging from 0.573 degrees to 5.73 degrees approximately. Here our interest is mainly in sines. Detailed treatments of tangents and radians will be given in §§30 and 38.

In order to set the hairline to an angle on the sine scale, it is necessary to determine the values of the angles represented by the subdivisions. Since there are ten primary intervals between 8° and 9° each represents 0.1°; since each of the primary intervals is subdivided into two secondary intervals each of the latter represents 0.05°. Again since there are five primary intervals between 20° and 25°, each represents 1°; since each primary interval here is subdivided into five secondary intervals each of the latter represents 0.2°. The third from the last mark on the scale represents 80°, the next to the last 85°, and the last 90°*. The *SRT* scale is a folded *D* scale and accordingly is read like the *D* scale.

When the hairline is set to an angle on the S or the SRT scale, the sine of the angle is on scale C at the hairline, and hence on scale D when the rule is closed. Also when the hairline is set to an angle on the cosine scale (S red) the cosine of the angle is on scale C at the hairline.

Each small inscription at the right end of a scale is called the **legend** of the scale. A legend of a scale specifies a range of values associated with the function represented by the scale. Thus *the legend* **0.1 to 1.0** *of scale S specifies that the sines of the angles on S and the cosines of angles on S red range from* 0.1 *to* 1, *and the legend* **0.01 to 0.1** *of the SRT scale indicates that sines (or radian equivalents and tangents) of angles on SRT range from* 0.01 *to* 0.1.

Example. Evaluate (*a*) sin 36.4°. (*b*) sin 3.40°.

Solution. (*a*) Opposite 36.4° on *S*,
read on *C* (or *D* when rule is closed) **0.593**.

The result 0.593 lies between 0.1 and 1.0, that is, within the range specified by the legend **0.1 to 1.0** of *S*.

Solution. (*b*) Opposite 3.40° on *SRT*,
read on *C* **0.0593**.

The result 0.0593 lies between 0.01 and 0.1, that is, within the range specified by the legend **0.01 to 0.1** of *SRT*.

*On the 20-inch rule divisions between 80° and 90° represent 82°, 84°, and 86°.

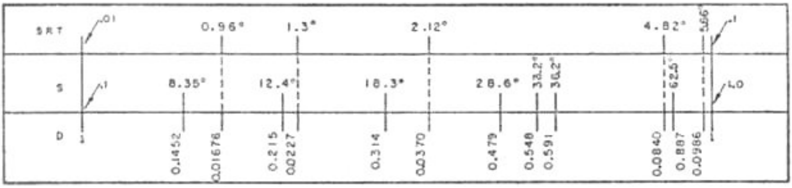


FIG. 3.

Fig. 3 shows scales SRT, S, and D on which certain angles and their sines are indicated. As an exercise close your slide rule and read the sines of the angles shown in the figure and compare your results with those given. Note that the values of sines appearing in Fig. 3 conform with the corresponding legends.

The S and SRT scales are essentially one continuous scale read against two continuous cycles of the C scale. Fig. 4 represents this relationship.

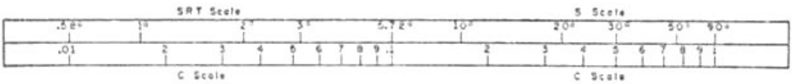


FIG. 4.

Each angle on S red is 90° minus the corresponding angle on S (black). Also equations (7) and (8) §25 are

$$\sin A = \cos (90° - A), \cos A = \sin (90° - A).$$

Hence when the hairline is set to an angle A on S it is set to sin A and to cos $(90° - A)$ on scale C. For example

set the hairline to 25° on S,

at the hairline read on C, 0.423 = sin 25° = cos 65°.

To find the cosine of an angle greater than 84.26°, use $\cos A = \sin (90° - A)$. Thus to find cosine 86.9°, write cosine 86.9° = sin 3.1° and opposite 3.1° on SRT read on C, 0.0541 = sin 3.1° = cos 86.9°

EXERCISES

1. By examination of the slide rule verify that on the S scale from the left index to 10° the smallest subdivision represents 0.05°; from 10° to 20° it represents 0.1°; from 20° to 30° it represents 0.2°; from 30° to 60° it represents 0.5°; from 60° to 80° it represents 1°; and from 80° to 90° it represents 5°.

2. Find the sine of each of the following angles:

(a) 30°. (b) 38°. (c) 3.33°. (d) 90°. (e) 88°.
(f) 1.583°. (g) 14.63°. (h) 22.4°. (i) 11.80°. (j) 51.5°.

3. Find the cosine of each of the angles in Exercise 2. Use the red numbers on the S scale.

4. Find x in each equation:

(a) $\sin x = 0.5$.	(d) $\sin x = 0.1$.	(g) $\sin x = 0.062$.
(b) $\sin x = 0.875$.	(e) $\sin x = 0.015$.	(h) $\sin x = 0.031$.
(c) $\sin x = 0.375$.	(f) $\sin x = 0.62$.	(i) $\sin x = 0.92$.

5. Find x in each equation:

(a) $\cos x = 0.5$.	(d) $\cos x = 0.1$.	(g) $\cos x = 0.062$.
(b) $\cos x = 0.875$.	(e) $\cos x = 0.015$.	(h) $\cos x = 0.031$.
(c) $\cos x = 0.375$.	(f) $\cos x = 0.62$.	(i) $\cos x = 0.92$.

27. Simple operations involving the *S* and *SRT* scales. If the reader will reflect that when the hairline is set to an angle A on scale S, it is also set to $\sin A$ on C, he can easily see that sines and cosines of angles can be used in combined operations and proportions by means of the S and SRT scales just as square roots and reciprocals were used in Chapter III by means of the B scale and the CI scale. Thus to find $8 \sin 40°$,

opposite 8 on D set index of C,
opposite 40° on S read on D, **5.14** $= 8 \sin 40°$.

The decimal point was placed after observing on the slide rule that $\sin 40°$ is approximately 0.6 and therefore that $8 \sin 40°$ is approximately $8 \times 0.6 = 4.8$. The legend of the S scale **0.1 to 1.0** indicates that the approximate value of $\sin 40°$ is 0.6, a value between 0.1 and 1.0.

To find $8/\cos 40°$,

opposite 8 on D set 40° of S red,
opposite index of C read on D, **10.44** $= \dfrac{8}{\cos 40°}$.

Here the decimal point was placed after observing on the slide rule that $\cos 40°$ is near 0.8 and therefore that $8/\cos 40°$ is nearly equal to $8/0.8 = 10$. Here again the legend **0.1 to 1.0** of S indicates that $\cos 40°$ is between **0.1** and 1.0.

The following examples illustrate the use of proportions involving trigonometric functions.

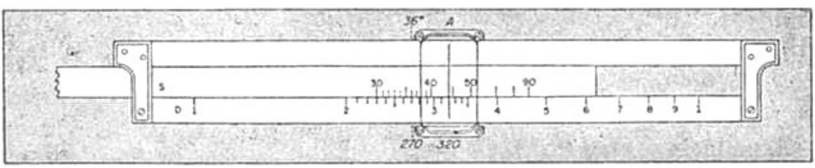


FIG. 5.

Example 1. Find A if $\dfrac{\sin 36°}{270} = \dfrac{\sin A}{320}$.

Solution. Here both parts in the first ratio are known. Hence write

$$\frac{S}{D}: \quad \frac{\sin 36°}{270} = \frac{\sin A}{320}$$

and

opposite 270 on D set 36° of S,
push hairline to 320 on D,
at hairline read on S, **44.2°** $= A$.

Example 2. Find A and x if $\frac{250}{\sin 32°} = \frac{330}{\sin A} = \frac{x}{\cos 80°}$.

Solution. Write

$$\frac{D}{S}: \quad \frac{250}{\sin 32°} = \frac{330}{\sin A} = \frac{x}{\cos 80°}$$

and

opposite 250 on D set 32° of S,
push hairline to 330 on D,
at hairline read on S, **44.40°** $= A$,
push the slide so that left index on C replaces right index,
push hairline to 80° red on S,
at hairline read on D, **81.9** $= x$.

Here the decimal point was located by noting that $\sin 32° = 0.5$ approx. and $\cos 80° = 0.17$ approx. Hence

$$x = \frac{250 \times 0.17 \text{ approx.}}{0.5 \text{ approx.}} = 80 \text{ approx.}$$

Example 3. Find θ if $\sin \theta = \frac{3}{5}$.

Solution. Write the given equation in the form

$$\frac{S}{D}: \quad \frac{\sin \theta}{3} = \frac{1\ (= \sin 90°)}{5}$$

and

set 90° of S opposite 5 on D,
opposite 3 on D read on S, **36.9°** $= \theta$.

Example 4. Find θ if $\cos \theta = \frac{2}{3}$.

Solution. Write the given equation in the form

$$\frac{S}{D}: \quad \frac{\cos \theta}{2} = \frac{1\ (= \sin 90°)}{3}$$

and

set 90° of S opposite **3** on D,
opposite 2 on D read on S red, **48.2°** $= \theta$.

EXERCISES

1. In each of the following proportions find the unknowns:

(a) $\dfrac{\sin 50.4^\circ}{7} = \dfrac{\sin 42.2^\circ}{x} = \dfrac{\sin \theta}{8}$. (b) $\dfrac{\sin \theta}{30.5} = \dfrac{\sin 55^\circ}{x} = \dfrac{\sin 60.5^\circ}{32.8}$.

(c) $\dfrac{\sin 25^\circ}{20} = \dfrac{\sin 40^\circ}{x} = \dfrac{\sin 70^\circ}{y}$. *(d) $\dfrac{\mathrm{Sin}\ \theta}{15.6} = \dfrac{\sin \phi}{25.6} = \dfrac{\sin 12.92^\circ}{40.7}$.

2. Find the value of each of the following:

(a) $5 \sin 30^\circ$. (e) $28 \cos 25^\circ$.
(b) $12 \sin 60^\circ$. (f) $35 \csc 52.3^\circ$.
(c) $22/\sin 30^\circ$. (g) $17 \sec. 16^\circ$.
(d) $15/\sin 20^\circ$. (h) $55 \sin 18^\circ$.

3. Find the value of θ in each of the following:

(a) $\sin \theta = \dfrac{307 \sin 42.5^\circ}{2030}$. (c) $\sin \theta = \dfrac{433 \sin 18.17^\circ}{136}$.

(b) $\sin \theta = \dfrac{413 \sin 77.7^\circ}{488}$. (d) $\sin \theta = \dfrac{156 \sin 12.92^\circ}{40.7}$.

4. Find the value of x in each of the following:

(a) $x = \dfrac{179.5 \sin 6.42^\circ}{\sin 34.5^\circ}$. (c) $x = \dfrac{123.4 \sin 8.20^\circ}{\sin 33.5^\circ}$.

(b) $x = \dfrac{3.27 \sin 73^\circ}{\sin 2.22^\circ}$. (d) $x = \dfrac{375 \sin 18.67^\circ}{\cos 62.7^\circ}$.

5. Find the value of x in each of the following:

(a) $x = \dfrac{4 \sin 35^\circ - 5.4 \sin 17^\circ}{\sin 47^\circ}$. (c) $x = \dfrac{18 \sin 52.5^\circ - 23.4 \cos 42.2^\circ}{\sin 22^\circ \sin 63^\circ}$.

(b) $x = \dfrac{8 - 6 \sin 70^\circ}{\sin 37^\circ - 0.21}$. (d) $x = \dfrac{(27.7 \sin 39.2^\circ)^2 - 16 \cos 12.67^\circ}{46.2 \sin 10.17^\circ + 32.1 \sin 17.27^\circ}$.

28. Law of sines applied to solve a triangle. In the conventional way of lettering a triangle, each side is represented by a small letter and the opposite angle by the same letter capitalized. Thus in Fig. 6, each of the pairs, a and A, b and B, c and C represents a side and the angle opposite. The law of sines (see equation 15 §25) is

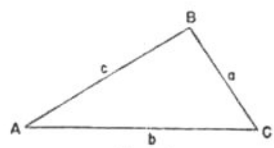


FIG. 6.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

Using this law and the method of solving proportions explained in §27, we can solve any triangle for which a side, the opposite angle, and another part are given.

Example 1. Given a triangle (see Fig. 7) in which $a = 50$, $A = 65°$ and $B = 40°$, find b, c, and C.

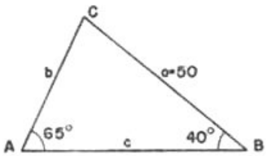


FIG. 7.

Solution. Since $A + B + C = 180°$, $C = 180° - (A + B) = 75°$.

Application of the law of sines to the triangle gives

$$\frac{\sin 65°}{50} = \frac{\sin 40°}{b} = \frac{\sin 75°}{c}.$$

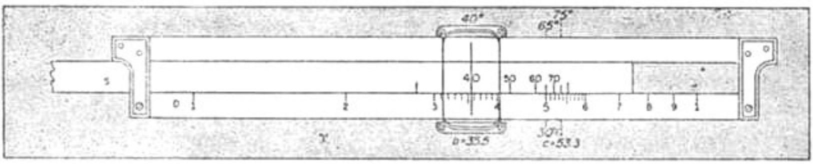


FIG. 8.

Accordingly,

opposite 50 on D set 65° of S,
push hairline to 40° on S,
at hairline read on D, **35.5** $= b$,
push hairline to 75° on S,
at hairline read on D. **53.3** $= c$.

Example 2. Find the unknown parts of the triangle in which $a = 38.3$, $A = 25°$, $B = 38°$.

Solution. In this solution, it is necessary to use $\sin C = \sin 117°$. By (11) of §25, $\sin 117° = \sin (180° - 117°) = \sin 63°$. Hence we shall use $\sin 63°$ instead of $\sin 117°$ since the S scale does not provide directly for 117°. In general, use **the exterior angle of a triangle in the law of sines when the interior angle is greater than 90°.**

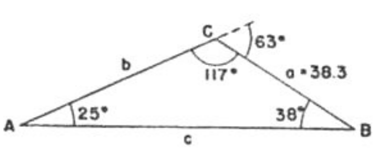


FIG. 9.

Hence from Fig. 9 write

$$\frac{S}{D}: \quad \frac{\sin 25^\circ}{38.3} = \frac{\sin 38^\circ}{b} = \frac{\sin 63^\circ}{c}$$

and

opposite 38.3 on D set 25° of S,
opposite 38° on S read b = **55.8** on D,
opposite 63° on S read c = **80.7** on D.

29. Short cut in solving a triangle. Observe that it is not necessary to write the law of sines in solving a triangle. In accordance with the setting based on the law of sines, **opposite parts on a triangle are set opposite on the slide rule.** The parts to be set opposite can be used directly from the figure. Thus from Fig. 10 it appears at once that the pairs of opposites are: 68.7, 47°; x, 62°; y, 71°.

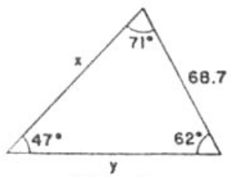


FIG. 10.

To solve the triangle

opposite 687 on D set 47° of S,
opposite 62° on S read x = **82.9** on D,
opposite 71° on S read y = **88.8** on D.

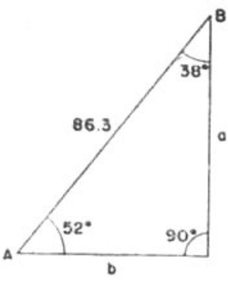


FIG. 11.

To solve the right triangle of Fig. 11, note that 90° and 86.3 are opposite and

opposite 863 on D set 90° of S,
opposite 52° on S read a = **68.0** on D,
opposite 38° on S read b = **53.1** on D.

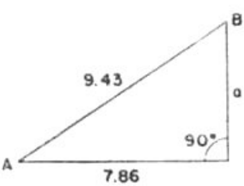


FIG. 12

To solve the right triangle of Fig. 12

opposite 943 on D set 90° of S,
opposite 786 on D read B = **56.5°** on S,
compute $A = 90° - B$ = **33.5°**,
opposite 33.5° on S read a = **5.21** on D.

In general, *to solve any triangle for which a side and the angle opposite are known,*

> *opposite the known side on D set opposite angle on S,*
> *opposite any known side on D read opposite angle on S,*
> *opposite any known angle on S read opposite side on D.*

EXERCISES

Solve the triangle having the given parts:

1. $a = 50$, $A = 65°$, $B = 40°$.

2. $c = 60$, $A = 50°$, $B = 75°$.

3. $a = 550$, $A = 10.2°$, $B = 46.6°$.

4. $a = 795$, $A = 79.98°$, $B = 44.68°$.

5. $a = 50.6$, $A = 38.67°$, $C = 90°$.

6. $a = 729$, $B = 68.83°$, $C = 90°$.

7. $b = 200$, $A = 64°$, $C = 90°$.

8. $c = 11.2$, $A = 43.5°$, $C = 90°$.

9. $b = 47.7$, $B = 62.93°$, $C = 90°$.

10. $a = 83.4$, $A = 72.12°$, $C = 90°$.

11. $a = 60$, $c = 100$, $C = 90°$.

12. $a = 0.624$, $c = 0.91$, $C = 90°$.

13. $b = 4250$, $A = 52.68°$, $C = 90°$.

14. $b = 2.89$, $c = 5.11$, $C = 90°$.

15. $b = 512$, $c = 900$, $C = 90°$.

16. $a = 52$, $c = 60$, $C = 90°$.

17. $a = 120$, $b = 80$, $A = 60°$.

18. $b = 91.1$, $c = 77$, $B = 51.1°$.

***19.** $a = 50$, $c = 66$, $A = 123.2°$.

20. $a = 8.78$, $c = 10$, $A = 61.4°$.

21. $b = 0.234$, $c = 0.198$, $B = 109°$.

****22.** $a = 21$, $A = 4.17°$, $B = 75°$.

*****23.** $b = 8$, $a = 120$, $A = 60°$.

24. $a = 40$, $b = 3$, $A = 75°$.

25. $c = 35.7$, $A = 58.65°$, $C = 90°$.

26. $c = 0.726$, $B = 10.85°$, $C = 90°$.

27. $a = 0.821$, $B = 21.57°$, $C = 90°$.

28. The length of a kite string is 250 yds., and the angle of elevation of the kite is 40°. If the line of the kite string is straight, find the height of the kite.

29. A vector is directed due N.E. and its magnitude is 10. Find the component in the direction of north.

30. Find the angle made by the diagonal of a cube with the diagonal of a face of the cube drawn from the same vertex.

31. A ship at point S can be seen from each of two points, A and B, on the shore. If $AB = 800$ ft., angle $SAB = 67.7°$, and angle $SBA = 74.7°$, find the distance of the ship from A.

32. To determine the distance of an inaccessible tower A from a point B, a line BC and the angles ABC and BCA were measured and found to be 1000 yd., 44°, and 70°, respectively. Find the distance AB.

* $\sin 123.2° = \sin(180° - 123.2°) = \sin 56.8°$.
** The SRT scale must be used for 4.17°.
*** The SRT scale must be used for angle B.

30. The T (Tangent) scale. The black numbers on the T scale represent angles from 5.71° to 45°, the red numbers represent angles from 45° to 84.29°.

When the hairline is set to an angle A on T (black), $\tan A$ is at the hairline on scale C, and hence on scale D when the rule is closed; when the hairline is set to an angle A on T red, $\tan A$ is at the hairline on CI.

Since

$$\tan 5.71° = 0.1, \quad \tan 45° = 1, \quad \tan 84.29° = 10,$$

the range of values on scale C for tangents of angles between 5.71° and 45° is 0.1 to 1, and on scale CI for tangents of angles between 45° and 84.29° is 1 to 10. The *black* legend **0.1 to 1.0** at the right end of the T scale indicates that tangents read on C (*black*) are between 0.1 and 1; the *red* legend **10.0 to 1.0** indicates that tangents read on CI *red* are between 1 and 10. The general rule governing the use of red and black numbers is given in the next article.

For example,

opposite 26° on T (black), read on C, **0.488** = tan 26°,
opposite 64° on T red, read on CI, **2.05** = tan 64°.

The cotangent of an angle may be found by first using either of the identities (6) and (10) §25, namely

$$\cot A = 1/\tan A, \quad \cot A = \tan (90° - A) \tag{18}$$

to express the cotangent as the tangent of an angle and then using the method outlined above. Thus to find cot 26°, write from (18) $\cot 26° = \tan (90° - 26°) = \tan 64°$ and

opposite 64° on T read on CI, **2.05** = cot 26°,

or write $\cot 26° = 1/\tan 26°$ and

opposite 26° on T read on CI, **2.05** = cot 26°.

To find cot 64°, write $\cot 64° = \tan (90° - 64°) = \tan 26°$ and

opposite 26° on T read on C, **0.488** = cot 64°.

In computing an expression involving the tangent of an angle greater than 45° or any cotangent of an angle, it is advisable before beginning the computation to replace the tangent or cotangent by the tangent of an angle less than 45°. Thus to evaluate $565 \tan 56° \div \cot 42°$ we would first write

$$\frac{565 \tan 56°}{\cot 42°} = \frac{565 \cot 34°}{\cot 42°} = \frac{565 \tan 42°}{\tan 34°}$$

and
push the hairline to 565 on D,
draw 34° of T under the hairline,
push the hairline to 42° on T,
at the hairline read on D, **754**.

The decimal point was placed after making the rough approximation

$600 \times 0.9 \div 0.6 = 900$. The numbers 0.9 and 0.6 lie between 0.1 and 1.0, that is, within the range specified by the legend **0.1 to 1.0** of T.

It is shown in trigonometry that the sine and the tangent of an angle less than 5.71° are so nearly equal that they may be considered equal for slide rule purposes. Thus to find tan 2.25° and cot 2.25°,

opposite 2.25° on SRT read on C, **0.0393** = tan 2.25°,
opposite 2.25° on SRT read on CI, **25.5** = 1/tan 2.25° = cot 2.25°.

The operator should be careful in finding an angle greater than 45° on the tangent scale from a ratio. Thus to find A where $\tan A = \frac{5.6}{3.1}$, it is essential that the setting be made as though $90° - A$ were to be found. In this case

$$\tan(90° - A) = \cot A = \frac{3.1}{5.6}, \text{ or } \frac{\tan(90° - A)}{3.1} = \frac{1}{5.6}.$$

Hence

opposite 56 on D set 1 (= tan 45°) of T,
opposite 31 on D read $90° - A$ = **29°** on T black,
or opposite 31 on D read A = **61°** on T red.

Note that the setting must be made as though $90° - A$, an angle less than 45°, were to be found.

EXERCISES

1. Fill out the following table:

ψ	8.1°	27.25°	62.32°	1.017°	74.25°	87°	47.47°
tan ψ							
cot ψ							

2. The following numbers are tangents of angles. Find the angles:

(a) 0.24. (b) 0.785. (c) 0.92. (d) 0.54. (e) 0.059.
(f) 0.082. (g) 0.432. (h) 0.043. (i) 0.0149. (j) 0.374.
(k) 3.72. (l) 4.67. (m) 17.01. (n) 1.03. (o) 1.232.

3. The numbers in Exercise 2 are cotangents of angles. Find the angles:

4. Find the angle x from each equation:

(a) $\tan x = \frac{3.7}{6.8}$. (b) $\tan x = \frac{287}{642}$. (c) $\tan x = \frac{5.72}{2.86}$.

(d) $\tan x = \frac{8.52}{6.73}$. (e) $\cot x = \frac{5}{6}$. (f) $\cot x = \frac{17.2}{143}$.

31. Other functions on the S and T scales. Because of the reciprocal relations (4), (5), and (6) of §25, the complementary relations (7), (8), (9) and (10) of §25, the fact that each red number n representing an angle on scales S and T is $90° - n$, and that the numbers on CI are reciprocals of their opposites on C, four functions or angles may be read at once when the hairline is set to an angle on S or T. To perceive this and a rather interesting color relation, let (B) represent black and (R) represent red, and

push the hairline to 30° (B) or 60° (R) on S,
at the hairline read sin 30° (B) = 0.5 (B) on C,
at the hairline read cos 60° (R) = 0.5 (B) on C,
at the hairline read csc 30° (B) = 2 (R) on CI,
at the hairline read sec 60° (R) = 2 (R) on CI.

Again push the hairline to 35° (B) or 55° (R) on T,
at the hairline read tan 35° (B) = 0.700 (B) on C,
at the hairline read cot 55° (R) = 0.700 (B) on C,
at the hairline read cot 35° (B) = 1.428 (R) on CI,
at the hairline read tan 55° (R) = 1.428 (R) on CI.

These two illustrations indicate that *whenever the value of a direct function (sin, tan, sec) is read, the colors of the angle and its function are the same; whenever the value of a co-function (cos, cot, csc) is read, the colors of the angle and its function are different.* In other words: *direct functions (sin, tan, sec) are read on like colors (black to black, or red to red); co-functions (cos, cot, csc) are read on opposite colors (black to red, or red to black).*

The reader can easily derive a method of reading any desired function without using the relation of colors just considered. However the relation is interesting and helpful to anyone who uses all six trigonometric functions frequently.

EXERCISES

Using the red numbers on the trigonometric scales, solve Exercises 3 and 5 of §26, and Exercises 1 and 3 of §30.

32. The law of sines applied to right triangles with two legs given. When the two legs of a right triangle are the given parts, we may first find the **smaller** acute angle from its tangent and then apply the law of sines to complete the solution.

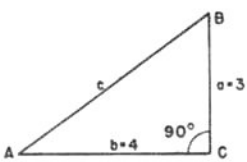


FIG. 13.

Example. Given the right triangle of Fig. 13 in which $a = 3$, $b = 4$; solve the triangle.

Solution. From the triangle we read $\tan A = \frac{3}{4}$. Hence write

$$\frac{T}{D}: \quad \frac{\tan A}{3} = \frac{1}{4}$$

and opposite 4 on D set index of C,
push hairline to 3 on D,
at hairline read A = **36.88°** on T,
at hairline read B = **53.12°** on T red.

Now complete the solution by using the method of §28. Since the hairline is set to 3 on D, draw the opposite angle 36.88° of S under the hairline, and opposite 1 (= sin 90°) on S read c = **5** on D. (See Fig. 14).

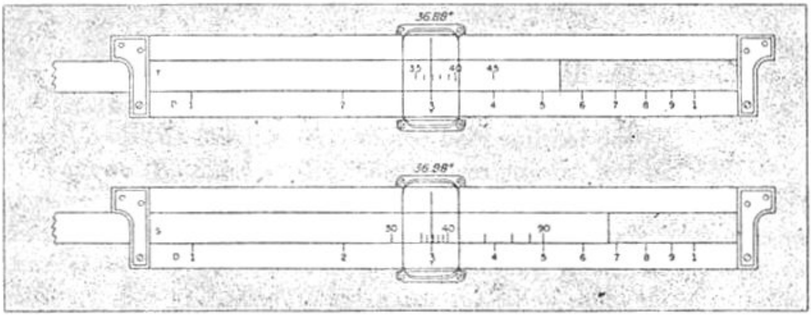


FIG. 14

The following rule is based on the solution just completed. Those operators who have occasion to solve many right triangles of the type under consideration should use the rule.

Rule.

To solve a right triangle for which two legs are given,
to larger leg on D set proper index of slide,
push hairline to smaller leg on D,
at the hairline read smaller acute angle of triangle on T,
draw this angle on S under the hairline,
at index of slide read hypotenuse on D.

The solution of the triangle of Fig. 15 in accordance with the rule is as follows:

to 8.62 on D set right index of C,
push hairline to 4.79 on D,
at hairline read B = **29.08°** on T,
draw 29.08° on S under the hairline,
at index of S read c = **9.86** on D.

Therefore $A = 90° - B =$ **60.92°**.

FIG. 15.

EXERCISES

Solve the following right triangles:

1. $a = 12.3$, $b = 20.2$.	**4.** $a = 273$, $b = 418$.	**7.** $a = 13.2$, $b = 13.2$.
2. $a = 101$, $b = 116$.	**5.** $a = 28$, $b = 34$.	**8.** $a = 42$, $b = 71$.
3. $a = 50$, $b = 23.3$.	**6.** $a = 12$, $b = 5$.	**9.** $a = 0.31$, $b = 4.8$.

10. The length of the shadow cast by a 10-ft. vertical stick on a horizontal plane is 8.39 ft. Find the angle of elevation of the sun.

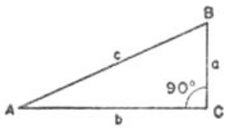
33. Use of the *DI* scale in solving right triangles.

The *DI* scale may be used to advantage in solving right triangles. For this purpose a useful proportion will be derived.

From the adjoining figure read

$$\frac{a}{c} = \sin A, \quad \text{or} \quad a = c \sin A,$$

$$\frac{a}{b} = \tan A, \quad \text{or} \quad a = b \tan A.$$



Equating these values of a obtain

$$a = b \tan A = c \sin A.$$

By applying rule B of §12, this may be written

$$\frac{1}{1/a} = \frac{\tan A}{1/b} = \frac{\sin A}{1/c}. \tag{19}$$

Because of limitations on the T scale, *formula* (19) *is used only when A* **is less than or equal to 45°**; that is, when *a represents the smaller leg of the triangle.* The following rule based on proportion (19), states a method of solving any right triangle for which two legs are known.

Rule. *To solve a right triangle when two legs are given*

opposite smaller leg on DI set index of C,
opposite longer leg on DI read smaller angle on T (black),
opposite this angle on S read hypotenuse on DI.

Example 1. Solve the right triangle having legs 3 and 4.

Solution. Setting $a = 3$, $b = 4$, in (19) obtain

$$\frac{1}{1/3} = \frac{\tan A}{1/4} = \frac{\sin A}{1/c}.$$

Accordingly:

opposite 3 on DI set 1(right) of C,
opposite 4 on DI read A = **36.9°** on T,
opposite 36.9° on S read c = **5** on DI,
$B = 90° - A$ = **53.1°**.

Example 2. Solve the right triangle having $a = 15$, $b = 8$.

Solution. Using the italicized rule,

opposite 8 on DI set 1(left) of C,
opposite 15 on DI read B = **28.1°** on T,
opposite 28.1° on S read c = **17** on DI,
$A = 90° - B$ = **61.9°**.

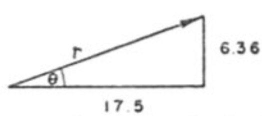
EXERCISES

Use the method involving the DI scale to solve the following right triangles:

1. $a = 12.3$, $b = 20.2$.
4. $a = 273$, $b = 418$.
7. $a = 13.2$, $b = 13.2$.
2. $a = 101$, $b = 116$.
5. $a = 28$, $b = 34$.
8. $a = 42$, $b = 71$.
3. $a = 50$, $b = 23.3$.
6. $a = 12$, $b = 5$.
9. $a = 0.31$, $b = 4.8$.

10. The length of a shadow cast by a 10 ft. vertical stick on a horizontal plane is 8.39 ft. Find the angle of elevation of the sun.

11. The rectangular components of a vector r are 17.5 and 6.36 as shown in the adjoining figure. Find the magnitude and direction of the vector.



12. Find the magnitude and direction of a vector having as the horizontal and vertical components 17.25 and 8.04, respectively.

34. Solution of a triangle for which two sides and the included angle are given. To solve an oblique triangle in which two sides and the included angle are given, it is convenient to divide the triangle into two right angles. The method is illustrated in the following example.

Example. Given an oblique triangle in which $a = 6$, $b = 9$, and $C = 32°$, solve the triangle.

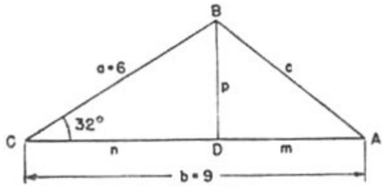


Fig. 16.

Solution. From B of Fig. 16, drop the perpendicular p to side b. Applying the law of sines to the right triangle CBD, we obtain

$$\frac{\sin 90^\circ}{6} = \frac{\sin 32^\circ}{p} = \frac{\sin 58^\circ}{n}.$$

Solving this proportion, we find $p = 3.18$ and $n = 5.09$. From the figure $m = 9 - 5.09 = 3.91$. Hence, in triangle ABD, we have

$$\tan A = \frac{p}{m} = \frac{3.18}{3.91},$$

or

$$\frac{\tan A}{3.18} = \frac{1}{3.91}.$$

Solving this proportion, we get $A = 39.1°$. Now applying the law of sines to triangle ABD, we obtain

$$\frac{\sin 39.1^\circ}{3.18} = \frac{\sin 90^\circ}{c}.$$

Solving this proportion, we find $c =$ **5.04.** Finally, using the relation $A + B + C = 180°$, we obtain $B =$ **108.9°.** The italicized rule of §32 could have been used in place of the last two proportions.

If the given angle is obtuse, the perpendicular falls outside the triangle, but the method of solution is essentially the same as that used in the above example.

The law of cosines (16) of §25 may also be used for the solution. To solve the triangle of Fig. 16, we have

$$c^2 = a^2 + b^2 - 2ab \cos C$$

or

$$c^2 = 6^2 + 9^2 - 2 \times 6 \times 9 \cos 32^\circ = 36 + 81 - 91.6 = 25.4$$

and $c =$ **5.04.** Now using the setting based on the law of sines

opposite 5.04 on D draw 32° of S,
opposite 6 on D read $A =$ **39.1°** on S,
$B = 180° - 32° - 39.1° =$ **108.9°.**

The solution is checked by pushing the hairline to 71.1° (=180° − 108.9°) on S and reading 9 on D at the hairline.

A third method of solving this case is considered in Ex. 14. It is based on the law of tangents.

EXERCISES

Solve the following triangles:

1. $a = 94$, $b = 56$, $C = 29°$.	4. $b = 2.30$, $c = 3.57$, $A = 62°$.	7. $a = 0.085$, $c = 0.0042$, $B = 56.5°$.
2. $a = 100$, $c = 130$, $B = 51.8°$.	5. $a = 27$, $c = 15$, $B = 46°$.	8. $a = 17$, $b = 12$, $C = 59.3°$.
3. $a = 235$, $b = 185$, $C = 84.6°$.	6. $a = 6.75$, $c = 1.04$, $B = 127.2°$.	9. $b = 2580$, $c = 5290$, $A = 138.3°$.

10. Solve exercises 1 to 5 by using the law of cosines to get the third side and then the law of sines to get the unknown angles.

11. The two diagonals of a parallelogram are 10 and 12 and they form an angle of 49.3°. Find the length of each side.

12. Two ships start from the same point at the same instant. One sails due north at the rate of 10.44 mi. per hr., and the other due northeast at the rate of 7.71 mi. per hr. How far apart are they at the end of 40 minutes?

13. In a land survey find the latitude and departure of a course whose length is 525 ft. and bearing N 65.5° E. (See Fig. 17).

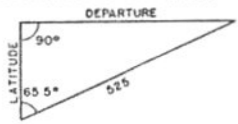


FIG. 17.

14. The law of tangents

$$\frac{\tan \frac{1}{2}(A - B)}{a - b} = \frac{\tan \frac{1}{2}(A + B)}{a + b} = \frac{\tan \frac{1}{2}(180° - C)}{a + b} \qquad (\alpha)$$

is used to solve a triangle for which two sides and the included angle are given. The three cases to which this leads with the slide rule are illustrated below.

(a) $C > 90°$. Use (α) directly. For example if $a = 6.75$, $b = 1.04$, $C = 127.15°$ write

$$\frac{\tan \frac{1}{2}(A - B)}{5.71} = \frac{\tan \frac{1}{2}(A + B)}{7.79} = \frac{\tan 26.43°}{7.79}$$

and

to 7.79 on D set 26.43° of T,
opposite 5.71 on D read $\frac{1}{2}(A - B) = 20.02°$ on T.

The simultaneous solution of $\frac{1}{2}(A - B) = 20.02°$ and $\frac{1}{2}(A + B) = 26.43°$ is $A = 46.45°$, $B = 6.41°$. Now using the method based on the law of sines find $c = 7.43$.

(b) $C < 90°$, $90° - \frac{1}{2}(A - B) < 45°$. Use ($\alpha$) in the form

$$\frac{\tan [90° - \frac{1}{2}(A - B)]}{a + b} = \frac{\tan [90° - \frac{1}{2}(A + B)]}{a - b}. \qquad (\beta)$$

For example if $a = 30.3$, $b = 2.5$, $C = 50°$, write from (β)

$$\frac{\tan [90° - \frac{1}{2}(A - B)]}{32.8} = \frac{\tan 25°}{27.8}$$

and

opposite 27.8 on D set 25° of T black,
opposite 32.8 on D read $90° - \frac{1}{2}(A - B) = 28.81°$ on T.

Now solve $90° - \frac{1}{2}(A - B) = 28.81°$ with $90° - \frac{1}{2}(A + B) = 25°$ to obtain $A = \mathbf{126.19°}$, $B = \mathbf{3.81°}$, and then find $c = \mathbf{28.8}$ by using the method based on the law of sines.

(c) $C < 90°$, $90° - \frac{1}{2}(A - B) > 45°$. In this case use (β).

For example if $a = 130$, $b = 100$, $C = 51°50'$, write from (β)

$$\frac{\tan [90° - \frac{1}{2}(A - B)]}{230} = \frac{\tan 25.92°}{30}$$

and

opposite 30 on D set 25.92° of T,
push hairline to right index of C,
draw left index of C to 230 on D,
at hairline read $90° - \frac{1}{2}(A - B) = 74.98°$ on T red.

Now find $A = \mathbf{79.10°}$, $B = \mathbf{49.06°}$, $c = \mathbf{104.1}$.

Solve each of the three illustrative examples of this exercise without referring to the solutions given.

35. Law of cosines applied to solve triangles for which three sides are given. When the three sides are the given parts of an oblique triangle, we may find one angle by means of the law of cosines $a^2 = b^2 + c^2 - 2bc \cos A$ and then complete the solution by using the law of sines.

Example. Given the oblique triangle of Fig. 18, in which $a = 15$, $b = 18$, and $c = 20$, find A, B, and C.

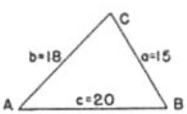


Fig. 18.

Solution. From the law of cosines we write $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$,

or
$$\cos A = \frac{18^2 + 20^2 - 15^2}{2 \times 18 \times 20} = \frac{499}{720}, \text{ or } \frac{\cos A}{499} = \frac{1}{720}.$$

Hence opposite 720 on D set right index of C,
opposite 499 on D read $A = \mathbf{46.1°}$ on S red.

Now apply the method based on the law of sines and

opposite 15 on D set 46.1° of S,
opposite 18 on D read $B = \mathbf{59.9°}$ on S,
opposite 20 on D read $C = \mathbf{74.0°}$ on S.

The relation $A + B + C = 46.1° + 59.9° + 74.0° = 180°$ serves as a check.

EXERCISES

Solve the following triangles:

1. $a = 3.41$, $b = 2.60$, $c = 1.58$.	4. $a = 61.0$, $b = 49.2$, $c = 80.5$.	7. $a = 97.9$, $b = 106$, $c = 139$.
2. $a = 111$, $b = 145$, $c = 40$.	5. $a = 7.93$, $b = 5.08$, $c = 4.83$.	8. $a = 57.9$, $b = 50.1$, $c = 35.0$.
3. $a = 35$, $b = 38$, $c = 41$.	6. $a = 21$, $b = 24$, $c = 27$.	9. $a = 13$, $b = 14$, $c = 15$.

10. The sides of a triangular field measure 224 ft., 245 ft., and 265 ft. Find the angles at the vertices.

11. Find the largest angle of the triangle whose sides are 13, 14, 16.

12. Solve Ex. 11 by means of the following formula:

$$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \text{ where } s = \frac{1}{2}(a+b+c).$$

13. In triangle ABC of Fig. 19 $p^2 = b^2 - m^2 = a^2 - n^2$. Hence $b^2 - a^2 = m^2 - n^2$. Factoring and replacing $(m + n)$ by c, we have

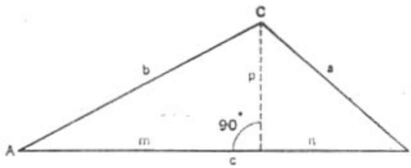


FIG. 19.

$$(b + a)(b - a) = (m + n)(m - n) = c(m - n),$$

or

$$\frac{b+a}{c} = \frac{m-n}{b-a}. \tag{a}$$

To solve the triangle ABC, find $m - n$ by using proportion (a). Combine this result with

$$m + n = c,$$

to find m and n. Then solve each of the right triangles of triangle ABC and use the results to find the angles A, B, and C.

Apply this method to solve Exs. 1, 2, 3.

14. Another method of solving for angle A when sides a, b, and c are given follows. From the law of cosines, equation (16) §25, get

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{2bc + (b^2 - 2bc + c^2) - a^2}{2bc}$$
$$= 1 + \frac{(b-c)^2 - a^2}{2bc} = 1 - \frac{a^2 - (b-c)^2}{2bc},$$

or

$$\cos A = 1 - \frac{(a - b + c)(a + b - c)}{2bc}. \tag{a}$$

Thus to solve the triangle in which $a = 21$, $b = 24$, $c = 27$ for A, substitute these numbers in (a) to obtain

$$\cos A = 1 - \frac{(21 - 24 + 27)(21 + 24 - 27)}{2(24)(27)} = \frac{2}{3}, \text{ or } \frac{\cos A}{2} = \frac{1}{3}.$$

Hence

opposite 3 on D set right index of S,
opposite 2 on D read $A = 48.2°$ on S red.

Now use the law of sines to get $B = $ **58.4°**, $C = $ **73.4°**.

Show that if $a = 97.9$, $b = 106$, $c = 139$, angle $A = 44.6°$.

Also use the method of this exercise to obtain angle A in Exercises 3, 4, 8, and 9.

36. Law of sines applied to oblique triangles, continued. The ambiguous case. When the given parts of a triangle are two sides and an angle opposite one of them, and when the side opposite the given angle is less than the other given side, there may be two triangles which have the given parts. We have already solved triangles in which the side opposite the given angle is greater than the other side. In this case there is always only one solution. Consider now a case where there are two solutions.

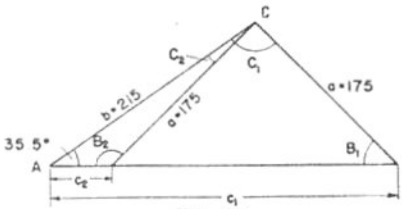


FIG. 20.

Example. Given $a = 175$, $b = 215$, and $A = 35.5°$; solve the triangle.

Solution. Fig. 20 shows the two possible triangles, AB_1C and AB_2C, having the given parts. To solve these triangles

opposite 175 on D set **35.5°** of S,
opposite 215 of D read $B_1 = $ **45.5°** on S,
$C_1 = 180° - A - B_1 = $ **99°**,
opposite $180° - 99°$ ($=81°$) on S read $c_1 = $ **298.**

From Fig. 20 it appears that $B_2 = 180° - B_1 = $ **134.5°**. $C_2 = 180° - A - B_2 = B_1 - A = $ **10°**. Since 175 of D is opposite 35.5° of S, push hairline to 10° on S and read $c_2 = $ **52.3** on D at the hairline.

It is instructive to observe that the slide was set only once, and that the required parts were obtained by pushing the hairline to parts already found and reading unknown parts at the hairline.

Let the three known parts of a triangle be a, b, and A. Fig. 21 represents the triangle with the given parts encircled. If a is less than b but greater than p, there are two triangles AB_1C and AB_2C having the given parts, if $a = p$ there is only one triangle ABC, and if a is less than p there will be no solution. Hence when p is found the computer knows the number of solutions to expect.

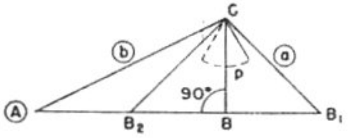


FIG. 21.

If a is greater than b, there will be one and only one triangle satisfying the given conditions.

EXERCISES

Solve the following oblique triangles:

1. $a = 18$, $b = 20$, $A = 55.4°$.	**3.** $a = 32.2$, $c = 27.1$, $C = 52.4°$.	**5.** $a = 177$, $b = 216$, $A = 35.6°$.
2. $b = 19$, $c = 18$, $C = 15.82°$.	**4.** $b = 5.16$, $c = 6.84$, $B = 44°$.	**6.** $a = 17{,}060$, $b = 14{,}050$, $B = 40°$.

7. Find the length of the perpendicular p for the triangle of Fig. 22. How many solutions will there be for triangle ABC if (a) $b = 3$? (b) $b = 4$? (c) $b = p$?

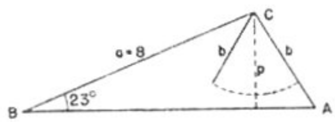


FIG. 22.

37. Combined operations. The method for evaluating expressions involving combined operations as stated in §§16 and 23 applies without change when some of the numbers are trigonometric functions. This is illustrated in the following examples:

Example 1. Evaluate $\dfrac{4 \sin 38°}{\tan 42°}$.

Solution. Push hairline to 4 on D,
draw 42° of T under the hairline,
push hairline to 38° on S,
at the hairline read **2.735** on D.

Example 2. Evaluate $\dfrac{6.1 \sqrt{17} \sin 72° \tan 20°}{2.2}$.

Solution. Use rule C §15 to write $\dfrac{\sqrt{17} \sin 72° \tan 20°}{2.2\left(\dfrac{1}{6.1}\right)}$.

Push hairline to 17 on A right,
draw 22 of C under the hairline,
push hairline to 20° on T,
draw 61 of CI under the hairline,
push hairline to left index of C,
draw right index of C under the hairline,
push hairline to 72° on S,
at the hairline read **3.96** on D.

Example 3. Evaluate $\dfrac{7.9 \csc 17^\circ \cot 31^\circ \cos 41^\circ}{18 \tan 48^\circ \sqrt{3.8}}$.

Solution. Replacing csc 17° by $\dfrac{1}{\sin 17^\circ}$, cot 31° by $\dfrac{1}{\tan 31^\circ}$, and tan 48° by $\dfrac{1}{\tan 42^\circ}$ and using rule C §15, we obtain

$$\frac{\left(\frac{1}{18}\right) 7.9 \tan 42^\circ \cos 41^\circ}{\sqrt{3.8} \sin 17^\circ \tan 31^\circ}.$$

Push hairline to 79 on D,
draw 38 of B left under the hairline,
interchange indices,
push hairline to 18 on CI,
draw 17° of S under the hairline,
push hairline to 42° on T,
draw 31° of T under the hairline,
push hairline to 41° on S red,
at the hairline read **0.871** on D.

The student could have avoided the use of red numbers by replacing in the given expression cos 41° by sin 49°.

The CF scale may often be used to avoid shifting the slide. In the process of evaluating a fraction consisting of a number of factors in the numerator over a number of factors in the denominator, the hairline may be pushed to a number of the numerator on the CF scale provided that a number of the denominator on the CF scale is drawn under the hairline later in the process, and conversely. In other words the CF scale may be used at any time for a multiplication (or division) if it is later used for a division (or multiplication).

Example 4. Evaluate $\dfrac{2.10 \times 2.54 \times \sqrt{45}}{\sin 70^\circ \times \tan 35^\circ \times 3.06}$.

Solution. Push hairline to 2 10 on D,
draw 70° of S under the hairline,
push hairline to 2 54 on CF,
draw 35 of T under the hairline,
push hairline to 45 on B right,
draw 3 06 of CF under the hairline,
under index of C read **17.77** on D.

Note that the folded scale was used twice, once in the third setting and once in the sixth.

EXERCISES

Evaluate the following:

1. $\dfrac{18.6 \sin 36^\circ}{\sin 21^\circ}$.
2. $\dfrac{32 \sin 18^\circ}{27.5}$.
3. $\dfrac{4.2 \tan 38^\circ}{\sin 45.5^\circ}$.
4. $\dfrac{34.3 \sin 17^\circ}{\tan 22.5^\circ}$.
5. $\dfrac{13.1 \cos 40^\circ}{\tan 35.2^\circ}$.
6. $\dfrac{17.2 \cos 35^\circ}{\cot 50^\circ}$.
7. $\dfrac{7.8 \csc 35.5^\circ}{\cot 21.4^\circ}$.
8. $\dfrac{63.1 \sec 80^\circ}{\tan 55^\circ}$.
9. $\dfrac{\sin 18^\circ \tan 20^\circ}{3.7 \tan 41^\circ \sin 31^\circ}$.
10. $\dfrac{\sin 62.4^\circ}{8.1 \tan 22.3^\circ}$.
11. $3.14 \sin 13.17^\circ \csc 32^\circ$.
12. $7.1\, \pi \sin 47.6^\circ$.
13. $\dfrac{0.61 \csc 12.25^\circ}{\cot 35.3^\circ}$.
14. $\dfrac{1 \sin 22.7^\circ}{\tan 28.2^\circ}$.
15. $\dfrac{3.1 \sin 61.6^\circ \csc 15.30^\circ}{\cos 27.7^\circ \cot 20^\circ}$.
16. $\dfrac{13.1 \sin 3.12^\circ}{\tan 30.2^\circ}$.
17. $\dfrac{0.0037 \sin 49.8^\circ}{\tan 2.10^\circ}$.
18. $\dfrac{\sqrt{16.5} \sin 45.5^\circ}{\sqrt{4.6}\; 41.2 \cot 71.2^\circ}$.
19. $\dfrac{\sqrt[3]{6.1}\; 4.91}{\tan 13.23^\circ}$.
20. $\dfrac{\sin 51.5^\circ}{(39.1)\,(6.28)}$.
21. $\dfrac{\csc 49.5^\circ}{(19.1)\,(7.61)\,\sqrt{69.4}}$.
22. $(48.1)\,(1.68) \sin 39^\circ$.
23. $0.0121 \sin 81^\circ \cot 41^\circ$.
24. $\dfrac{1.01 \cos 71.2^\circ \sin 15^\circ}{\sqrt{4.81} \cos 27.2^\circ}$.

25. Solve for the unknowns in the following equations:

(*a*) $\dfrac{\tan \theta}{27} = \dfrac{\tan \alpha}{49} = \dfrac{\tan 33.2^\circ}{38}$.

(*b*) $y = \dfrac{\tan 24.2^\circ}{6.15} = \dfrac{\tan \theta}{1.07}$.

(*c*) $y = (407 \cot 82.88^\circ)^2$.

(*d*) $y = \dfrac{17.2}{\tan 34.2^\circ}$.

(*e*) $y = \dfrac{84.1 \tan 75^\circ}{27.4}$.

(*f*) $y = \dfrac{9.32 \tan 17^\circ}{32.2}$.

(*g*) $y = \dfrac{10.7}{15.1 \cot 42^\circ}$.

(*h*) $\tan \theta = \dfrac{4.77 \tan 21.2^\circ}{25.7}$.

(*i*) $\tan \theta = \dfrac{472 \tan 11.75^\circ}{333}$.

38. Radians. Small Angles. A radian is an angular unit equal to $\left(\frac{180}{\pi}\right)^\circ$, or 57.3° accurate to three figures. The *SRT* scale is a *C* scale whose marks represent numbers of degrees ranging from 0.573° to 5.73° approximately. It is so folded that the following rule holds.

Rule. *When the hairline is set to an angle in degrees on the SRT scale, it is also set to the same angle in radians on the C* scale, provided the number on the C scale is prefixed by "0.0" as indicated by the legend* 0.01 *to* 0.1 *at the end of the SRT scale.*

For example, in accordance with the rule,

push hairline to 3.56° on *SRT*,
at hairline read 621 on *C*.

Therefore 3.56° = **0.0621 radian.**

Observe that if we multiply both members of the equation

$$3.56^\circ = 0.0621 \text{ radian}$$

by 10, 10^2, $\frac{1}{10}$, and $\frac{1}{10^2}$ in succession, we get

$$(10)\,(3.56^\circ) = (10)\,(0.0621), \text{ or } 35.6^\circ = 0.621 \text{ radian},$$
$$(100)\,(3.56^\circ) = (100)\,(0.0621), \text{ or } 356^\circ = 6.21 \text{ radians},$$
$$\left(\frac{1}{10}\right)(3.56^\circ) = \left(\frac{1}{10}\right)(0.0621), \text{ or } 0.356^\circ = 0.00621 \text{ radian},$$
$$\left(\frac{1}{100}\right)(3.56^\circ) = \left(\frac{1}{100}\right)(0.0621), \text{ or } 0.0356^\circ = 0.000621 \text{ radian}.$$

In general for any integer k, positive or negative

$$10^k(3.56^\circ) = 10^k 0.0621 \textit{ radian}.$$

Now using the rule in reverse,

push the hairline to 1176 on *C*,
at hairline read 0.674° on *SRT*,

and conclude that

$$0.01176 \text{ radian} = \mathbf{0.674^\circ}.$$

Multiplying this through by 10^2, $\frac{1}{10}$, and 10^k in succession, we get

$$1.176 \text{ radians} = 67.4^\circ,$$
$$0.001176 \text{ radian} = 0.0674^\circ,$$
$$10^k\,(0.01176) \text{ radians} = 10^k\,(0.674^\circ).$$

* Of course the *D* scale may be used instead of the *C* scale when the rule is closed.

For angles θ in radians, where θ is less than 0.1 radian (or 5.73°), the following relation holds

$$\theta \text{ radians} \cong \sin\theta \cong \tan\theta, \tag{20}$$

where the symbol "$\cong$" means "*approximately equals*". In other words, *the value of an angle in radians found by means of the italicized rule is also its sine and its tangent to slide rule accuracy.**

For example:

push hairline to 3.84° on SRT,
at hairline read 670 on C.

Therefore, in accordance with the italicized rule,

$$\sin 3.84° \cong \tan 3.84° \cong 0.0670,$$

and, in agreement with equation (20),

$$\sin 0.384° \cong \tan 0.384° \cong 0.00670,$$
$$\sin 0.0384° \cong \tan 0.0384° \cong 0.000670,$$

and so on.

By using the relations of §25, the italicized rule, and (20), we can find the values of other trigonometric functions of small angles.

For example

$$\cot 1.352° = 1/\tan 1.352° \cong 1/\sin 1.352° = \csc 1.352°.$$

Hence

push hairline to 1.352° on SRT,
at hairline read on C, $0.0236 \cong \sin 1.352°$,
at hairline read on CI, $40.31 \cong \csc 1.352° \cong \cot 1.352°$.

Also to find cos 88.76°, use (8) §25 to get

$$\cos 88.76° \cong \sin(90° - 88.76°) = \sin 1.24°,$$

push hairline to 1.24° on SRT,
at hairline read 0.0216 on C.

Therefore

$$\cos 88.76° = \sin 1.24° \cong \mathbf{0.0216}.$$

Then without moving the slide

at hairline read 462 on CI

and conclude that

$$\sec 88.76° = 1/\cos 88.76° \cong \mathbf{46.2},$$
$$\tan 88.76° = 1/\cot 88.76° \cong 1/\cos 88.76° \cong \mathbf{46.2}.$$

* The greatest error inherent in formula (20) is at $\theta = 0.1$ radian; it is nearly $+0.00017$ for sin 0.1 and -0.00033 for tan 0.1. These errors are comparable in magnitude with other errors occurring in slide rule computation.

Before beginning the exercises, the student should use the slide rule, the italicized rule of this section and (20) to verify the following approximate equations:

(*a*) $1.272° = 0.0222$ radian.
(*b*) $12.72° = 0.222$ radian.
(*c*) 0.0531 radian $= 3.04°$.
(*d*) 5.31 radians $= 304°$.
(*e*) $\sin 2.86° = 0.0499$.
(*f*) $\sin 0.286° = 0.00499$.
(*g*) $\tan 0.286° = 0.00499$.
(*h*) $\csc 0.286° = 20.0$.
(*i*) $\cot 0.286° = 20.0$.
(*j*) $\sec 87.25° = 20.8$.

EXERCISES

1. Express in radians:
(*a*) 1.416°. (*b*) 0.833°. (*c*) 2.5°. (*d*) 2.67°.

2. Express in degrees:
(*a*) 0.01823 radian. (*b*) 0.0462 radian. (*c*) 0.0865 radian.

3. Express in radians:
(*a*) 3.59°. (*b*) 0.0359°. (*c*) 35.9°. (*d*) 359°.

4. Express in degrees:
(*a*) 0.0296 radian. (*b*) 0.296 radian. (*c*) 0.000296 radian.

5. Express in radians:
(*a*) 912°. (*b*) 435°. (*c*) 0.000314°. (*d*) 2900°.

6. Find $\sin 3.42°$, $\tan 3.42°$, $\csc 3.42°$, $\cot 3.42°$.

7. Find $\sin 0.056°$, $\tan 0.056°$, $\csc 0.056°$, $\cot 0.056°$.

8. Find $\cos 89.75°$, $\sec 89.75°$, $\tan 89.75°$, $\cot 89.75°$.

9. Express in degrees the following angles expressed in radians:
(*a*) $\frac{\pi}{3}$. (*b*) $\frac{3\pi}{4}$. (*c*) $\frac{\pi}{72}$. (*d*) $\frac{\pi}{180}$. (*e*) $\frac{5\pi}{6}$.

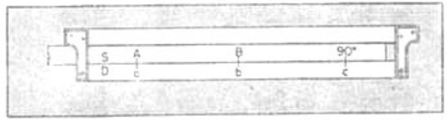
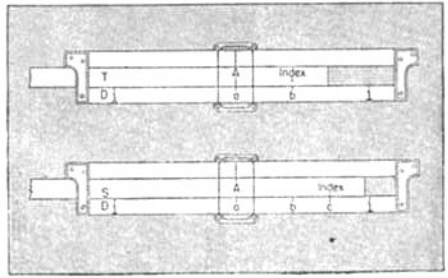
Hint: Since π radians $= 180°$, replace π by 180°. However to change $\frac{5\pi}{6}$ radians to degrees

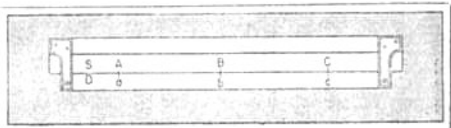
opposite 6 on *DF* set 5 of *C*,
opposite index of *D* read 15 on *SRT*, and
$5\pi/6$ radians $= 150$ degrees.

10. Evaluate the following:
(*a*) $83 \sin 0.0144°$.
(*b*) $500 \tan 0.0097°$.
(*c*) $432 \sin 0.716°$.
(*d*) $\dfrac{\tan 0.2°}{0.0001745}$.
(*e*) $\dfrac{\sin 0.3°}{0.131}$.
(*f*) $\dfrac{8 \sec 88.25°}{4.72}$.

11. The angles in the following exercises are in radians. Change these angles to degrees and then find the values of the functions:
(*a*) $\sin 0.345$. (*b*) $\tan 0.524$. (*c*) $\sin 1$. (*d*) $\cos 1$.
(*e*) $\sin 0.628$. (*f*) $\csc 2$. (*g*) $\cot 4.17$. (*h*) $\cos 2.81$.

39. Summary. The following tables summarize the methods of solution of triangles.

	Known	Solve by	In general the setting will be
RIGHT TRIANGLES	Any two parts other than two legs §28	Law of sines $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin 90°}{c}$	
	*Two legs §32	Rule of §32 or The proportion $\frac{\tan A}{a} = \frac{1}{b}$, and the law of sines	

OBLIQUE TRIANGLES	Three parts, two of which are a side and angle opposite §28	Law of sines $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	
	Two sides and the included angle §34	Dropping a perpendicular and solving the two right triangles thus formed	FIGS. 23, 24 & 25.
	Three sides §35	Law of cosines $a^2 = b^2 + c^2 - 2bc \cos A$ and law of sines	

*Also see §33.

40. Miscellaneous Exercises. Solve the following triangles:

1. $c = 80$, $A = 20°$, $C = 90°$.
2. $b = 30$, $A = 10°$, $C = 90°$.
3. $a = 80$, $A = 75°$, $C = 90°$.
4. $a = 10.11$, $b = 17.3$, $C = 90°$.
5. $a = 2$, $b = 3$, $c = 4$.
6. $a = 5.6$, $b = 4.3$, $c = 4.9$.
7. $c = 1$, $A = 36°$, $C = 90°$.
8. $a = 795$, $A = 80°$, $B = 44.7°$.
9. $a = 500$, $A = 10.2°$, $B = 46.6°$.
10. $b = 29.0$, $A = 87.7°$, $C = 33.2°$.
11. $a = 55.6$, $c = 66.7$, $C = 77.7°$.
12. $a = 51.38$, $c = 67.94$, $B = 79.2°$.
13. $a = 0.321$, $b = 0.361$, $c = 0.402$.
14. $a = 4$, $b = 7$, $c = 6$.
15. $a = 78$, $b = 83.4$, $B = 56.5°$.
16. $b = 8000$, $A = 24.5°$, $B = 86.5°$.
17. $a = 42{,}930$, $c = 73{,}480$, $C = 127.6°$.
18. $a = 61.3$, $b = 84.7$, $c = 47.6$.

19. If the sides of a triangular field are 70 ft., 110 ft., and 96 ft. long, find the angle opposite the longest side.

20. The diagonals of a parallelogram are 5 ft. and 6 ft. in length. If the angle they form is 49.12°, find the sides of the parallelogram.

21. A car is traveling at a rate of 44 ft. per second up a grade which makes an angle of 10° with the horizontal. Find how long it takes for the car to rise 200 ft.

22. A lighthouse is 16 mi. in the direction 29.5° east of north from a cliff. Another lighthouse is 12 mi. in the direction 72.8° west of south from the cliff. What is the direction of the first lighthouse from the second?

23. A 52-ft. ladder is placed 20 ft. from the foot of an inclined buttress, and reaches 46 ft. up its face. What is the inclination of the buttress?

24. If in a circle a chord of 41.36 ft. subtends an arc of 145.6°, find the radius of the circle.

41. Applications involving vectors. Since vectors are used in the solution of a great number of the problems of science, a few applications involving vectors will be considered at this time.

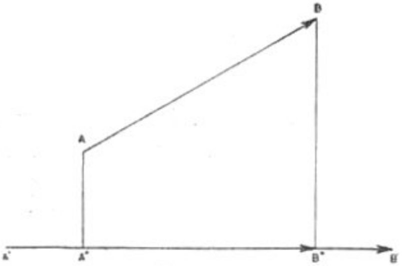


Fig. 26.

A vector $A\ B$ (see Fig. 26) is a segment of a straight line containing an arrowhead pointed toward B to indicate a direction from its initial point A to its terminal point B. The length of the segment indicates the *magnitude* of the vector and the line with attached arrowhead indicates direction. If from the ends A and B of the vector, perpendiculars be dropped to the line of a vector $A'\ B'$ and meet it in the points A'' and B'', respectively, then the vector $A''\ B''$ directed from A'' to B'' is called the component of vector AB in the direction of $A'\ B'$.

A force may be represented by a vector, the length of the vector representing the magnitude of the force, and the direction of the vector the direction of the force. In fact, many quantities defined by a magnitude and a direction can be represented by vectors.

In each of the following applications, two mutually perpendicular components of a vector are considered. Evidently these components may be thought of as the legs of a right triangle having as hypotenuse the vector itself.

For convenience the rule for solving a right triangle when two legs are given is repeated here.

Rule. *To solve a right triangle for which two legs are given,*
to larger leg on D set proper index of slide,
push hairline to smaller leg on D,
at the hairline read smaller acute angle of triangle on T,
draw this angle on S under the hairline,
at index of slide read hypotenuse on D.

Example 1. Find the magnitude and the angle of the vector representing the complex number $3.6 + j\,1.63$ where $j = \sqrt{-1}$.

Solution. If the numbers x and y be regarded as the rectangular coordinates of a point, the complex number $x + jy$ is represented by the vector from the origin to the point (x, y). Hence we must find R and θ in Fig. 27. Therefore, in accordance with the italicized rule stated above,

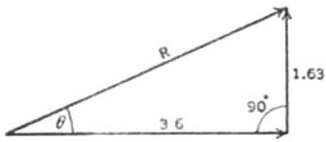


Fig. 27.

to 36 on D set right index of slide,
push hairline to 163 on D,
at the hairline read θ = **24.36°** on T,
draw 24.36° of S under the hairline,
at index of slide read R = **3.95** on D.

Example 2. A force of 26.8 lb. acts at an angle of 38° with a given direction. Find the component of the force in the given direction, and also the component in a direction perpendicular to the given one.

Solution. Denoting the required components by x and y (see Fig. 28), we write

$$\frac{26.8}{\sin 90^\circ} = \frac{y}{\sin 38^\circ} = \frac{x}{\sin 52^\circ},$$

make the corresponding setting, and read x = **21.1**, y = **16.5**.

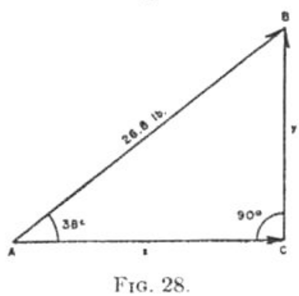


Fig. 28.

Example 3. A certain circuit consists of a resistance $R = 3.6$ and an inductive reactance $X = 2.7$ in series. Find the impedance z, the susceptance B, and the conductance G.

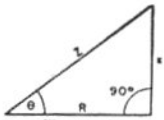


Fig. 29.

Solution. The quantities R, X and z have relations which may be read from Fig. 29. Conductance G and susceptance B are found from the relations

$$G = \frac{R}{R^2 + X^2}, \quad B = \frac{X}{R^2 + X^2},$$

or, since $z = \sqrt{R^2 + X^2}$,

$$\left.\begin{aligned} G &= \frac{R}{\sqrt{R^2 + X^2}\sqrt{R^2 + X^2}} = \frac{\cos\theta}{z}, \\ B &= \frac{x}{\sqrt{R^2 + X^2}\sqrt{R^2 + X^2}} = \frac{\sin\theta}{z}. \end{aligned}\right\} \qquad (a)$$

From equations (a) we obtain

$$\frac{z}{1} = \frac{\sin\theta}{B} = \frac{\cos\theta}{G}. \qquad (b)$$

First apply the italicized rule stated above to find z and θ of Fig. 29, and then use the proportion principle to find B and G from (b). Hence

to 36 on D set right index of slide,
push hairline to 27 on D,
at the hairline read $\theta = 36.9°$ on T,
draw 36.9° on S under the hairline,
at index of slide read **z = 4.5** on D,
draw 45 of C opposite left index of D,
push hairline to 36.9° on S,
at the hairline read B = **0.1333** on D,
push hairline to 36.9° *red* on S,
at the hairline read G = **0.178** on D.

EXERCISES

1. Find the magnitudes of the unknown vectors and of the unknown angles θ in Figs. 30, 31 and 32.

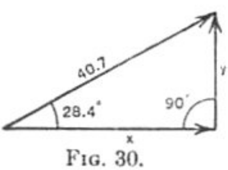


Fig. 30.

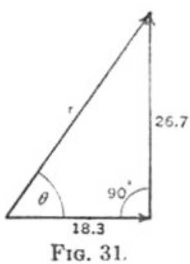


Fig. 31.

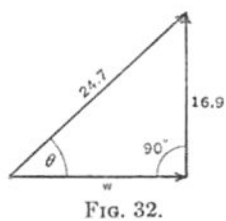


Fig. 32.

2. The rectangular components of a vector are 15.04 and 5.47 (see Fig. 33). Find the magnitude r and direction angle θ of the vector.

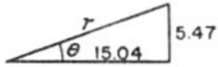


FIG. 33.

3. Find the magnitude and direction of a vector having as the horizontal and vertical components 18.12 and 8.45, respectively.

4. Find the horizontal and vertical components of a vector having magnitude 2.5 and making an angle of 16.25° with the horizontal.

5. A force of magnitude 28.8 lb. acts at an angle of 68° with the horizontal. Find its horizontal component, and its vertical component.

6. A 12-inch vector and an unknown vector r have as a resultant a 16-inch vector which makes an angle of 28° with the 12-inch vector as shown in Fig. 34. Find the unknown vector r.

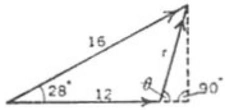


FIG. 34.

7. Find the magnitude and the angle of the vector representing the imaginary number $-2.7 + j3.6$. Hint. Use Fig. 35.

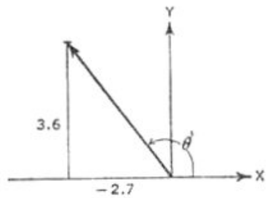


FIG. 35.

8. Through what angle θ measured counter-clockwise must a vector whose complex expression is $-10 - j5$ be rotated to bring it into coincidence with the vector whose complex expression is $3 + j4$? (See Fig. 36.)

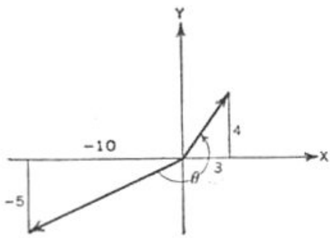


FIG. 36

9. The complex expressions for two vectors (see Fig. 37) are $v_1 = 7 - j14$ and $v_2 = -6 - j8$. From the tip of v a line is drawn perpendicular to v_1. Find the length m of this perpendicular, and the length n of the line from the origin to the foot of the perpendicular.

10. A certain circuit consists of a resistance of 8.24 ohms and an inductive reactance of 4.2 ohms, in series. Find the impedance, the susceptance, and the conductance. (See Example 3.)

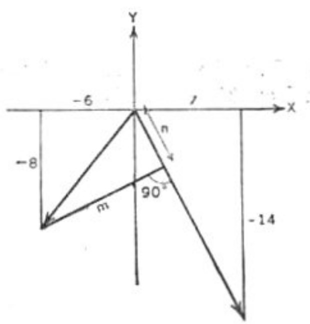


FIG. 37.

11. Find the impedance, the susceptance, and the conductance of a circuit which consists of a resistance of 8.76 ohms and an inductive reactance of 11.45 ohms in series.

42. Applications. The solutions of many practical problems are obtained by dealing with rectilinear figures. In finding the length of a specified line segment of a rectilinear figure, the beginner is likely to read a number of lengths which are not needed. This may be well at first, but the efficient operator reads and tabulates only useful numbers. The following examples and solutions indicate efficient methods of finding desired parts of rectilinear figures.

Example 1. Find the line segment marked x in Fig. 38.

Solution. By using the law of sines, we write

$$\frac{368}{\sin 39^\circ} = \frac{y}{\sin 65^\circ}, \quad \frac{y}{\sin 50^\circ} = \frac{x}{\sin 28^\circ}$$

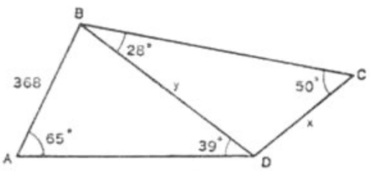


FIG. 38.

and then find x by making the following settings:
push hairline to 368 on D,
draw 39° of S under the hairline,
push hairline to 65° on S,
draw 50° of S under the hairline,
push hairline to 28° on S,
at the hairline read x = **325** on D.

The value of y was not tabulated, but it could have been read at the hairline on scale D when the hairline was set to 65° of scale S. Also it was not necessary to write the ratios; for, when one remembers that each ratio is that of a side of a triangle to the sine of the opposite angle, he has no difficulty in perceiving, from an inspection of the figure, the settings to be made.

Generally it is necessary to compute the magnitudes of a number of angles before the slide rule computation can be carried out. This process is illustrated in Example 2.

Example 2. Find the length of the side marked z in Fig. 39(a).

Solution. To find the length of the side marked z in Fig. 39 (a), first draw Fig. 39 (b), compute the angles shown in the figure, and push the hairline to 289 on D, draw 77° (= 180° − 103°) of S under the hairline,
push hairline to 32° on S,
draw 38° of S under the hairline,
push hairline to 65° on S,
draw 45° of S under the hairline,
push hairline to 77° on S,
at the hairline read z = **319** on D.

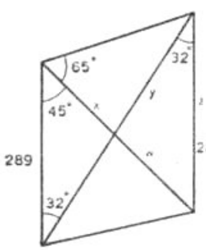


FIG. 39 (a).

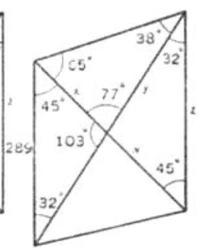


FIG. 39 (b).

In some problems it is necessary to perform some of the earlier settings in a chain of settings, compute some parts on the basis of the results, make some more settings, compute more parts, etc. This process is illustrated in Example 3.

Example 3. Find the side x of the inscribed quadrilateral shown in Fig. 40(a).

Solution. Angles Q and S are right angles because each is inscribed in a semicircle. Knowing two legs of right triangle PQR we first find its hypotenuse and then deal with triangle PSR. Accordingly
to 184 on D set left index of slide,
push hairline to 781 on D,
at the hairline read A [Fig. 40 (b)] = 23° on T,
draw 23° of S under the hairline,
compute B [Fig. 40 (b)] = 65° − A = 42°,
interchange indices (see § 6),
push hairline to 42° on S,
at the hairline read x = **13.37** on D.

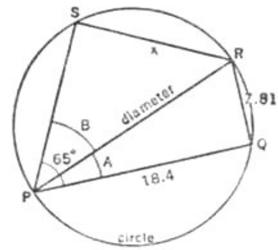


FIG. 40 (a).

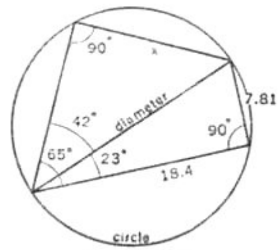


FIG. 40 (b).

The following example illustrates more in detail the same method of procedure.

Example 4. An engineer in a level country wishes to find the distance between two inaccessible points C and D and the direction of the line connecting them. He runs the line AB [Fig. 41 (a)] due north and measures the side and angles as indicated. Using his data solve his problem.

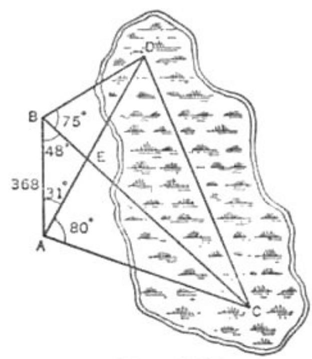


FIG. 41 (a).

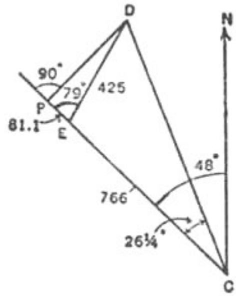


FIG. 41 (*b*).

Solution. First find EA (but do not write it), and then find $EC = 766$; afterwards find BE (but do not write it) and then $ED = 425$. In the triangle DEC [see Fig. 41 (*b*)] two sides and the included angle are now known: hence the method of §33 may be applied to it to find $DC =$ **944** and angle $ECD = 26\frac{1}{4}°$. Therefore the angle $NCD = 48° - 26\frac{1}{4}° = 21\frac{3}{4}°$, and line CD makes an angle of $21\frac{3}{4}°$ with a line directed due north. The operator may check these answers by making the suggested settings.

43. Miscellaneous Exercises.

1. Find the length of the line segment BC in Fig. 38.

2. Find the length of the line segment marked w in Fig. 39a.

3. In Fig. 42 find the length of the line segment marked x.

4. Line segment AB in Fig. 43 is horizontal and CD is vertical. Find the length of CD.

5. In the statement of Ex. 4, replace "Fig. 43" by "Fig. 44" and solve the resulting problem.

FIG. 42.

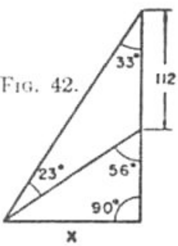


FIG. 43.

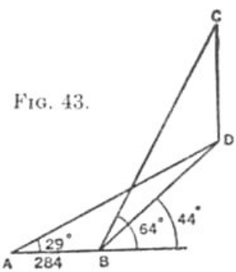
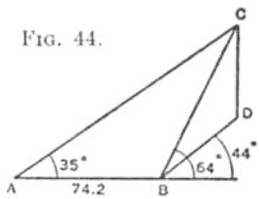


FIG. 44.



6. Find the length of the line segment marked x in Fig. 45.

7. If in Fig. 46 line segment BD is perpendicular to plane ABC, find its length.

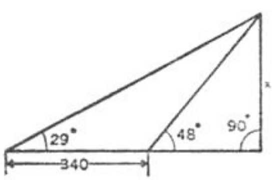


FIG. 45.

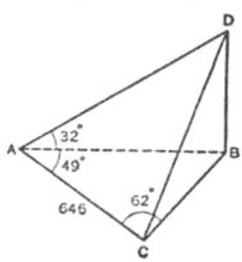


FIG. 46.

8. A tower and a monument stand on a level plane. (See Fig. 47). The angles of depression of the top and bottom of the monument viewed from the top of the tower are 13° and 31° respectively; the height of the tower is 145 ft. Find the height of the monument.

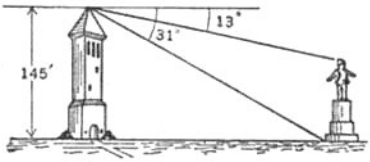


FIG. 47.

9. The captive balloon C shown in Fig. 48 is connected to a ground station A by a cable of length 842 ft. inclined 65° to the horizontal. In a vertical plane with the balloon and its station and on the opposite side of the balloon from A a target B was sighted from the balloon on a level with A. If the angle of depression of the target from the balloon is 4° find the distance from the target to a point C directly under the balloon.

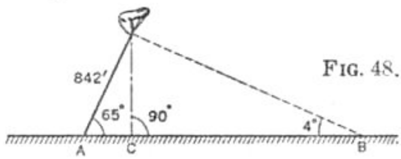


FIG. 48.

10. A lighthouse standing on the top of the cliff shown in Fig. 49 is observed from two boats A and B in a vertical plane through the lighthouse. The angle of elevation of the top of the lighthouse viewed from B is 16° and the angles of elevation of the top and bottom viewed from A are 40° and 23°, respectively. If the boats are 1320 ft. apart find the height of the lighthouse and the height of the cliff.

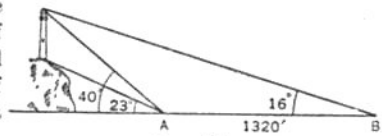


FIG. 49.

11. Fig. 50 represents a 600 ft. radio tower. AC and AD are two cables in the same vertical plane anchored at two points C and D on a level with the base of the tower. The angles made by the cables with the horizontal are 44° and 58° as indicated. Find the lengths of the cables and the distance between their anchor points.

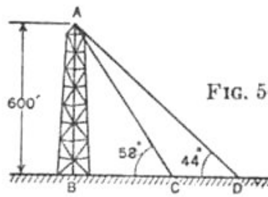


FIG. 50.

12. Two fixed objects, A and B of Fig. 51, were observed from a ship at point S to be on a straight line passing through S and bearing N 15° E. After sailing 5 miles on a course N 42° W the captain of the ship found that A bore due east and B bore N 40°E. Find the distance from A to B.

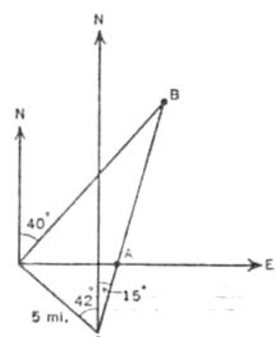


FIG. 51.

CHAPTER V

THE LOG LOG SCALES

44. Introduction. One of the most difficult problems of elementary mathematics is that of finding the value of a power of a number. By means of the scales considered in this chapter the process of doing this is as easy as the process of multiplication or division. Not only are such simple expressions as 2^5, 1.5^7, and 0.85^4 easily evaluated but also the more complex expressions such as $1.89^{4.2}$, $0.59^{1.25}$, $e^{5.6}$ and $1.036^{-0.34}$ are computed with the same ease.

In what follows we shall refer to the six scales labeled $LL1$, $LL2$, $LL3$, $LL01$, $LL02$, $LL03$ as the Log Log scales and to any one of these six as a Log Log scale. Also we shall refer to the group $LL1$, $LL2$, $LL3$ which exhibit a continuum of numbers ranging from 1.01 to 22000 as the LL scales and to the group $LL01$, $LL02$, $LL03$, which exhibit a continuum of decimal fractions ranging from 0.99 to 0.00005 as the $LL0$ scales. Thus it appears that the LL scales represent numbers greater than 1 and the $LL0$ scales represent numbers less than 1.

The next article deals with a very important basic relation among these scales.

45. Mated scales. *A pair of Log Log scales having the same number in their labels are called* **mated scales,** *and each of a pair is called the* **mate** *of the other.* The mated pairs are:

$$\begin{bmatrix} LL3 \\ LL03 \end{bmatrix} \qquad \begin{bmatrix} LL2 \\ LL02 \end{bmatrix} \qquad \begin{bmatrix} LL1 \\ LL01 \end{bmatrix}$$

The following statement indicates the relation between mated scales.

Opposite numbers on mated scales are reciprocals of each other.

Thus to find the value of $\frac{1}{2}$, the reciprocal of 2,

push the hairline to 2 on $LL2$,

at the hairline read on $LL02$, **0.500** $= \frac{1}{2}$.

To find the value of $\frac{1}{0.552}$, the reciprocal of 0.552, opposite 0.552 on $LL02$, read on $LL2$, **1.812** $= \frac{1}{0.552}$.

Note that in finding reciprocals by means of mated Log Log scales there is no question as to the position of the decimal point. A method giving reciprocals to a high degree of accuracy is discussed in §56; this method uses only scales $LL1$, $LL2$, and their mates.

EXERCISES

1. Set the hairline to 5 on $LL3$, read the reciprocal of 5 on $LL03$; set the hairline to 1.25 on $LL2$, read the reciprocal of 1.25 on $LL02$; set the hairline to 1.04 on $LL1$, read the reciprocal of 1.04 on $LL01$.

2. Using the Log Log scales find the reciprocal of each of the following:

(*a*) 16.	(*b*) 3.52.	(*c*) 0.0155.
(*d*) 1.95.	(*e*) 0.752.	(*f*) 1.163.
(*g*) 1.0142.	(*h*) 0.9515.	

46. Powers of e. The constant e (2.718 approximately) and its powers are closely associated with the Log Log scales. This constant is the base of the system of natural logarithms and it appears again and again in the mathematics of science and engineering. By using the Log Log scales in conjunction with the D scale we can find powers of e.

The Log Log scales are so constructed that when the hairline is set to a number n on scale D, it is also set to e^n on a Log Log scale.*

To use this fact in evaluating e^2,
set the hairline to 2 on D,
at the hairline read on $LL3$, **7.39** $= e^2$.

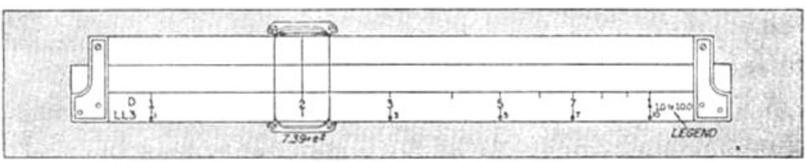


Fig. 1

The legends at the right end of the Log Log scales serve as guides to indicate the scale on which an answer is to be read. The leftmost

*This statement is fundamental; the complete development of the theory of the Log Log scales can be derived by the use of this one fact.

mark on scale $LL3$ represents e^1, the rightmost mark represents e^{10}, and the legend of scale $LL3$ is **1.0 to 10.0.** It now appears that if $e^x = N$, then the legend numbers 1.0 and 10.0 of $LL3$ are the limits of x when N is on scale $LL3$. In like manner the legend of $LL03$, **−1.0 to −10.0** is based on the exponents of e^{-1} and e^{-10} represented by the end points of the scale. The legend of each Log Log scale has a similar relation to the numbers represented on the scale. The answer **7.39** above is read on $LL3$ because the exponent 2 on e^2 is between the legend numbers of $LL3$. The rule for finding powers of e may be stated as follows:

Rule. *To find the value of e^n, set the hairline to n on D, at the hairline read the value of e^n on the Log Log scale containing n between its legend numbers.*

To gain familiarity with the process of finding powers of e, the reader should make the suggested settings and check all results:

push the hairline to 3 on D

at the hairline read $\begin{cases} \text{on } LL3,\ 20.1 = e^3, \\ \text{on } LL03,\ 0.0498 = e^{-3}; \end{cases}$

at the hairline read $\begin{cases} \text{on } LL2,\ 1.350 = e^{0.3}, \\ \text{on } LL02,\ 0.741 = e^{-0.3}; \end{cases}$

at the hairline read $\begin{cases} \text{on } LL1,\ 1.0305 = e^{0.03}, \\ \text{on } LL01,\ 0.9704 = e^{-0.03}. \end{cases}$

It appears from the italicized rule that x on the D scale and e^x on a Log Log scale are opposites. Hence we can set the hairline to e^x. on a Log Log scale* and obtain the value of x from the reading on D. Thus to find x when

$$e^x = 0.122,$$

opposite 0.122 on $LL03$ read on D, **210.**

Since 0.122 was found on $LL03$, the value of x must lie between the legend numbers −1.0 and −10.0 of scale $LL03$; hence $x =$ **−2.10.**

To become familiar with the process, find x satisfying

$$e^x = 0.002,\ e^x = 0.2,\ e^x = 2.0,\ e^x = 20,\ e^x = 200$$

by making the following settings and placing the decimal point in each by means of the appropriate legend:

*The case of operations involving numbers beyond the range of the Log Log scales is considered in §§ 54 and 55.

opposite 0.002 on $LL03$ read on D, 621 and obtain $x = -6.21$,
opposite 0.2 on $LL03$ read on D, 161 and obtain $x = -1.61$,
opposite 2.0 on $LL2$ read on D, 693 and obtain $x =$ **0.693**,
opposite 20 on $LL3$ read on D, 2995 and obtain $x =$ **2.995**,
opposite 200 on $LL3$ read on D, 530 and obtain $x =$ **5.30**.

Example 1. Evaluate $e^{3.5}$ and $e^{-3.5}$.

Solution. Push hairline to 350 on D,
at the hairline read on $LL3$, **33.1** $= e^{3.5}$,
at the hairline read on $LL03$, **0.0302** $= e^{-3.5}$.

Scale $LL3$ was selected for the value of $e^{3.5}$ because the exponent 3.5 lies on the range specified by the legend of the $LL3$ scale. Similarly scale $LL03$ was chosen because -3.5 lies on the range specified by the legend of the $LL03$ scale.

Fig. 2 shows the setting.

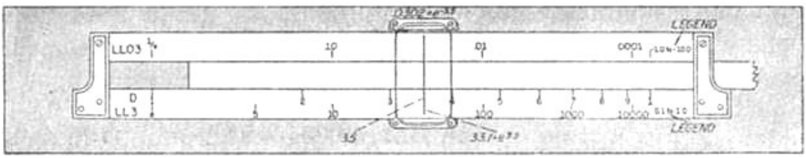


FIG. 2

Example 2. Evaluate: (*a*) e^2, e^{-2}. (*b*) $e^{0.2}$, $e^{-0.2}$. (*c*) $e^{0.02}$, $e^{-0.02}$.

Solution. Push hairline to 2 on D,

$$\text{at the hairline read} \begin{cases} \text{on } LL3,\ 7.39 = e^2, \\ \text{on } LL03,\ 0.135 = e^{-2}; \end{cases}$$

$$\text{at the hairline read} \begin{cases} \text{on } LL2,\ 1.221 = e^{0.2}, \\ \text{on } LL02,\ 0.8187 = e^{-0.2}; \end{cases}$$

$$\text{at the hairline read} \begin{cases} \text{on } LL1,\ 1.0202 = e^{0.02}, \\ \text{on } LL01,\ 0.9802 = e^{-0.02}. \end{cases}$$

Example 3. Find the value of $e^{\sin 37°}$.

Solution. Close the rule,
push hairline to 37° on S,
at the hairline read on $LL2$, **1.825** $= e^{\sin 37°}$.

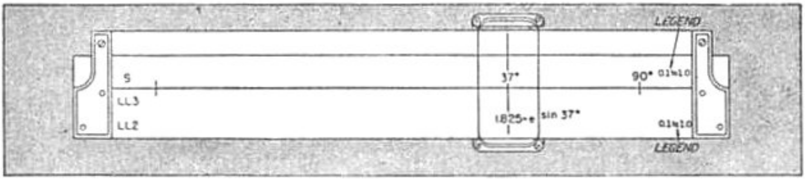


FIG. 3

The answer was read on scale $LL2$ because it is the Log Log scale which carries the same legend as that of the S scale, namely 0.1 to 1.0. Fig. 3 shows the setting.

EXERCISES

1. Evaluate:

(a) e^3.	(e) $e^{0.035}$.	(i) $e^{-2.46}$.
(b) e^{-3}.	(f) $e^{-0.035}$.	(j) $e^{3.55}$.
(c) $e^{0.4}$.	(g) $e^{1.342}$.	(k) $e^{-0.0264}$.
(d) $e^{-0.4}$.	(h) $e^{-1.342}$.	(l) $e^{0.0833}$.

2. Find e^x when: (a) $x = 2.12$. (b) $x = -2.12$. (c) $x = 0.212$. (d) $x = -0.212$. (e) $x = 0.0212$. (f) $x = -0.0212$.

3. Evaluate:

(a) e^4.	(d) $e^{0.0214}$.	(g) $e^{-0.0185}$.
(b) $e^{8.2}$.	(e) $e^{-3.4}$.	(h) $e^{-6.2}$.
(c) $e^{0.43}$.	(f) $e^{-0.163}$.	

4. Evaluate:

(a) $e^{\sin 45°}$.	(f) $e^{-\tan 40°}$.	(k) $e^{\cot 60°}$.
(b) $e^{-\sin 45°}$.	(g) $e^{\sqrt{5}}$.	(l) $e^{\cot 120°}$.
(c) $e^{-\cos 65°}$.	(h) $e^{-\sqrt{5}}$.	(m) $e^{\sqrt[3]{33.3}}$.
(d) $e^{\cos 65°}$.	(i) $e^{-\sqrt{0.142}}$.	(n) $21e^{\tan 12°}$.
(e) $e^{\tan 40°}$.	(j) $e^{\sqrt{0.142}}$.	(o) $e^{1.8 \sin 26°}$.

5. Find the value of x from each of the following equations:

(a) $e^x = 1.974$.	(d) $e^x = 1.270$.
(b) $e^x = 0.3362$.	(e) $e^x = 0.346$.
(c) $e^x = 6.54$.	(f) $e^x = 0.945$.

6. The damping factor f for a certain oscillatory motion is given by the formula

$$f = e^{-0.04t},$$

where t is the time in seconds. Find the time elapsed while the damping factor changes from 1 to $\frac{1}{2}$.

7. The three hyperbolic functions of x most frequently used are called hyperbolic sine of x, hyperbolic cosine of x, and hyperbolic tangent of x. They are designated by $\sinh x$, $\cosh x$, and $\tanh x$ respectively, and are defined by

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2}, \quad \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^{2x}-1}{e^{2x}+1}.$$

Using the above definitions evaluate the following:

(a) sinh 1.04. (d) sinh 0.75.
(b) cosh 2.52. (e) cosh 0.55.
(c) tanh 1.41. (f) tanh 0.63.

8. The formulas for the length l and the sag s (see Fig. 4) of a uniform chain hung from two points on the same level are

$$l = 2\frac{H}{w}\sinh\frac{w\,b}{H}, \qquad s = \frac{H}{w}\left(\cosh\frac{w\,b}{H} - 1\right),$$

where w is the wt. per ft. of the chain, H is the tension at the lowest point, and $2b$ is the distance between the points of suspension. Using the definitions for the $\cosh x$ and the $\sinh x$ as given in Exercise 7, find the length l and the sag s if $w = 2$ lb./ft., $H = 26$ lb., and $b = 30$ ft.

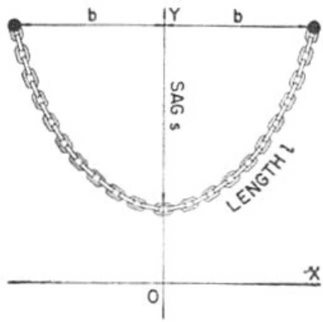


FIG. 4

47. Powers of any number. Elementary. In § 46 powers of e were found. A similar method enables us to find powers of any number. To illustrate the process consider the problem of finding the value of 3^2 and of 3^{-2}.

Push hairline to 3 on $LL3$,
draw left index of C under hairline,
push the hairline to 2 on C,
at hairline read $\begin{cases} \text{on } LL3,\ 9 = 3^2, \\ \text{on } LL03,\ 0.1111 = 3^{-2}. \end{cases}$

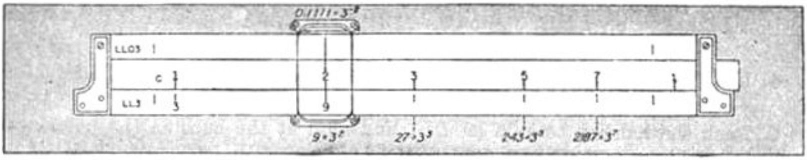


Fig. 5

Fig. 5 represents the setting. Without changing the position of your slide push the hairline to 3, 5 and 7 on C and at the hairline read on $LL3$, $27 = 3^3$, **243** $= 3^5$, and **2187** $= 3^7$ respectively.

The following rule for the process just illustrated will be helpful at first.

Rule. *To raise a number d to a power n set the index of C to d on a Log Log scale, opposite n on C read d^n on a Log Log scale. The CF scale may be used instead of the C scale in the process.*

In this article the scale for each answer will be specified. The process of determining the scale on which an answer is to be read will be discussed in the next article.

Example. Evaluate the following:

$$3^4,\ 3^{-4},\ 0.2^5,\ 0.2^{-5},\ 1.25^{0.932},\ 1.25^{-0.932},\ 0.9615^{12.5},\ 0.9615^{-12.5}.$$

Solution. Push left index of C to 3 on $LL3$,
push the hairline to 4 on C,
at the hairline read $\begin{cases}\text{on } LL3,\ \mathbf{81} = 3^4, \\ \text{on } LL03,\ \mathbf{0.0123} = 3^{-4};\end{cases}$
push left index of C to 0.2 on $LL03$,
opposite 5 on C read $\begin{cases}\text{on } LL03,\ \mathbf{0.00032} = 0.2^5, \\ \text{on } LL3,\ \mathbf{3100} = 0.2^{-5};\end{cases}$
push right index of C to 1.25 on $LL2$,
opposite 932 on C read $\begin{cases}\text{on } LL2,\ \mathbf{1.231} = 1.25^{0.932}, \\ \text{on } LL02,\ \mathbf{0.8124} = 1.25^{-0.932};\end{cases}$
push left index of C to 0.9615 on $LL01$,
opposite 125 on C read $\begin{cases}\text{on } LL02,\ \mathbf{0.612} = (0.9615)^{12.5}, \\ \text{on } LL2,\ \mathbf{1.633} = (0.9615)^{-12.5}.\end{cases}$

EXERCISES

1. Using the settings indicated by the italicized rule find on $LL3$ the values of 2^2, 5^2, 7^2, 8^2 and on $LL03$ the values of 2^{-2}, 5^{-2}, 7^{-2}, 8^{-2}.

2. Using the settings indicated in the italicized rule find on $LL02$ the values of $0.88^{2.1}$, $0.80^{2.1}$, $0.65^{2.1}$, and on $LL2$ the values of $0.88^{-2.1}$, $0.80^{-2.1}$, $0.65^{-2.1}$.

3. Push the hairline to 3 on $LL3$, at the hairline read the values of: 3^{-1} on $LL03$, $3^{0.1}$ on $LL2$, $3^{-0.1}$ on $LL02$, $3^{0.01}$ on $LL1$, $3^{-0.01}$ on $LL01$.

4. Use the rule of this article to find: 3^2, 3^{-2}, 4^2, 4^{-2}, 6^2, 6^{-2}, 9^2, 9^{-2}, 5.5^3, 5.5^{-3}.

5. Push the hairline to 1.06 on $LL1$ and read at the hairline the values of: 1.06^{-1} on $LL01$, 1.06^{10} on $LL2$, 1.06^{-10} on $LL02$, 1.06^{100} on $LL3$, 1.06^{-100} on $LL03$.

6. Push the hairline to 0.1 on $LL03$, at the hairline read the values of: $(0.1)^{-1}$ on $LL3$, $(0.1)^{0.1}$ on $LL02$, $(0.1)^{-0.1}$ on $LL2$, $(0.1)^{0.01}$ on $LL01$, $(0.1)^{-0.01}$ on $LL1$.

7. Push the hairline to 25 on $LL3$, draw the right index of C under the hairline, push hairline to 5 on C, at the hairline read the values of:
$25^{0.5}$ on $LL3$, $25^{-0.5}$ on $LL03$, $25^{0.05}$ on $LL2$, $25^{-0.05}$ on $LL02$.

8. Set index of C opposite 0.84 on $LL02$, push hairline to 3 on C and read the values of $(0.84)^3$ on $LL02$, $(0.84)^{-3}$ on $LL2$, $(0.84)^{30}$ on $LL03$, $(0.84)^{-30}$ on $LL3$, $(0.84)^{0.3}$ on $LL01$, $(0.84)^{-0.3}$ on $LL1$.

48. Powers of any number. General. In the problems of § 47 the scale on which each answer is to be read is specified. In this article we discuss the complete process of finding any power of any number provided that no step in the process involves numbers beyond the range of the Log Log scales. In §§ 54 and 55 the problem of dealing with numbers beyond the range of the Log Log scales is considered.

The following example illustrates important relations involved in the process of raising a number to a power. The operator should make all indicated settings, check the results, and read carefully the comments on the solution.

Example 1. Find the values of 9 raised to the following powers: 0.545, 2.13, −2.13, 0.213, −0.213, 0.0213, and −0.0213.

Solution.

Opposite 9 on $LL3$ draw the index 1 of CF,
push hairline to 545 on CF,
at the hairline read on $LL3$, **3.32** $= 9^{0.545}$,
push hairline to 213 on CF,
at the hairline read on $LL3$, **108** $= 9^{2.13}$.
Also read on $LL03$, **0.0093** $= 9^{-2.13}$;
on $LL2$, **1.597** $= 9^{0.213}$; on $LL02$, **0.626** $= 9^{-0.213}$;
on $LL1$, **1.0479** $= 9^{0.0213}$ on $LL01$, **0.954** $= 9^{-0.0213}$.

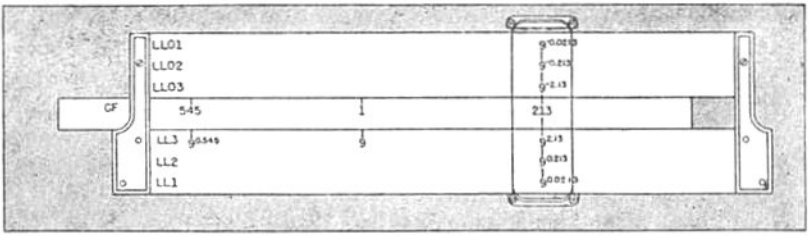


FIG. 6

Comments. Fig. 6 is a diagram of the solution. First observe that $9^{0.545}$ is on $LL3$ to the left of 9 on $LL3$, and that $9^{2.13}$ is on $LL3$ to the right of 9. Next observe that the last six exponents in the example have the same significant digits, and that the position of the decimal point in the exponent of 9 for readings on successive scales in either of the groups $LL1$, $LL2$, $LL3$ or $LL01$, $LL02$, $LL03$ differs by one place, *rightward displacement of the decimal point going with increasing scale number and leftward displacement with decreasing scale number.* Finally observe that powers having exponents equal but opposite in sign are on mated scales.

The italicized rule of § 47 applies generally and the comments on the example call attention to fundamental relations. The following statements, numbered (I), (II), and (III) for convenience of reference, are basic in the process of scale determination and location of the decimal point.

(I) If N is a number on any Log Log scale then any number to the right of N on the same scale is a power of N having an exponent between 1 and 10, and any number to the left of N on the same scale is a power of N having an exponent between 0.1 and 1. This relation is illustrated by powers of numbers on the $LL3$ scale of Fig. 6.

(II) If at the hairline on two adjacent scales in either of the groups $LL1$, $LL2$, $LL3$, or $LL01$, $LL02$, $LL03$, are found d^m on the lower numbered scale and d^n on the higher numbered scale, then the exponents m and n have the same significant digits and the decimal point in n is one place farther to the right than the decimal point in m. Fig. 6 illustrates this relation.

(III) If the hairline is set to N^m on any Log Log scale, it is set to N^{-m} on the mate of that scale. Fig. 6 illustrates this relation also.

Example. 2. Evaluate

(*a*) $(5.27)^{0.044}$. (*b*) $(0.955)^{186.3}$. (*c*) $(1.456)^{-0.054}$.

Solution. (*a*) Draw left index of C opposite 5.27 on $LL3$,
push hairline to 440 on C,
at hairline read on $LL1$, **1.0759** $= (5.27)^{0.044}$.

Note that, after the setting was made, in accordance with (I) $5.27^{4.4}$ was on $LL3$ at the hairline, and then by relation (II) $5.27^{0.44}$ was on $LL2$, and $5.27^{0.044}$ was on $LL1$. Fig. 7 shows the setting.

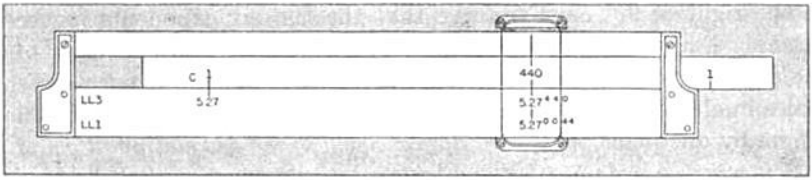


Fig. 7

Solution. (*b*) Draw left index of C opposite 0.955 on $LL01$,
push hairline to 1863 on C,
at hairline read on $LL03$, **0.000188** $= (0.955)^{186.3}$.

Note that, after the setting was made, in accordance with (I) $0.955^{1.863}$ was on $LL01$ at the hairline, and then by relation (II) $0.955^{18.63}$ was on $LL02$ and $\mathbf{0.955^{186.3}}$ was on $LL03$. Fig. 8 shows the setting.

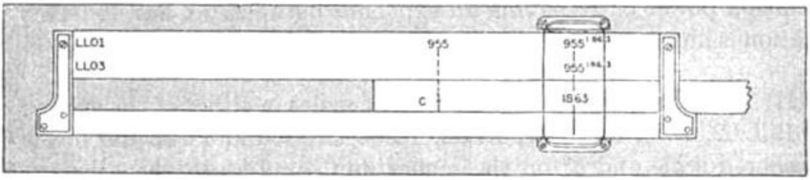


Fig. 8

Solution. (*c*) Set right index of C opposite 1.456 on $LL2$,
push hairline to 540 on C,
at hairline read on $LL01$, **0.9799** $= (1.456)^{-0.054}$.

Note that, after the setting was made, $1.456^{-0.54}$ was at the hairline on $LL02$ by relations (III) and (I), and then by relation (II) that $1.456^{-0.054}$ was at the hairline on $LL01$. Fig. 9 shows the setting.

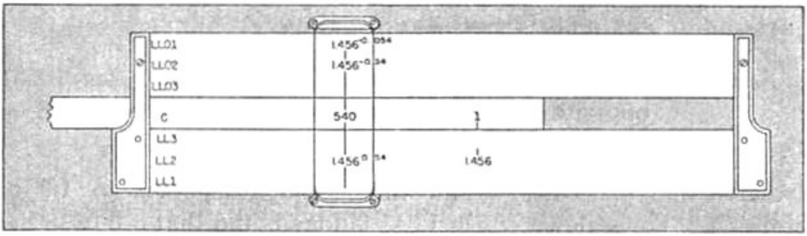


FIG. 9

EXERCISES

1. Find the values of: $(1.056)^{0.85}$, $(1.056)^{8.5}$, $(1.056)^{-85}$.

2. Find the values of:

(*a*)	$1.03^{1.81}$.	$1.03^{-1.81}$.	$1.03^{-18.1}$.
(*b*)	$8.55^{1.81}$.	$8.55^{-1.81}$.	$8.55^{0.0181}$.
(*c*)	$0.77^{2.11}$.	$0.77^{21.1}$.	$0.77^{-0.211}$.
(*d*)	$0.224^{0.843}$.	$0.224^{-0.00843}$.	$0.224^{-0.843}$.

Evaluate the following expressions:

3. $1.03^{18.1}$.	12. $74^{0.004}$.	21. $9^{\sin 30°}$.
4. $1.0163^{57.5}$.	13. $74^{-0.04}$.	22. $9^{-\sin 30°}$.
5. $8.55^{0.18}$.	14. 0.74^{10}.	23. $5^{\cos 28°}$.
6. $0.98^{1.783}$.	15. $0.74^{-2.88}$.	24. $5^{-\cos 28°}$.
7. $0.98^{178.3}$.	16. $0.74^{0.657}$.	25. $1.6^{\sqrt[3]{5}}$.
8. $0.98^{-178.3}$.	17. $1.035^{-1.685}$.	26. $0.675^{\sqrt[3]{0.00465}}$.
9. $2.72^{2.43}$.	18. $1.035^{168.5}$.	27. 0.983^{150}.
10. $2.72^{-2.43}$.	19. $1.035^{-0.865}$.	28. 0.983^{-15}.
11. $74^{0.04}$.	20. 0.988^{243}.	29. $1.0325^{42.5}$.

30. A flyer's chance of returning safely from n missions is 0.96^n. Find the chance that he will return safely from:

(*a*) 100 missions. (*b*) 50 missions. (*c*) 15 missions.

49. Logarithms to the base e. In the equation

$$e^x = N^*$$

x is the logarithm of N to the base e. Since by § 46, x on D and e^x on that Log Log scale having a legend containing x between its numbers are opposites, the following rule applies to find logarithms to the base e:

*The equations $e^x = N$ and $\log_e N = x$ are equivalent and together constitute the definition of the natural logarithm of N.

Rule. *To find $\log_e N$, set the hairline to N found on a Log Log scale, read the number on scale D at the hairline, and so place the decimal point in this number that the result lies between the legend numbers of the Log Log scale used.*

Example. Find $\log_e 1.345$ and $\log_e 0.9196$.

Solution. Opposite 1.345 on $LL2$ obtain on C, $0.296 = \log_e 1.345$, opposite 0.9196 on $LL01$ obtain on C, $-0.0838 = \log_e 0.9196$.

Observe that 0.296 lies within the legend range **0.1 to 1.0** of scale $LL2$, the scale on which 1.345 is found, and that -0.0838 lies within the legend range **-0.01 to -0.1** of scale $LL01$, the scale on which 0.9196 is found.

EXERCISES

1. Find the logarithms to the base e of: 500, 50, 2, 1.4, 1.043.

2. Find the logarithms to the base e of: 0.002, 0.02, 0.5, 0.714, 0.9091, 0.9804.

3. Set the hairline to 3 on D and read in order the six numbers which have the respective logarithms to the base e: 3, 0.3, 0.03, -3, -0.3, and -0.03.

4. Find the values of:

(*a*) $\log_e 76$.	(*d*) $\log_e 0.84$.	(*g*) $\log_e 0.909$.
(*b*) $\log_e 7.6$.	(*e*) $\log_e 0.145$.	(*h*) $\log_e 1.43$.
(*c*) $\log_e 9.2$.	(*f*) $\log_e 0.893$.	(*i*) $\log_e 1.043$.

5. Find the numbers which have the following logarithms to the base e:

(*a*) 3.4.	(*d*) -0.36.	(*g*) -1.058.
(*b*) 0.34.	(*e*) -0.55.	(*h*) 2.22.
(*c*) -1.74.	(*f*) 1.058.	(*i*) -0.113.

6. Show that the first three significant digits in the logarithms to the base e of the following numbers are 693: 1.0718, 0.9330, 2, 0.5, 1024, 0.000977.

7. (*a*) Find $\log_e 4$. (*b*) Find five numbers other than 4 each having as its logarithm to the base e the same first three significant figures as $\log_e 4$.

50. Logarithms to any base. The logarithm of N to the base d is the exponent x in the equation

$$d^x = N. \tag{1}$$

Thus if $x = \log_3 81$, then $3^x = 81$; since $3^4 = 81$, it appears that $\log_3 81 = 4$.

In accordance with § 47 when the index of C is opposite a number d on a Log Log scale then d^x on a Log Log scale is opposite x on C; using this fact we can solve (1) in which d and N are given, to find x the required logarithm. The following rule may be helpful.

Rule. *To find $\log_d N$, set the index of C to d on a Log Log scale, push the hairline to N on a Log Log scale, at the hairline read $\log_d N$ on C. Place the decimal point in accordance with the statements (I) (II) and (III) of* § 48.

Example. Find: (*a*) $\log_3 81$. (*b*) $\log_{0.623} 0.9718$. (*c*) $\log_8 0.726$.

Solution. (*a*) From $L = \log_3 81$, $3^L = 81$. Hence
set left index of C opposite 3 on $LL3$,
push hairline to 81 on $LL3$,
at hairline on C read $4 = L$.

The position of the decimal point could have been found by inspection, or by relation (I) § 48.

Solution. (*b*) From $L = \log_{0.623} 0.9718$, $0.623^L = 0.9718$. Hence
set right index of C opposite 0.623 on $LL02$,
push hairline to 0.9718 on $LL01$,
at hairline read on C, $\mathbf{0.0605} = \log_{0.623} 0.9718$.

The decimal point was placed in accord with relations (I) and (II) of § 48.

Solution. (*c*) From $L = \log_8 0.726$, $8^L = 0.726$. Hence
set left index of C opposite 8 on $LL3$,
push hairline to 0.726 on $LL02$,
at hairline read on C, $L = \mathbf{-0.154}$.

The position of the decimal point was found in accordance with statements (I), (II), and (III) of § 48.

EXERCISES

Find the value of L in each of the equations numbered 1 to 18:

1. $L = \log_7 100$.
2. $L = \log_{27} 81$.
3. $L = \log_2 32$.
4. $L = \log_{4.4} 0.471$.
5. $L = \log_{0.650} 0.962$.
6. $L = \log_e 1.682$.
7. $L = \log_{10} 27.2$.
8. $L = \log_{10} 49{,}600$.
9. $L = \log_{10} 343{,}000$.
10. $L = \log_{3.1} 7.34$.
11. $L = \log_e 10$.
12. $L = \log_{0.2} 0.8$.
13. $L = \log_{1.05} 0.8$.
14. $L = \log_{0.95} 3.47$.
15. $L = \log_{0.93} 25.7$.
16. $\log_L 10 = 2.78$.
17. $\log_L 68 = 0.91$.
18. $\log_{10} 1.015 = L$.

19. Find the logarithm to the base 3.34 of each of the following numbers: 42.5, 167, 0.96, 0.267, 0.045.

20. Find the logarithm to the base 0.45 of each of the following numbers: 0.682, 50, 100, 0.945.

51. Continuous relation of C scale to Log Log scales. The discussion of this article will explain basic continuous relations between the C scale and the Log Log scales from a new point of view.

Imagine the LL scales to be placed end to end in one continuous scale opposite to similarly placed $LL0$ scales and to four lengths of the C scale. Fig. 10 shows the arrangement in skeleton form for a special case. In Fig. 10 the index of the C scale is set opposite 1.015 of the $LL1$ scale and hence opposite $0.9852 = 1.015^{-1}$ of the $LL01$ scale. Note that the numbers on the successive C scales must then be considered as varying from 0.1 to 1, from 1 to 10, from 10 to 100, and from 100 to 1000.

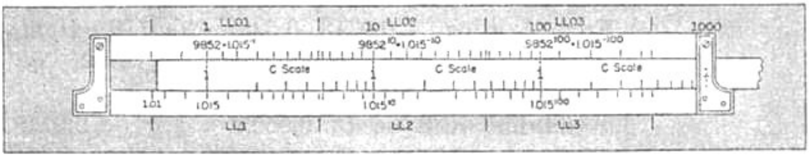


Fig. 10

The index 1 of C is opposite 1.015 on $LL1$. Also opposite numbers on scale C and the Log Log scales have the relations of 1.015^a on an LL scale, a on C, and 1.015^{-a} on an $LL0$ scale. A visualization of the general continuous scale relation just illustrated is useful in the process of scale determination and location of the decimal point.

52. The proportion principle for Log Log scales. Fig. 11 indicates a slide rule with an index of the C scale set opposite N on an LL scale, n on C opposite P on LL, and m on C opposite Q on LL. Applying the principle of § 48 we get from Fig. 11

$$P = N^n, \quad Q = N^m.$$

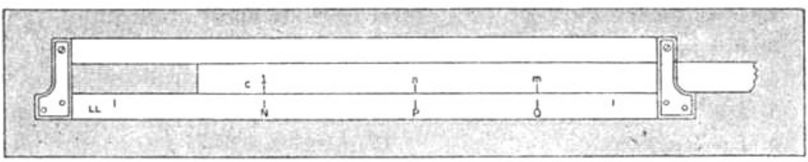


Fig. 11

From these equations we get $\log P = n \log N$, $\log Q = m \log N$, or, equating the values of $\log N$,

$$\frac{\log P}{n} = \frac{\log Q}{m}. \ * \tag{2}$$

*This relation holds, whatever base of logarithms is used. The reader may well think of e as the base.

Hence when three of the quantities in a proportion of the form (2) are known, the fourth one can be found. The decimal point is placed in accordance with the relations between numbers on the Log Log scales and the C scale set forth in § § 48 and 51.

Example 1. Find x in the proportion

$$\frac{LL3}{C} : \frac{\log 3.84}{3} = \frac{\log 9.63}{x}.$$

Solution. Set 3 of C opposite 3.84 on $LL3$,
push the hairline to 9.63 on $LL3$,
at the hairline on C read **5.05** $= x$.

The decimal point in 5.05 was placed in accord with the continuous relation, explained in § 51, between the C scale and the $LL3$ scale.

Example 2. Find the value of $x = 8.32^{7.2/2.8}$.

Solution. Equate the logarithms of the two members to obtain

$$\log x = \frac{7.2}{2.8} \log 8.32,$$

or

$$\frac{LL3}{C} : \frac{\log x}{7.2} = \frac{\log 8.32}{2.8},$$

and

opposite 8.32 on $LL3$ set 28 on C,
opposite 72 of C read on $LL3$, **232** $= x$.

Here the relation (I) of § 48 indicates that x is to be read on $LL3$.

Example 3. Find x from $x = \sqrt[51]{0.8^{6.4}} = (0.8)^{6.4/51}$.

Solution. Equate the logarithms of the two members and obtain

$$\log x = \frac{6.4}{51} \log 0.8,$$

or

$$\frac{LL0}{C} : \frac{\log x}{6.4} = \frac{\log 0.8}{51},$$

and opposite 0.8 on $LL02$ set 51 of C,
opposite 64 on C read on $LL01$, **0.9724** $= x$.

A brief study of Fig. 12 will show the reason for reading the result on $LL01$. Whenever the operator is in doubt as to the position of a decimal point or the scale to be used for a reading, he should draw a rough sketch like Fig. 10 showing essential relations.

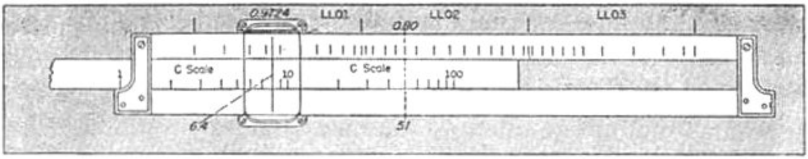


FIG. 12

Example 4. Solve $0.72^x = 28.7^{1.34}$.

Solution. Equate the logarithms of the two members to obtain $x \log 0.72 = 1.34 \log 28.7$,

or

$$\frac{\log 0.72}{1.34} = \frac{\log 28.7}{x},$$

and

opposite 0.72 on $LL02$ set 1.34 of C,
opposite 28.7 on $LL3$ read on C, $-$ **13.7** $= x$.

The minus sign on 13.7 was chosen because 0.72, being less than 1, has a negative logarithm to base e and 28.7, being greater than 1, has a positive logarithm to base e. A brief study of Fig. 13 in the light of § 51 will indicate the reason for the position of the decimal point in 13.7.

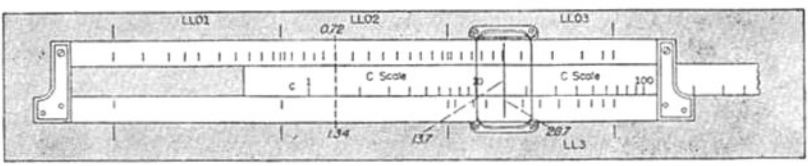


FIG. 13

EXERCISES

1. $3.82^{\frac{3.82}{1.71}}$.
2. $148^{\frac{72}{343}}$.
3. $7.545^{0.38}$.
4. $1.177^{5.2}$.
5. $1.0733^{\frac{587}{62.9}}$.
6. $0.476^{3/7}$.
7. $0.865^{\frac{42.6}{2.17}}$.
8. $0.865^{\frac{4.26}{2.17}}$.
9. $0.953^{\frac{0.851}{2.64}}$.
10. $0.953^{\frac{5.51}{2.64}}$.
11. $0.953^{\frac{85.1}{2.64}}$.
12. $0.0553^{\frac{2.80}{1.49}}$.
13. $0.0553^{\frac{0.280}{1.49}}$.
14. $0.0553^{\frac{0.280}{14.9}}$.
15. $1.456^{\frac{28.6}{1.172}}$.
16. $1.456^{\frac{2.86}{1.172}}$.
17. $1.456^{\frac{2.86}{11.72}}$.
18. $53.6^{\frac{2.763}{3.07}}$.

Solve the following equations for the unknown quantities:

19. $1.14^{0.72} = x$.
20. $4.02^{x} = 8.4$.
21. $12^{y} = 7.137$.
22. $81^{t} = 10$.
23. $y^{2.14} = 4140$.
24. $2.06^{3t} = 12$.
25. $4.02 = (2.37)^{\frac{1}{y-1}}$.
26. $S^{5.32} = 9.96$.
27. $2.37^{\frac{t}{5.3}} = 17.4$.
28. $x^{1.732} = 24.6$.

Solve each equation for x:

29. $0.311^{x} = 10.2$.
30. $5.75^{x} = 0.556$.
31. $1.043^{x} = 0.759$.
32. $0.955^{x} = 25.9$.
33. $x^{-2.14} = 0.617$.
34. $0.435^{x} = 1.475$.
35. $1.056^{-2/x} = 0.623$.
36. $0.054^{-5/x} = 1.355$.

53. Powers-of-ten notation. Problems of science often involve very large numbers and very small numbers. To gain a quick understanding of, and ease in combining such numbers, a computer often writes them in the form

$$m \times 10^{k} \tag{3}$$

where m is an ordinary number, generally 1 or a number between 1 and 10, and k is an integer positive or negative.

Illustrations of numbers in this form are:

$$384 = 3.84 \times 10^{2}, \qquad 384000000 = 3.84 \times 10^{8},$$
$$0.0384 = 3.84 \times 10^{-2}, \qquad 0.0000384 = 3.84 \times 10^{-5}.$$

Observe that *in powers-of-ten notation the factor* 10^{k} *has the effect of moving the decimal point* k *decimal places rightward or leftward according as* k *is a positive or a negative integer.*

In combining numbers as in (3), the following laws of exponents apply:

$$10^{k} \times 10^{r} = 10^{k+r}, \quad \frac{10^{k}}{10^{r}} = 10^{k-r}, \quad (10^{k})^{r} = 10^{kr}.$$

For example

$$\frac{832000 \times 0.00324}{5230} = \frac{8.32 \times 10^{5} \times 3.24 \times 10^{-3}}{5.23 \times 10^{3}}$$

$$= \frac{8.32 \times 3.24}{5.23} \times 10^{5+(-3)-3} = 5.15 \times 10^{-1} = \mathbf{0.515}$$

and

$$\frac{(160000)^{5} \times (0.00000195)^{6}}{(0.00545)^{2}} = \frac{(1.6 \times 10^{5})^{5} \times (1.95 \times 10^{-6})^{6}}{(5.45 \times 10^{-3})^{2}}$$

$$= \frac{(1.6)^{5}\,(1.95)^{6}}{(5.45)^{2}} \times 10^{5(5)+6(-6)-2(-3)} = \frac{10.49 \times 55.0 \times 10^{-5}}{29.7}$$

$$= 19.4 \times 10^{-5} = 1.94 \times 10^{-4} = \mathbf{0.000194}.$$

EXERCISES

1. Express in the powers-of-ten notation:

(a) 5860. (b) 675000. (c) 0.0623. (d) 0.479. (e) 28 million. (f) 2.76 billion. (g) 0.00000091. (h) 0.00495×10^6. (i) 8645×10^{-6}.

2. Simplify, and give the answer in the powers-of-ten notation:

(a) $(3 \times 10^2)(6 \times 10^4)$. (b) $(7 \times 10^{-3})^3$. (c) $(3 \times 10^{-2})^3$. (d) $(5 \times 10^{-6})(7 \times 10^4)^2$. (e) $\sqrt{64 \times 10^8}$. (f) $\sqrt[3]{8 \times 10^{-12}}$.

3. To make each of the following evaluations, express the numbers involved in the powers-of-ten notation, simplify, make slide rule computations, and finally write the answer in the powers-of-ten notation:

(a) $(8.31 \times 10^{-4})^2$. (b) $(7.45 \times 10^5)^2 \sqrt{2.65 \times 10^{-4}}$. (c) $\dfrac{\sqrt{3.68 \times 10^6}}{\sqrt{5.12 \times 10^{-4}}}$. (d) $(3.16 \times 10^4)^{0.75}$. (e) $\dfrac{(2.30 \times 10^2)^3 (1.42 \times 10^{-3})^4}{(1.96 \times 10^{-3})^2}$. (f) $\sqrt{61 \times 10^2 \times 5.2 \times 10^{-4}}$.

4. Evaluate and give the answer in the powers-of-ten notation:

(a) $\sqrt[3]{64 \times 10^3 \times 0.125 \times 10^{-5}}$. (b) $6.8 \times 10^4 \sqrt{2.96 \times 10^{-4}}$. (c) $\dfrac{\sqrt{72.4 \times 10^3}}{\sqrt{0.00042 \times 0.0683}}$. (d) $\sqrt[4]{25600 \times 0.00816}$. (e) $\dfrac{(0.000659)^{1/4}}{0.00673}$. (f) $\dfrac{(2675000)^{5/16}}{(0.00234)^{1/5}}$.

5. A *light year* is the distance traveled by light in a year. If the speed of light is 186,300 mi/sec., find in the powers-of-ten notation the number of miles in a light year.

54. Numbers less than 0.00005 or greater than 22000. The methods developed in this article and the next are to be used when the operator, in attempting to solve a problem by previously discussed methods, finds that a required reading involves an extension of the Log Log scales. These scales represent numbers from 0.00005 to 22000 except for a gap between 0.99 and 1.01. This discussion applies to the computation of numbers outside this range. This article will consider methods to be applied for very large numbers or very small numbers.

The method of attack in finding powers d^m of numbers is to write the base d in the powers-of-ten notation, then, by using the law of exponents, resolve the power into several parts, one an integral power of 10 and the others within the range of the rule, and finally make the computation. The following example will illustrate the method.

Example. Evaluate: (*a*) $24^{5.32}$. (*b*) $247^{5.32}$. (*c*) $(0.0000042)^{2.31}$.

Solution. (*a*) $24^{5.32} = (2.4 \times 10)^{5.32} = 2.4^{5.32} \times 10^{0.32} \times 10^5$.

Hence

to 2.4 on $LL2$ set right index of C,
opposite 532 on C read on $LL3$, **105.4** $= 2.4^{5.32}$,
to 10 on $LL3$ set left index of C,
opposite 320 on C read on $LL2$, **2.09** $= 10^{0.32}$.*

Therefore

$$24^{5.32} = 105.4 \times 2.09 \times 10^5 = 220.5 \times 10^5 = \mathbf{2.205 \times 10^7}.$$

(*b*) $247^{5.32} = (2.47 \times 10^2)^{5.32} = 2.47^{5.32} \times 10^{10.64} = 2.47^{5.32} \times 10^{0.64} \times 10^{10} = 122.8 \times 4.37 \times 10^{10} = 536 \times 10^{10} = \mathbf{5.36 \times 10^{12}}$.

(*c*) $(0.0000042)^{2.31} = (4.2 \times 10^{-6})^{2.31} = 4.2^{2.31} \times 10^{-13.86} = 4.2^{2.31} \times 10^{-0.86} \times 10^{-13} = 27.6 \times 0.138 \times 10^{-13} = \mathbf{3.81 \times 10^{-13}}$.

To find the factor $10^{-0.86}$, either

set left index of C to 10 on $LL3$,
opposite 86 on C read on $LL03$, **0.138**,

or

write $\log_{10} 10^{-0.86} = -0.86 = 9.14 - 10$ and
opposite 14 on L read on D, **0.138**.

EXERCISES

1. 125.6^4. *Hint*: $(1.256 \times 10^2)^4 = 1.256^4 \times 10^8$.
2. $85.6^{4.32}$. *Hint*: $0.856^{4.32} \times 10^{8.64} = 0.856^{4.32} \times 10^{0.64} \times 10^8$.
3. $0.0001346^{2.65}$. *Hint*: $(1.346 \times 10^{-4})^{2.65} = 1.346^{2.65} \times 10^{-10.60}$.
4. $0.000894^{0.03}$. *Hint*: $(8.94 \times 10^{-4})^{0.03} = 8.94^{0.03} \times (0.1)^{0.12}$.
5. (*a*) $135.8^{4.1}$. (*b*) $78.7^{4.56}$. (*c*) $0.0001257^{2.73}$. (*d*) $0.0008275^{0.032}$.
6. (*a*) $7.84^{6.3}$. (*b*) $9.36^{5.7}$. (*c*) $4.2^{0.0042}$. (*d*) $1.021^{0.32}$.
7. $\log_{10} 29{,}300$. *Hint*: $\log_{10} 29{,}300 = \log_{10} 293 + \log_{10} 100$.
8. Find x if $x^{3.1} = 72{,}000$. *Hint*: $\left(\frac{x}{10}\right)^{3.1} = \frac{72{,}000}{10^{3.1}} = \frac{72}{10^{0.1}}$.
9. Find x if $6.4^x = 42{,}000$. *Hint*: $6.4^{x-2} = \frac{42000}{6.4^2}$
10. Find x if $10^x = 580{,}000$.
11. Find x if $5.83^x = 1.005$.

*Since 0.32 is $\log_{10} 10^{0.32}$ opposite .32 on the L scale read on D, $2.09 = 10^{0.32}$.

55. Numbers between 0.99 and 1.01. If z is any real number then

$$e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \cdots. \tag{4}$$

Hence, if x is a positive number less than 0.01,

$$e^x = 1 + x \text{ approximately.}$$

Imagine a Log Log scale representing numbers from 1.001 to 1.01 and call it the LLD scale. Then when the hairline is set on D to x lying in the range $0.001 < x < 0.01$, it is also set to e^x ($= 1 + x$ approximately) on the LLD scale. Hence the D scale may be used as the LLD scale by adding 1 to each reading x on the D scale. For this purpose the left index of D represents 1.001, 4 on D represents 1.004, 6 represents 1.006, and the right index represents 1.01. Similarly, calling the Log Log scale representing numbers between $1 - 0.01 = 0.99$ and $1 - 0.001 = 0.999$ the $LLOD$ scale we see that the D scale may be used as the $LLOD$ scale. In this case the right index of D represents $1 - 0.01 = 0.99$ on $LLOD$, 4 on D represents $1 - 0.004 = 0.996$ on $LLOD$, and so on. Also the D scale may be used to represent a whole series of Log Log scales which might be called LLD_{-1}, $LLOD_{-1}$, LLD_{-2}, $LLOD_{-2}$, . . . each pair dealing with numbers ten times as close to 1 as its predecessor. Fig. 14 illustrates the relations of the additional imaginary Log Log scales to the D scale.

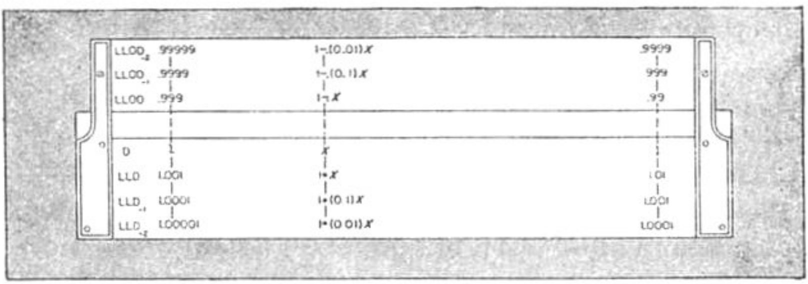


FIG. 14. The letter x in this figure represents a number in the interval $0.001 < x < 0.01$.

The following examples will illustrate methods of using the D scale as a Log Log scale.

Example 1. Evaluate:

(*a*) $1.005^{3.4}$ and 1.005^{34}. (*b*) $0.995^{3.4}$ and 0.995^{34}.

Solution. (*a*) If $1 + x = 1.005$, $x = 0.005$. Therefore set right index of C to 5 on D (considered as LLD), opposite 34 on C read on $LL1$, **1.01715** $= 1.005^{3.4}$, opposite 34 on C read on $LL2$, **1.1853** $= 1.005^{34}$.

Solution. (*b*) If $1 - x = 0.995$, $x = 0.005$. Therefore
set right index of C to 5 on D (considered as $LLOD$),
opposite 34 on C read on $LL01$, **0.98317** $= 0.995^{3.4}$,
opposite 34 on C read on $LL02$, **0.8436** $= 0.995^{34}$.

Example 2. Find the set of values obtained by raising 127 to each of the powers:

0.1240, 0.0124, 0.00124, 0.000124,
−0.0124, −0.00124, −0.000124.

Solution. To 127 on $LL3$ set left index of C,
opposite 124 on C read
on $LL2$, **1.824** $= 127^{0.124}$,
on $LL1$, **1.0619** $= 127^{0.0124}$,
on LLD, 601; therefore **1.00601** $= 127^{0.00124}$,
on LLD_{-1}, 601; therefore **1.000601** $= 127^{0.000124}$,
on $LL01$, **0.9417** $= 127^{-0.0124}$,
on $LLOD$, 601; therefore $1 - .00601 =$ **0.99399** $= 127^{-0.00124}$,
on $LLOD_{-1}$, 601; therefore $1 - .000601 =$ **0.999399** $= 127^{-0.000124}$.

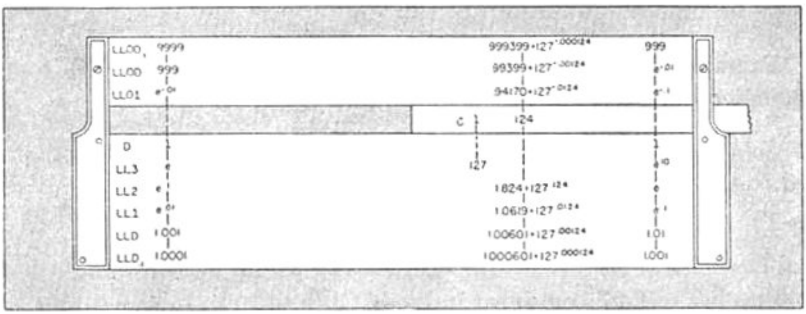


FIG. 15

Fig. 15 represents the solution of Example 2 as it would be made on a rule having all the Log Log scales involved.

If z is numerically less than 0.001 the first two terms of (4) give e accurate to seven figures. Hence, the slight error involved in using $1 + x$ for e^x in the method just considered is entirely negligible for slide rule purposes when the positive number x, as used above, is less than 0.001. The approximate magnitude of the error in the position of $1 + x$ on D considered as a Log Log scale is $x^2/2$. The greatest theoretical error with x between 0.01 and 0.001, that is for

LLD and $LLOD$, is the linear space corresponding to the distance on the slide rule between the mark for 1.01 on $LL1$ and the hairline set to 1 left on scale D. By considering the scale value of this linear space near a result on the scale where the result is read, one can estimate the maximum theoretical error for the result and multiply this by 100 x to find the estimated theoretical error in the result.

To eliminate the error just considered, when x is between 0.001 and 0.01, proceed as above using

$$\left.\begin{array}{l} x\,(1 - x/2) \text{ on } D \text{ to get } 1 + x \text{ on } D \text{ considered as scale } LLD, \\ x\,(1 + x/2) \text{ on } D \text{ to get } 1 - x \text{ on } D \text{ considered as scale } LLOD \end{array}\right\} \quad (5)$$

Fig. 16 shows the indicated relation.

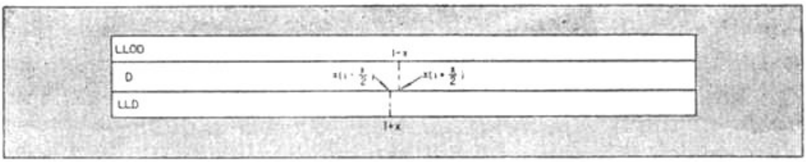


Fig. 16

Example 3. Find the principle and interest on \$1000 at 3% compounded quarterly for 10 years.

Solution. The compound interest formula $S = P\,(1 + r/t)^{nt}$ adapted to the given example becomes

$$S = 1000\left(1 + \frac{.03}{4}\right)^{10\times 4} = 1000\,(1.0075)^{40}.$$

To 75 on D set index of C,
opposite 4 on C read 1.35 on $LL2$,
$S = 1000 \times 1.35 = \$1350.$

This answer is slightly in error. To take account of the slight error note from (5) that in this problem

$$x\,(1 - x/2) = 0.0075\left(1 - \frac{0.0075}{2}\right) = .0075\,(0.99625)$$

and

set the right index of C to 0.996 on D,
push hairline to 75 on C,
draw right index of C under hairline,
opposite 4 on C read 1.3483 on $LL2$.

Therefore the correct answer is \$1000 (1.3483) = \$1348.30.

Example 4. Find 0.991^{32} using (5).

Solution. From (5), $x\,(1 + x/2) = 0.009\left(1 + \frac{0.009}{2}\right) = .009\ (1.0045)$,

set left index of C to 1.0045 on D,
push hairline to 9 on C,
draw right index of C under the hairline,
opposite 32 on C read on $LL02$, **0.7488** $= (0.991)^{32}$.

EXERCISES

1. Evaluate:
1.004^{22}, 0.996^{22}, 1.004^{220}, 0.996^{220}.

2. Evaluate:
$9.55^{0.03}$, $9.55^{-0.03}$, $9.55^{0.003}$, $9.55^{-0.003}$.

Set left index of C to 9.55 on $LL3$, opposite 3 on C read the first answer on $LL1$, the second on $LL01$, 677 on D. Then write for the third and fourth answers 1.00677, $1 - 0.00677 = 0.99323$.

3. Evaluate:
$1.0455^{0.52}$, $1.0455^{-0.52}$, $1.0455^{0.052}$, $1.0455^{0.052}$, $1.0455^{0.0052}$, $1.0455^{-0.0052}$.

4. Evaluate:
$9.55^{0.0031}$, $9.55^{-0.0031}$, $9.55^{0.00031}$, $9.55^{-0.00031}$.

5. Evaluate: 1.006^5, 0.994^5, 1.004^8, 0.996^8.

6. Find: 1.009^{45} and 0.991^{45} first by using the D scale in the usual way as an LL scale and also as an $LL0$ scale, and then by the method using the expressions (5).

7. Using the method suggested by expressions (5) find: 1.008^6, $1.008^{0.6}$, $1.008^{0.06}$.

8. Using the method suggested by expressions (5) find: 0.996^6, $0.996^{0.6}$, $0.996^{0.06}$.

9. Using the expressions (5) evaluate:
(*a*) $1.009^{29.6}$, $1.009^{2.96}$, $1.009^{0.296}$, $1.009^{0.0296}$.
(*b*) $0.991^{29.6}$, $0.991^{2.96}$, $0.991^{0.296}$, $0.991^{0.0296}$.

10. Evaluate by the method not using expressions (5):
1.000642 and 0.999358 to each of the powers: 0.065, 0.65, 6.5, 65 and 650.

56. Reciprocals. The method of this article may be used to find the reciprocal of any number with a high degree of accuracy. The accuracy is indicated by the number of figures which may be read from the part of the scale involved.

Rule I. *To find the reciprocal of a number in decimal notation, move the decimal point in it to form a number in the range 0.3 to 2.9999, set the hairline to the result found on a Log Log scale, read the number at the hairline on the mated Log Log scale, and in this number move the decimal point the same number of places as before and in the same direction.*

Thus to find the reciprocal of 6850, write 0.6850,
set hairline to 0.685 on $LL02$,
at hairline read on $LL2$, 1.460,
set the decimal 4 places to the left to get **0.0001460.**

Observe that the decimal point was moved four places to the left twice.

When the process of Rule I involves setting the hairline to a number $1 \pm a$ in the range 0.99 to 1.01, a range not found on the Log Log scales, the following rule may be used.

Rule II. *When a number* $1 \pm a$ *lies in the range 0.99 to 1.01, then*
the reciprocal of $1 + a$ *is* $1 - a$,
the reciprocal of $1 - a$ *is* $1 + a$,
the results will be accurate to three decimal places, and the fourth place will generally be correct also.

For example,
reciprocal of 1.0094 is $1 - 0.0094 =$ **0.9906** (approx.),
reciprocal of 0.9917 is $1 + 0.0083 =$ **1.0083** (approx.).

Rules I and II may be used to find the reciprocal of any number. However, much greater accuracy can be found in most cases by using the following rule.

Rule III. *To find the reciprocal of a given number, first move the decimal point in it to form a new number in the range 1 to 10. If this new number lies in the range 1.5 to 9.5*, divide the number by the digit 2, 3, 4, 5, 6, 7, 8, or 9 nearest to its value, find the reciprocal of the result by Rule I or II, and divide this by the same digit. Finally move the decimal point the same number of places and in the same direction as it was first moved.*

*If the number mentioned in Rule III is not in the range 1.5 to 9.5, use Rule I or II.

To become familiar with the process the reader should carry out the following suggested operations in detail. To find the reciprocal of 0.685 write 6.850; then

$6.850 \div 7 = 0.97857.$
Opposite 0.97857 on $LL01$ read on $LL1$, 1.02185,
$1.02185 \div 7 = 0.14598$, and the answer is **1.4598.**

This result is in error by 1 in the last place. The required value accurate to five figures is 1.4599.

To find the reciprocal of 0.00048700, write

$4.8700 \div 5 = 0.9740,$
opposite 0.9740 on $LL01$ read on $LL1$, 1.02665,
$1.02665 \div 5 = 0.20533$, and the answer is **2053.3.**

The required value accurate to five figures is 2053.4.

The following outline of solutions will furnish drill work to give familiarity with the method.

Example. Find the reciprocal of (*a*) 0.002376. (*b*) 2762.3. (*c*) 8004.76.

Solution (*a*). To find the reciprocal of 0.002376, write 2.376; then
$2.376 \div 2 = 1.188.$
Opposite 1.188 on $LL2$ read on $LL02$, $0.8418 = 1/1.188$,
$0.8418 \div 2 = 0.4209$, and the answer is **420.9.**

Solution (*b*). To find the reciprocal of 2762.3, write 2.7623; then
$2.7623 \div 3 = 0.92077.$
Opposite 0.92077 on $LL01$ read on $LL1$, 1.0859.
$1.0859 \div 3 = 0.3620$, and the answer is **0.0003620.**

Solution (*c*). To find the reciprocal of 8004.76, write 8.00476; then
$8.00476 \div 8 = 1.000595.$
Reciprocal of $1 + .000595$ is 0.999405, (by Rule II.)
$0.999405 \div 8 = 0.124926$, and the answer is **0.00124926.**

EXERCISES

1. Find the reciprocals of the following numbers:
   (*a*) 1.653, 13.74, 1.0557, 0.0010236.
   (*b*) 576, 8173, 0.009555, 0.98635.
   (*c*) 2453, 0.02964, 0.20964, 22.56.
   (*d*) 0.364, 0.00462, 3789, 4937.

2. Find the reciprocals of the following numbers:
   523, 0.01036, 9568, 1286, 0.000248, 0.008635, 36.74, 0.07931, 0.00625, 4260.

3. Find the reciprocals of the following numbers:
   1.0032, 0.994, 1.008, 0.999, 99.83, 0.010042.

57. Miscellaneous Exercises. Find the value of unknown quantities represented by x, y, and z in the following equations:

1. $x = 3.15^{2.16}$.
2. $x = 3.15^{0.216}$.
3. $y = e^{-1.74}$.
4. $y = e^{-0.36}$.
5. $x = 0.55^{2.10}$.
6. $z = 0.55^{0.21}$.
7. $x = \sqrt[4]{18.0}$.
8. $x = \sqrt[5]{18.0}$.
9. $y = e^{-0.55}$.
10. $y = 0.35^{0.55}$.
11. $x = e^{1.058}$.
12. $x = e^{-1.058}$.
13. $y = \sqrt[4]{0.698}$.
14. $y = \sqrt[5]{0.645}$.
15. $z = 0.978^{1.80}$.
16. $z = 0.978^{0.180}$.
17. $x = 1.35^{1.92}$.
18. $x = 1.35^{19.2}$.
19. $y = 6.1^{0.48}$.
20. $y = \sqrt[8]{6.1}$.
21. $z^{0.78} = 2.35$.
22. $3.22^{y} = 11.0$.
23. $x^{1.55} = 0.29$.
24. $0.315^{y} = 0.830$.

CHAPTER VI

LOGARITHMS AND THE SLIDE RULE

58. Construction of the *D* scale. Perhaps the simplest explanation of the construction of the scales of the slide rule can be made in terms of logarithms. Since nearly all the scales are constructed by the same method, a detailed consideration of the construction of the D scale will indicate how most of the other scales are made.

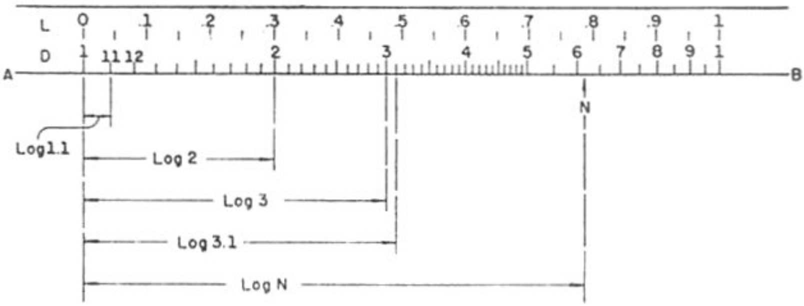


FIG. 1.

To construct a D scale, first reproduce the L scale (see Fig. 1). Since it is a uniformly marked and numbered 10-inch† scale, it may be used for finding lengths in terms of ten inches as the unit of measure. Next draw a line AB parallel to the L scale. Opposite 0 on L make a mark on AB and letter it 1. This mark will be referred to as the left index. Opposite log* 2 (= 0.3010 approximately) on L make a mark on AB and letter it 2. Similarly opposite log 3 (= 0.4771 approximately) on L make a mark and letter it 3, etc., until a mark has been made on AB for each of the digits 1 to 9. Instead of marking the right index 10 as we should expect, since log 10 = 1.0, number it 1. This gives the ten primary divisions. The other division marks are located in a similar manner. Thus *to each division mark is associated a number and this mark is situated at a distance from the left index equal to the mantissa of the logarithm of that number.*

†Nominally the D scale is 10 inches long. Its exact length however is 25 centimeters. On the 20-inch rule the D scale is 50 centimeters long.

*The symbol log N will be understood to mean the mantissa of $\log_{10} N$ unless otherwise specified.

The mantissa, or fractional part of the logarithm of a number, is independent of the position of the decimal point. Hence if we think of the distances from the left index as the mantissas of the logarithms of the numbers represented by the divisions, it appears that we can think of the primary divisions as representing the range of numbers 1, 2, 3, . . . 10, the range 10, 20, 30, . . . 100, the range 100, 200, 300, . . . 1000, etc. Naturally, in each of these cases, we think of the secondary divisions as representing appropriate numbers lying between the numbers represented by adjacent primary divisions.

59. Accuracy. From §50 we write

$$\log_{10} N = d \tag{1}$$

where N represents the number associated with any specified mark on the D scale and d is the distance of the mark from the left index. By applying calculus to equation (1) we easily prove that for small errors in d

$$\text{Relative error in } N = \frac{(\text{error in } N)}{N} = 2.3026 \text{ (error in } d). \tag{2}$$

Now the error in d is the error made in making the reading. The right-hand member is independent of N. Therefore the relative error in the number read does not depend on its size and hence is the same for all parts of the scale. Near the left end of the D scale a careful reading should be in error by no more than 1 in the fourth place i.e. the relative error should be no greater than 1 in 1000. Hence the error of a reading made on any part of the D scale should not be much greater than 1 in 1000 or one tenth of one per cent.

60. Multiplication and division. The middle part of the rule which may be moved back and forth relative to the other part is referred to as the *slide*; the outer or fixed part of the rule is called the *body*. The D scale is located on the body and the C scale is the same as the D scale except that it is located on the slide. Hence the C scale may be moved relative to the D scale, and we are able to add distances as indicated in Fig. 2.

From this figure and the considerations of §58, it appears that

$$\log P = \log N + \log M. \tag{3}$$

But the sum of two logarithms is equal to the logarithm of their product. Accordingly we get from (3)

$$\log P = \log MN, \text{ or } P = MN. \tag{4}$$

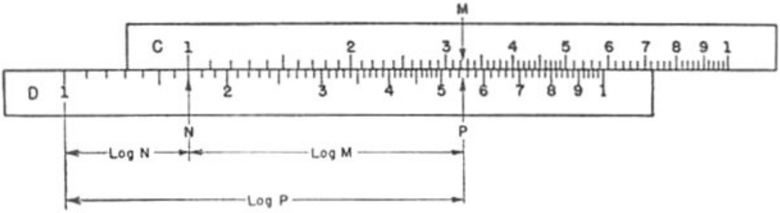


FIG. 2.

Hence Fig. 2 shows the setting to be used for multiplying numbers.

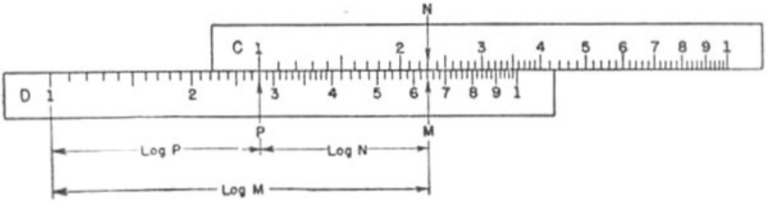


FIG. 3.

From Fig. 3 and the considerations of §58 it appears that

$$\log P = \log M - \log N, \tag{5}$$

or since

$$\log M - \log N = \log (M/N),$$

we have

$$\log P = \log \frac{M}{N}, \text{ and } P = \frac{M}{N}. \tag{6}$$

Thus Fig. 3 shows the setting to be used for dividing numbers.

The rule for multiplication §5 and the rule for division §7 are justified by the principles set forth above.

Observe that when the slide is set with M and N as opposites on the C and D scales, any other pair of opposites on the C and D (or CF and DF) scales have the same ratio P. The proportion principle is based on this fact.

61. The inverted scales. The CI scale is constructed in the same manner as the D scale except that the distances are measured leftward from the right index, and the numbers associated with the primary division marks are in red.

Let N be the number associated with a position on the C scale and K the number on the CI scale associated with the same position.

Then, in accordance with §60,

$$\log N + \log K = 1.$$

Hence we may write

$$\log K = 1 - \log N = \log 10 - \log N = \log \frac{10}{N},$$

or

$$K = \frac{10}{N}.$$

Therefore, except for the position of the decimal point, K is the reciprocal of N. In other words, *when the hairline is set to a number on the CI scale, it is automatically set to the reciprocal of that number on the C scale.*

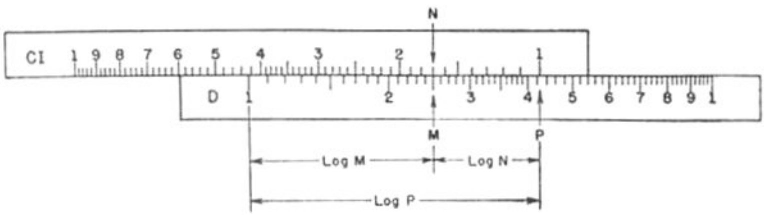


FIG. 4.

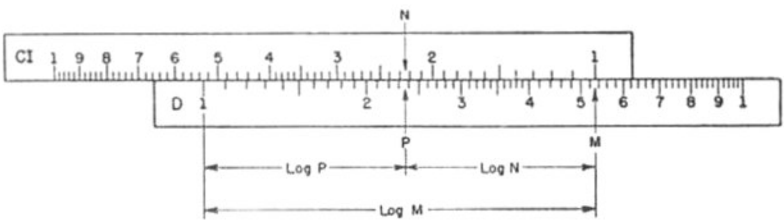


FIG. 5.

Fig. 4 indicates how multiplication may be accomplished by using the CI scale in conjunction with the D scale while Fig. 5 indicates how division may be accomplished. From Fig. 4, we have

$$\log P = \log M + \log N, \text{ or } P = MN,$$

and from Fig. 5, we have

$$\log P = \log M - \log N, \text{ or } P = M/N.$$

62. The *A* scale, the *B* scale, and the *K* scale. The A scale is constructed by the method used in the case of the D scale except that the unit of measure employed is 5 inches instead of 10 inches and the scale is repeated. The B scale is the same as the A scale except that it is situated on the slide while the A scale is on the body.

When the hairline is set to a number N on the A scale it is automatically set to a number M on the D scale, see Fig. 6. The two

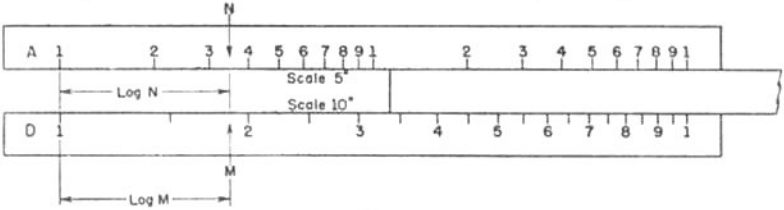


FIG. 6.

lengths marked log N and log M in the figure are equal. However since the unit in the case of log N is half the unit in the case of log M, we have

$$\log M = \frac{1}{2} \log N = \log N^{\frac{1}{2}} = \log \sqrt{N},$$

and $$M = \sqrt{N}.$$

Hence, *a number on scale D is the square root of the opposite number on scale A*. A similar relation exists between numbers on scales C and B.

The K scale is constructed by the method used in the case of the D scale except that the unit of measure employed is one third of 10 inches instead of 10 inches. The argument used above may be employed to show that *when the hairline is set to a number on the K scale it is automatically set to the cube root of the number on the D scale.*

63. The trigonometric scales. The general plan of constructing the S (sine) scale is the same as that for the D scale. Here again 10 inches is taken as the unit of measure. To each division mark on the S scale is associated an acute angle (in black) such that the distance of the division from the left index is equal to the mantissa of the logarithm of the sine of the angle. Thus Fig. 7 shows the

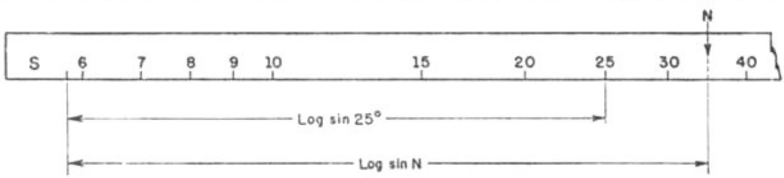


FIG. 7.

division marked 25 at a distance from the left index of the mantissa of log sin 25°. Hence when the hairline is set to an angle on the sine scale, it is automatically set to the sine of the angle on the

C scale. Fig. 8 shows a setting for finding $P = \dfrac{16 \sin 68^\circ}{\sin 27^\circ}$.

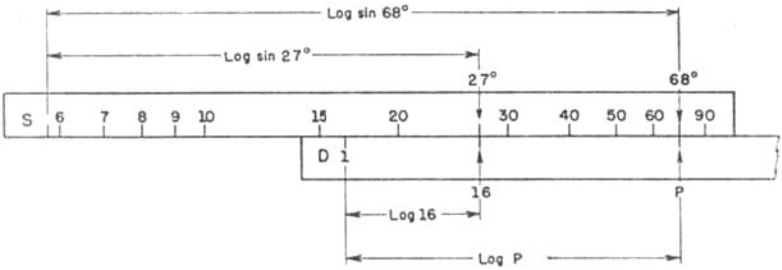


FIG. 8.

From this figure it appears that

$$\log P = \log 16 - \log \sin 27^\circ + \log \sin 68^\circ = \log \frac{16 \sin 68^\circ}{\sin 27^\circ},$$

or

$$P = \frac{16 \sin 68^\circ}{\sin 27^\circ}.$$

Since the slide rule does not take account of the characteristics of the logarithms, the position of the decimal point is determined in accordance with the result of a rough approximation.

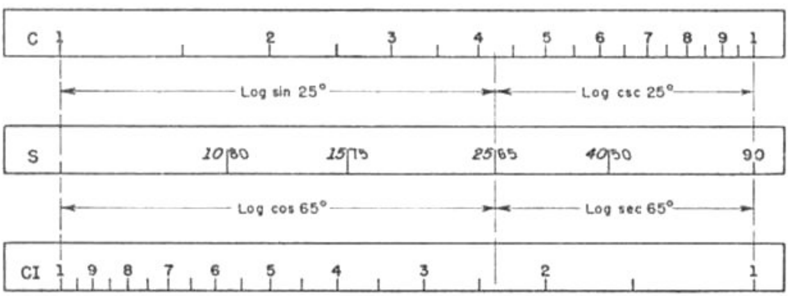


FIG. 9.

If the learner will note that the angles designated by red numbers are the complements of the angles in black, and remember that the distance from a division on the C scale to the right index is the logarithm of the reciprocal of the number represented by the division, and also that

$$\sin \theta = \cos (90^\circ - \theta),$$
$$\csc \theta = 1/\sin \theta,$$
$$\sec \theta = 1/\cos \theta,$$

he will easily see the relations indicated in Fig. 9 for the representative angle 25°.

The T scale is constructed by taking 10 inches as the unit of measure and associating to each division mark on it an acute angle such that the distance of the mark from the left index is equal to the mantissa of the logarithm of the tangent of the angle. Recalling that

$$\cot (90° - \theta) = \tan \theta = 1/\cot \theta,$$

the student will easily see the relations indicated in Fig. 10 for the representative angle 25°.

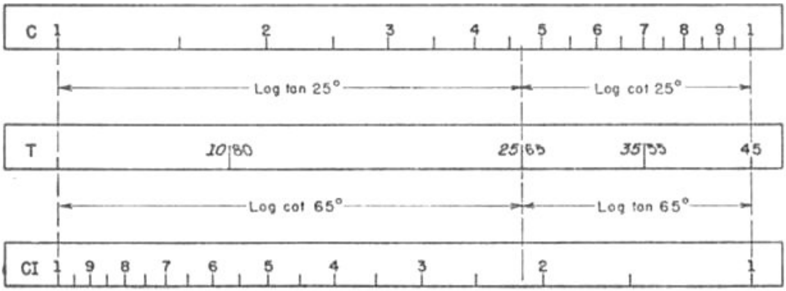


FIG. 10.

The facts illustrated in Figs. 9 and 10 are the basis of the following rule:

If the hairline be set to an angle on a trigonometric scale, it is automatically set to the *complement* of this angle. One of these angles is expressed in black type, the other in red. From what has been said it appears that we read, at the hairline on the C scale or on the CI scale, a figure expressing a direct function (sine, tangent, secant) by reading a figure of the same color as that representing the angle, a co-function (cosine, cosecant, cotangent) by reading a figure of the opposite color. In other words, *associate direct function with like colors, co-function with opposite colors.*

The S scale applies to angles ranging from 5.74° to 90°; the sines of these angles range from 0.1 to 1 as indicated by its legend. Any angle in the range from 0.573° to 5.73° has a sine approximately equal to its tangent. The SRT scale is related to the angles ranging from 0.573° to 5.73° just as the S scale is related to the angles ranging from 5.74° to 90°. Since any angle greater than 0.573° but less than 5.73° has its sine approximately equal to its tangent, the SRT scale may be used for tangents as well as for sines.

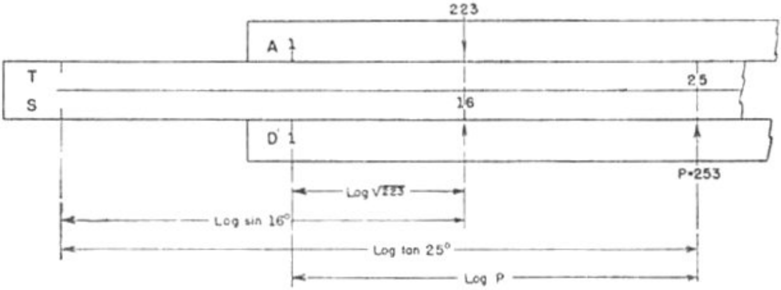


FIG. 11.

64. An application. Fig. 11 indicates the logarithmic basis of a setting which may be used to evaluate $\dfrac{\sqrt{223}\tan 25^\circ}{\sin 16^\circ}$. From the figure it appears that $\log P = \log\sqrt{223} - \log\sin 16^\circ + \log\tan 25^\circ = \log\dfrac{\sqrt{223}\tan 25^\circ}{\sin 16^\circ}$, or $P = \dfrac{\sqrt{223}\tan 25^\circ}{\sin 16^\circ}$. Since the reading at P is 253, we have $\dfrac{\sqrt{223}\tan 25^\circ}{\sin 16^\circ} = 25.3$.

65. The Log Log scales. In this article we shall use logarithms to the base 10 and also logarithms to the base e ($= 2.7183$ approximately). For convenience we shall indicate the *mantissa* of $\log_{10} N$ by $\log N$, and $\log_e N$ by $\ln N$.

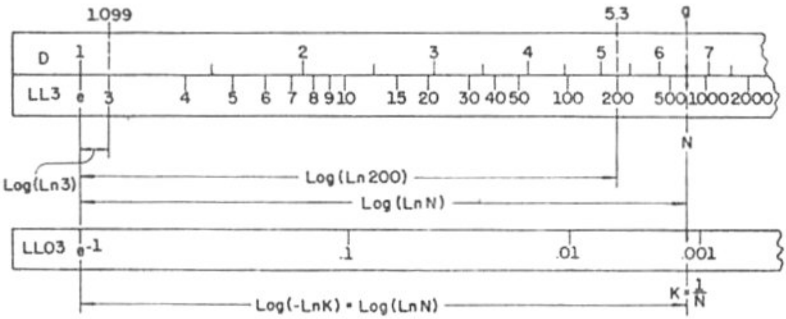


FIG. 12.

The mark at the extreme left of scale $LL3$, see Fig. 12, is opposite the left index of the D scale and is numbered e. If 10 inches is taken as the unit of measure, the mark on $LL3$ numbered 3 is distant log (ln 3) from the e mark, the mark numbered 200 is distant log (ln 200) from the e mark, and, in general, to any mark on scale $LL3$ distant log (ln N) from the e mark is associated the number N. To the point on scale $LL2$ opposite the left index of the D scale is assigned the number $e^{0.1}$ and to any mark on $LL2$ distant 10 log (ln K) from the left-hand mark is assigned the number K. Similarly to the point on $LL1$ opposite the index of D is assigned the number $e^{0.01}$ and to any mark on $LL1$ distant 100 log (ln K) from the $e^{0.01}$ point is assigned the number K.

The lower part of Fig. 12 shows a skeleton form of the $LL03$ scale. Any number K on this scale is the reciprocal of its opposite N, $K = \frac{1}{N}$, on the $LL3$ scale. The distance of any mark numbered $K\ (= \frac{1}{N})$ from the left most mark of $LL03$, numbered $e^{-1} = 0.3679$, is $\log\ (-\ \ln\ K) = \log\ (\ln\ N)$. Similarly, to each mark on scale $LL02$ or $LL01$ is assigned the reciprocal of its opposite on $LL2$ or $LL1$ respectively.

Let g be the number on the D scale (see Fig. 12) opposite N on $LL3$. Then since the distance from the point on $LL3$ opposite the index on D to the point on $LL3$ representing the number N is log (ln N) and the same distance on the D scale is log g, we have

$$\log g = \log\ (\ln\ N),\ \text{or}\ g = \ln\ N.$$

Hence, *when the hairline is set to a number N on the $LL3$ scale, it is automatically set to ln N on the D scale.* A similar statement applies to scales $LL2$ and $LL1$. From the discussion above it appears that ln N is on the range 1.0 to 10.0, 0.1 to 1.0, or 0.01 to 0.1 according as N is a number on $LL3$, $LL2$, or $LL1$. Legends indicating these ranges are written at the right end of the corresponding scales on the slide rule. Also since the number on the $LL0$ scales are the reciprocals of their opposites on the LL scales, their logarithms are the negatives of the logarithms of their opposites on the LL scales. Hence $-\ln K$ lies on the range -1.0 to -10.0, -0.1 to -1.0, or -0.01 to -0.1 according as K lies on $LL03$, $LL02$, or $LL01$, and appropriate legends are found on the rule.

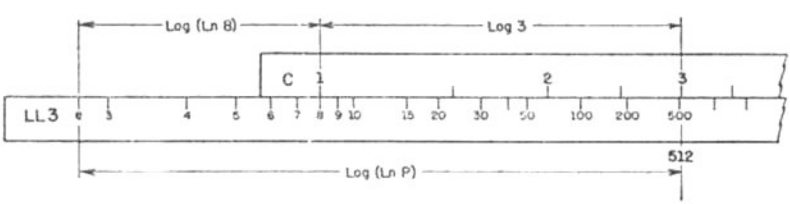


Fig. 13.

Fig. 13 shows a setting for evaluating 8^3. By equating two expressions for the distance from the left end of the $LL3$ scale (see

Fig. 13) to the mark associated with P, we obtain

$$\log(\ln P) = \log(\ln 8) + \log 3 = \log(3 \ln 8) = \log(\ln 8^3).$$

Hence

$$\ln P = \ln 8^3, \text{ or } P = 8^3.$$

Since the reading at P on the $LL3$ scale is 512, we have $512 = 8^3$.

Similarly, Fig. 14 shows a setting for evaluating $N = B^L$.

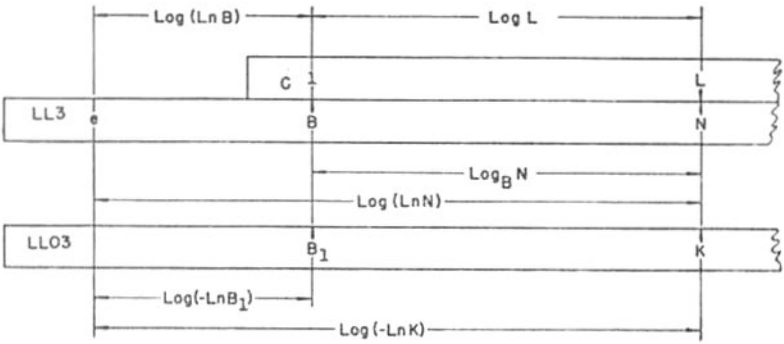


FIG. 14.

By following the procedure exhibited in the case of 8^3, we have

$$\log(\ln N) = \log(\ln B) + \log L = \log(L \ln B) = \log(\ln B^L).$$

When the logarithms of two numbers are equal the numbers are equal. Therefore

$$\ln N = \ln B^L, \; N = B^L.$$

From Fig. 14 and special reference to the $LL03$ scale, we have $\log(-\ln K) = \log(-\ln B_1) + \log L = \log(-L \ln B_1) = \log(-\ln B_1{}^L)$. Hence $-\ln K = -\ln B_1{}^L$, and $K = B_1{}^L$.

It is interesting to note, in this connection, how the Log Log scales may be used to find the logarithm of any number N to any base B. If we take the logarithms to the base B of both members of the equation $N = B^L$, we obtain

$$\log_B N = L.$$

Interpreting this equation in the light of Fig. 14, we see that *when the index of scale C is set opposite a number B on the LL scale, $\log_B N$ is on scale C opposite N on the LL scale. A similar statement applies to the LL0 scales.*

HISTORICAL NOTES ON THE SLIDE RULE

Since logarithms are the foundation on which the slide rule is built, the history of the slide rule rightly begins with John Napier of Merchiston, Scotland, the inventor of logarithms. In 1614 his "Canon of Logarithms" was first published. In presenting his system of Logarithms, Napier sets forth his purpose in these words:

> "Seeing there is nothing (right well beloved Students of Mathematics) that is so trublesome to mathematical practice, nor doth more molest and hinder calculators, than the multiplications, divisions, square and cubical extractions of great numbers, which besides the tedious expense of time are for the most part subject to many slippery errors, I began therefore to consider in my mind by what certain and ready art I might remove those hindrances."

From Napier's early conception of the importance of simplifying mathematical calculations resulted his invention of logarithms. This invention in turn made possible the slide rule as we know it today. Other important milestones in slide rule history follow.

In 1620 Edmund Gunter, of London, invented the straight logarithmic scale, and effected calculation with it by the aid of compasses.

In 1630 William Oughtred, the English mathematician, arranged two Gunter logarithmic scales adapted to slide along each other and kept together by hand. He thus invented the first instrument that could be called a slide rule.

In 1675 Sir Isaac Newton solved the cubic equation by means of three parallel logarithmic scales, and made the first suggestion toward the use of an indicator.

In 1722 John Warner, a London instrument dealer, used square and cube scales.

Circular slide rules and rules with spiral scales were made before 1733, but their inventors are unknown.

In 1775 Thomas Everard, an English Excise Officer, inverted the logarithmic scale and adapted the slide rule to gauging.

In 1815 Peter Roget, an English physician, invented a Log Log scale.

In 1859 Lieutenant Amédée Mannheim, of the French Artillery, invented the present form of the rule that bears his name.

Cylindrical calculators with extra long logarithmic scales were invented by George Fuller, of Belfast, Ireland, in 1878 and Edwin Thacher, of New York, in 1881.

A revolutionary slide rule construction, with scales on both the front and back surfaces of body and slide and with a double faced indicator referring to all scales simultaneously, was patented in 1891, by William Cox, who was mathematical consultant to Keuffel & Esser Co. With the manufacture of Mannheim rules and this new rule, K & E became the first commercial manufacturer of slide rules in the United States. These had previously all been imported from Europe.

Folded scales *CF*, *DF* and *CIF* were put on slide rules about 1900, to reduce the amount of movement and frequency of resetting the slide. At first the scales were folded at $\sqrt{10}$ but K & E later folded such scales at π so that π could be used as a factor without a resetting. Log Log scales in three sections were put on K & E rules about 1909.

The Parsons invention of about 1919, which included special scales for finding the hypotenuse of a right triangle was incorporated in a rule made in Japan. This rule later included a Gudermannian scale, patented by Okura, enabling the user to read hyperbolic functions.

A scale referring to the *A* or *B* scales to give the logarithms of the co-logarithms of decimal fractions was introduced on K & E slide rules about 1924. Puchstein's scales for hyperbolic functions, patented in 1923, were put on commercial K & E slide rules in 1929. The trigonometric scales were divided into degrees and decimals of a degree, thus making it possible to eliminate all non-decimal sub-divisions from the rule.

K & E introduced a slide rule (patented in 1939) in which all of the trigonometric scales are on the slide and refer to the full length *C* and *D* scales. In solving vector problems on this rule or other similar problems involving continuous operations and progressive manipulation, only the final answer needs to be read.

In 1947, on the basis of Bland's invention, the scales of the logarithms of the co-logarithms of decimal fractions were referred to the *C* and *D* scales, correlated with the Log Log scales and also with all of the other scales of the rule, thereby increasing the power of the slide rule by simplifying the solution of exponential or logarithmic problems, the determination of hyperbolic functions, reciprocals, etc.

It was about 1910 when the slide rule first began to come into general use in the United States. In the years that followed, K & E introduced many improvements in the rigidity of frame, indicator design, the precision of graduations, as well as a variety of new scale arrangements. All these have contributed to the wide popularity of the slide rule and its many uses in the mathematics of science and engineering, as well as for calculations of all kinds in business and industry.

Many types of slide rules have been devised and made in small quantities for the particular purposes of individual users. Rules have likewise been made specially for chemistry, surveying, artillery ranging, steam and internal combustion engineering, hydraulics, reinforced concrete work, air conditioning, radio and other special fields. However, the acceptance of such rules has been relatively limited.

The slide rule has a long and distinguished ancestry. The rule described in this manual incorporates the most valuable features that have been invented from the beginning of slide rule history, right up to date.

ANSWERS

Answers read between 2 and 4 on the C scale or D scale contain four significant figures, the last one being 0 or 5. Hence such answers have the fourth significant digit accurate to the nearest 5.

§5. Page 8

1. 6	**4.** 9.1	**7.** 49.8	**10.** 0.0826	**13.** 9.87
2. 7	**5.** 6.75	**8.** 340	**11.** 3220	**14.** 3.08
3. 10	**6.** 9.62	**9.** 47.0	**12.** 0.836	

§6. Page 9

1. 15	**5.** 0.001322	**9.** 244	**13.** 170.5
2. 15.77	**6.** 1737	**10.** 57.1	**14.** 5630
3. 3530	**7.** 9.98	**11.** 0.1621	**15.** 6890
4. 42.1	**8.** 1341	**12.** 0.201	**16.** 2870

§7. Page 10

1. 2.32	**5.** 0.000713	**9.** 1.154	**13.** 616
2. 165.2	**6.** 77.5	**10.** 0.0419	**14.** 0.0298
3. 0.0767	**7.** 1861	**11.** 0.936	**15.** 4.96
4. 106.1	**8.** 26.3	**12.** 1.535	**16.** 0.332

§8. Page 11

1. (*a*) 1576	(*c*) 220%	**4.** (*a*) 9.22 yds./sec.
(*b*) 2.60	(*d*) 2.73%	(*b*) 15.02 ft./sec.
(*c*) 5.25	**3.** (*a*) 178.9 mi.	(*c*) 186,000 mi./sec.
(*d*) 4.59	(*b*) 121.1 mi.	**5.** (*a*) 10.13 sec.
2. (*a*) 26.1%	(*c*) 2140 mi.	(*b*) 29.5 hrs.
(*b*) 64.4%		(*c*) 323 hrs.

§9. Page 14

1. 36.7	**5.** 0.00357	**9.** 0.01311	**13.** 249
2. 8.35	**6.** 13,970	**10.** 2.35	**14.** 0.275
3. 0.0000632	**7.** 1586	**11.** 0.0414	**15.** 0.1604
4. 3400	**8.** 0.0223	**12.** 2460	**16.** 0.0977
			17. 2.68 in.
			18. 86.8 sq. in.

§11. Page 18

1. $x = 43.3$

2. $x = 169.4$

3. $x = 285$

4. $x = 5.22$

5. $x = 2.30, \quad y = 31.8$

6. $x = 51.7, \; y = 3370$

7. $\begin{cases} x = 106.2 \\ y = 30.4 \end{cases}$

8. $\begin{cases} x = 1.586 \\ y = 41.4 \end{cases}$

9. $\begin{cases} x = 0.1013 \\ z = 0.0769 \end{cases}$

10. $\begin{cases} x = 3.97 \\ y = 0.984 \\ z = 0.272 \end{cases}$

11. $\begin{cases} x = 1.315 \\ y = 1.525 \end{cases}$

12. $\begin{cases} y = 37.8 \\ y = 69.5 \end{cases}$

13. $\begin{cases} x = 0.00416 \\ y = 0.0828 \end{cases}$

14. $\begin{cases} x = 0.1170 \\ y = 0.927 \end{cases}$

15. $\begin{cases} x = 186 \\ y = 13.42 \\ z = 50.3 \end{cases}$

§12. Page 20

1. 13.71	**5.** 0.3960	**9.** 3.20	**12.** 4.07
2. 23.0	**6.** 9.46	**10.** 0.1265	**13.** 9.68
3. 85.0	**7.** 42.0	**11.** 104.6	**14.** 47.6
4. 48.7	**8.** 3.14		

§13. Page 21

1. 167.6 cm.
   249 cm.
   980 cm.
2. (*a*) 274 m.
   (*b*) 800 m.
   (*c*) 2.54 m.
3. (*a*) 720 lb.
   (*b*) 2060 lb.
   (*c*) 31.4 lb.
4. (*a*) 235 sq. cm.
   (*b*) 929 sq. cm.
   (*c*) 421 sq. cm.
5. (*a*) 25,700 watts
   (*b*) 3,940,000 watts
   (*c*) 621 watts
6. (*a*) 1.121 gal.
   (*b*) 0.00255 gal.
   (*c*) 0.1504 gal.
7. 29.0 in., 584 in.,
   62.7 lb. per sq. in.
8. (*a*) 6.12 lb. per sq. in., 7.35 lb. per sq. in., 24.5 lb. per sq. in.
   (*b*) 21.6 cu. in., 33.4 cu. in., 79.9 cu. in., 183.8 cu. in.

§14. Page 23

1. 0.0625, 0.00385, 1.389, 15.38, 0.0575, 0.0541, 0.01490
2. 0.00253, 0.000559, 6.21
3. 2,160
4. 74.0, 10.97
5. 199.5, 8.55
7. 2.74, 0.364, 0.392, 2.56, 12.54, 0.0797

§15. Page 25

1. $x = 16.98,\quad y = 12.74$
2. $x = 0.0640,\quad y = 1.415$
3. $x = 154.9,\quad y = 6950$
4. $x = 0.00247,\quad y = 645$
5. $\begin{cases} x = 0.0481 \\ y = 0.0435 \\ z = 44.9 \end{cases}$
6. $\begin{cases} x = 11.07 \\ y = 0.0483 \\ z = 0.465 \end{cases}$

§16. Page 28

1. 11.2
2. 2.36
3. 39.4
4. 0.001155
5. 1.512
6. 1.015
7. 17.2
8. 96.1
9. 0.1111
10. 150,800
11. 15.32
12. 9.76
13. 0.00288
14. 144,800
15. 0.0267
16. 0.279
17. 41.3
18. 111.4
19. 3430
20. 4.75
21. 0.481
22. 3.76×10^{-6}
23. 7.05×10^{-7}
24. 2.82
25. 0.0527
27. 2.91
28. 3.55
29. 7.83
30. 184.4
31. 75.3
32. 12.0
33. 0.0252
34. 7.49
35. 17.52
36. 2180
37. 1741
38. 1766

§17. Page 30

625, 1024, 3720, 5620, 7920, 537,000, 204,000, 4.33, 3.07, 0.1116, 0.00001267, 0.908, 27,800,000, 2.24×10^{13}

§18. Page 32

1. 2.83, 3.46, 4.12, 9.43, 2.98, 29.8, 0.943, 85.3, 0.252, 0.00797, 252, 316
2. (*a*) 231 ft. (*b*) 0.279 ft. (*c*) 5720 ft.
3. (*a*) 18.05 ft. (*b*) 0.992 ft. (*c*) 49.8 ft.

§19. Page 34

1. 24.2
2. 0.416
3. 8.54
4. 0.0698
5. 4.43
6. 4.01
7. 6.68
8. 6.14
9. 0.427
10. 1.176
11. 32.8
12. 398
13. 43.7
14. 29.4
15. (*a*) 5.94 ft.^{2} (*b*) 3500 ft.^{2} (*c*) 0.445 ft.^{2} (*d*) 2.76 ft.^{2}
16. (*a*) 37.6 ft.^{2} (*b*) 0.00597 ft.^{2} (*c*) 965 ft.^{2} (*d*) 2.35×10^8 ft.^{2}

§20. Page 36

1. 64.2
2. 11.41
3. 109.1
4. 0.428
5. 9.65
6. 0.0602
7. 1.525×10^5
8. 1.589

§21. Page 36

1. 9.25, 32.8, 238,000, 422,000, 705,000, 3.94×10^8, 0.0925, 29.2, 5.39, 0.0000373, 0.84, 1.464×10^{11}, 5.71×10^{19}
2. 76

§22. Page 37

2.06, 3.11, 9.00, 9.47, 19.69, 0.1969, 0.424, 0.914, 44.7, 0.855, 909, 2.15, 4.64, 46.4

§23. Page 39

1. 2.19
2. 30.9
3. 54.3
4. 0.974
5. 1.52
6. 0.0577
7. 43,100
8. 1.745
9. 1.156
10. 1.193
11. 90.7
12. 12.76
13. 76.3
14. 2.12
15. 1.281×10^8
16. 0.00369
17. 5.03
18. 2290
19. 0.0544
20. 3.29
21. 0.000867
22. 27.3

§24. Page 40

1.515, 0.814, 5.991, 9.830-10, 8.022-10, 6.615-10, 1.861, 9.427-10, 7.904-10, 2.635

§26. Page 44

2. (*a*) 0.5 (*b*) 0.616 (*c*) 0.0581 (*d*) 1 (*e*) 0.999
   (*f*) 0.0276 (*g*) 0.253 (*h*) 0.381 (*i*) 0.204 (*j*) 0.783
3. (*a*) 0.866 (*b*) 0.788 (*c*) 0.998 (*d*) 0 (*e*) 0.0349
   (*f*) 1.00 (*g*) 0.968 (*h*) 0.925 (*i*) 0.979 (*j*) 0.623
4. (*a*) 30° (*b*) 61° (*c*) 22° (*d*) 5.74° (*e*) 0.86°
   (*f*) 38.3° (*g*) 3.55° (*h*) 1.775° (*i*) 66.9°
5. (*a*) 60° (*b*) 29° (*c*) 68° (*d*) 84.26° (*e*) 89.14°
   (*f*) 51.7° (*g*) 86.45° (*h*) 88.22° (*i*) 23.1°

§27. Page 47

1. (*a*) $x = 6.10$, $\theta = 61.7°$ (*b*) $\theta = 54°$, $x = 21.6$ (*c*) $x = 30.4$, $y = 44.5$ (*d*) $\theta = 4.92°$, $\phi = 8.08°$
2. (*a*) 2.5 (*b*) 10.39 (*c*) 44 (*d*) 43.9
   (*e*) 25.4 (*f*) 44.2 (*g*) 17.69 (*h*) 17.0
3. (*a*) 5.86° (*b*) 55.8° (*c*) 83° (*d*) 59°
4. (*a*) 35.4 (*b*) 80.7 (*c*) 31.9 (*d*) 262
5. (*a*) 0.978 (*b*) 6.02 (*c*) − 9.14 (*d*) 16.45

§29. Page 50

1. $C = 75°$, $b = 35.5$, $c = 53.3$
2. $C = 55°$, $b = 70.7$, $a = 56.1$
3. $C = 123.2°$, $b = 2260$, $c = 2600$
4. $C = 55.34°$, $b = 568$, $c = 664$
5. $B = 51.33°$, $c = 81.0$, $b = 63.2$
6. $A = 21.17°$, $b = 1884$, $c = 2020$
7. $B = 26°$, $a = 410$, $c = 456$
8. $B = 46.5°$, $a = 7.71$, $b = 8.12$
9. $A = 27.07°$, $a = 24.4$, $c = 53.6$
10. $B = 17.88°$, $b = 26.9$, $c = 87.6$
11. $A = 36.9°$, $B = 53.1°$, $b = 80$
12. $A = 43.3°$, $B = 46.7°$, $b = 0.662$
13. $B = 37.32°$, $a = 5570$, $c = 7010$
14. $B = 34.4°$, $A = 55.6°$, $a = 4.22$
15. $A = 55.3°$, $B = 34.7°$, $a = 740$
16. $A = 60.1°$, $B = 29.9°$, $b = 29.9$

§29. Page 50

17. $B = 35.3^\circ$
$C = 84.7^\circ$
$c = 138$

18. $A = 87.8^\circ$
$C = 41.1^\circ$
$a = 117$

19. Impossible

20. $B = 28.6^\circ$
$C = 90^\circ$
$b = 4.79$

21. $A = 17.9^\circ$
$C = 53.1^\circ$
$a = 0.076$

22. $b = 279$
$c = 284$
$C = 100.83^\circ$

23. $c = 123.8$
$B = 3.31^\circ$
$C = 116.69^\circ$

24. $B = 4.15^\circ$
$C = 100.85^\circ$
$c = 40.7$

25. $B = 31.35^\circ$
$a = 30.5$
$b = 18.57$

26. $A = 79.15^\circ$
$a = 0.713$
$b = 0.1367$

27. $A = 68.43^\circ$
$b = 0.325$
$c = 0.883$

28. 160.7 yd.

29. 7.07

30. 35.3°

31. 1265 ft.

32. 1029 yds.

§30. Page 52

1. 0.142, 0.515, 1.907, 0.0177, 3.55, 19.1, 1.09
7.03, 1.94, 0.524, 56.40, 0.282, 0.0524, 0.918.

2. (*a*) 13.50° (*b*) 38.15° (*c*) 42.60° (*d*) 28.37° (*e*) 3.38°
(*f*) 4.69° (*g*) 23.36° (*h*) 2.465° (*i*) 0.855° (*j*) 20.5°
(*k*) 74.95° (*l*) 77.91° (*m*) 86.63° (*n*) 45.85° (*o*) 50.95°

3. (*a*) 76.5° (*b*) 51.85° (*c*) 47.40° (*d*) 61.63° (*e*) 86.6°
(*f*) 85.3° (*g*) 66.6° (*h*) 87.54° (*i*) 89.145° (*j*) 69.5°
(*k*) 15.05° (*l*) 12.09° (*m*) 3.37° (*n*) 44.15° (*o*) 39.05°

4. (*a*) 28.55° (*b*) 24.09° (*c*) 63.4°
(*d*) 51.7° (*e*) 50.2° (*f*) 83.14°

§32. Page 55

1. $A = 31.3^\circ$
$B = 58.7^\circ$
$c = 23.7$

2. $A = 41.05^\circ$
$B = 48.95^\circ$
$c = 153.8$

3. $A = 65^\circ$
$B = 25^\circ$
$c = 55.2$

4. $A = 33.15^\circ$
$B = 56.85^\circ$
$c = 499$

5. $A = 39.5^\circ$
$B = 50.5^\circ$
$c = 44$

6. $A = 67.38^\circ$
$B = 22.62^\circ$
$c = 13$

7. $A = 45^\circ$
$B = 45^\circ$
$c = 18.67$

8. $A = 30.6^\circ$
$B = 59.4^\circ$
$c = 82.5$

9. $A = 3.7^\circ$
$B = 86.3^\circ$
$c = 4.8$

10. 50°

§33. Page 56

1. $A = 31.3^\circ$
$B = 58.7^\circ$
$c = 23.7$

2. $A = 41.05^\circ$
$B = 48.95^\circ$
$c = 153.8$

3. $A = 65^\circ$
$B = 25^\circ$
$c = 55.2$

4. $A = 33.15^\circ$
$B = 56.85^\circ$
$c = 499$

5. $A = 39.5^\circ$
$B = 50.5^\circ$
$c = 44$

6. $A = 67.38^\circ$
$B = 22.62^\circ$
$c = 13$

7. $A = 45^\circ$
$B = 45^\circ$
$c = 18.67$

8. $A = 30.6^\circ$
$B = 59.4^\circ$
$c = 82.5$

9. $A = 3.7^\circ$
$B = 86.3^\circ$
$c = 4.8$

10. 50°

11. 18.6, 20°

12. 19.02, 25°

§34. Page 58

1. $A = 119.9°$, $B = 31.1°$, $c = 52.6$
2. $A = 49.05°$, $C = 79.15°$, $b = 104$
3. $A = 55°$, $B = 40.4°$, $c = 285$
4. $B = 39.2°$, $C = 78.8°$, $a = 3.21$
5. $A = 100.95°$, $C = 33.05°$, $b = 19.8$
6. $A = 46.4°$, $c = 6.4°$, $b = 7.43$
7. $A = 121.1°$, $C = 2.4°$, $b = 0.0828$
8. $A = 77.2°$, $B = 43.5°$, $c = 14.99$
9. $B = 13.38°$, $C = 28.32°$, $a = 7420$
10.
11. 10 and 4.68
12. 4.93 mi.
13. Lat. = 218 ft. Dep. = 478 ft.

§35. Page 60

1. $A = 106.77°$, $B = 46.9°$, $C = 26.33°$
2. $A = 27.35°$, $B = 143.1°$, $C = 9.55°$
3. $A = 52.4°$, $B = 59.4°$, $C = 68.2°$
4. $A = 49.2°$, $B = 37.6°$, $C = 93.2°$
5. $A = 106.3°$, $B = 37.9°$, $C = 35.8°$
6. $A = 48.2°$, $B = 58.4°$, $C = 73.4°$
7. $A = 44.6°$, $B = 49.5°$, $C = 85.9°$
8. $A = 83.7°$, $B = 59.3°$, $C = 36.9°$
9. $A = 53.1°$, $B = 59.5°$, $C = 67.4°$
10. 51.9°, 59.4°, 68.7°
11. 72.58°

§36. Page 62

1. $B_1 = 66.1°$, $C_1 = 58.5°$, $c_1 = 18.6$, $B_2 = 113.9°$, $C_2 = 10.7°$, $c_2 = 4.08$
2. $B_1 = 16.72°$, $A_1 = 147.46°$, $a_1 = 35.5$, $B_2 = 163.28°$, $A_2 = 0.9°$, $a_2 = 1.04$
3. $A_1 = 70.3°$, $B_1 = 57.3°$, $b_1 = 28.8$, $A_2 = 109.7°$, $B_2 = 17.9°$, $b_2 = 10.51$
4. $A_1 = 69°$, $C_1 = 67°$, $a_1 = 6.93$, $A_2 = 23°$, $C_2 = 113°$, $a_2 = 2.91$
5. $B_1 = 45.3°$, $C_1 = 99.1°$, $c_1 = 300$, $B_2 = 134.7°$, $C_2 = 9.7°$, $c_2 = 51.1$
6. $A_1 = 51.3°$, $C_1 = 88.7°$, $c_1 = 21{,}900$, $A_2 = 128.7°$, $C_2 = 11.3°$, $c_2 = 4290$
7. $p = 3.13$; (*a*) none, (*b*) 2, (*c*) 1

§37. Page 64

1. 30.5
2. 0.36
3. 4.6
4. 24.2
5. 14.23
6. 16.79
7. 5.26
8. 254
9. 0.0679
10. 0.267
11. 1.35
12. 16.47
13. 2.04
14. 0.720
15. 4.25
16. 1.225
17. 0.0771
18. 0.0963
19. 38.2
20. 0.00319
21. 0.001086
22. 50.9
23. 0.01375
24. 0.0432
25. (*a*) $\theta = 24.94°$, $\alpha = 40.15°$ (*b*) $y = 0.0731$, $\theta = 4.485°$ (*c*) $y = 2580$ (*d*) $y = 25.3$ (*e*) $y = 11.45$ (*f*) $y = 0.0885$ (*g*) $y = 0.638$ (*h*) $\theta = 4.13°$ (*i*) 16.43°

§ 38. Page 67

1. (*a*) 0.0247 (*b*) 0.01454 (*c*) 0.0436 (*d*) 0.0466
2. (*a*) 1.044° (*b*) 2.65° (*c*) 4.96°
3. (*a*) 0.0627 (*b*) 0.000627 (*c*) 0.627 (*d*) 6.27
4. (*a*) 1.696° (*b*) 16.96° (*c*) 0.01696°
5. (*a*) 15.92 (*b*) 7.59 (*c*) 0.00000548 (*d*) 50.6
6. 0.0597, 0.0597, 16.75, 16.75
7. 0.000977, 0.000977, 1023, 1023
8. 0.00436, 229, 229, 0.00436
9. (*a*) 60°, (*b*) 135° (*c*) 2.5° (*d*) 1° (*e*) 150°
10. (*a*) 0.0209 (*b*) 0.0846 (*c*) 5.40 (*d*) 20.0 (*e*) 0.0400 (*f*) 55.5
11. (*a*) 0.338 (*b*) 0.578 (*c*) 0.841 (*d*) 0.540 (*e*) 0.588 (*f*) 1.100 (*g*) 0.603 (*h*) − 0.946

§40. Page 69

1. $B = 70°$, $a = 27.4$, $b = 75.2$
2. $B = 80°$, $a = 5.29$, $x = 30.5$
3. $B = 15°$, $b = 21.4$, $c = 82.8$
4. $A = 30.3°$, $B = 59.7°$, $c = 20.0$
5. $A = 29°$, $B = 46.6°$, $C = 104.4°$
6. $A = 74.6°$, $B = 47.8°$, $C = 57.6°$
7. $B = 54°$, $a = 0.588$, $b = 0.809$
8. $C = 55.3°$, $b = 568$, $c = 664$
9. $C = 123.2°$, $b = 2050$, $c = 2360$
10. $B = 59.1°$, $a = 33.8$, $c = 18.51$
11. $A = 54.5°$, $B = 47.8°$, $b = 50.5$
12. $A = 40.9°$, $C = 59.9°$, $b = 77.1$
13. $A = 49.4°$, $B = 58.6°$, $C = 72°$
14. $A = 34.8°$, $B = 86.4°$, $C = 58.8°$
15. $C = 72.2°$, $A = 51.3°$, $c = 95.2$
16. $A = 69°$, $a = 3320$, $c = 7480$
17. $B = 24.8°$, $b = 38{,}900$, $A = 27.6°$
18. $A = 45.2°$, $B = 101.4°$, $C = 33.4°$
19. 81.4°
20. 5 ft., 2.34 ft.
21. 26.2 sec.
22. 47.9° E. of N.
23. 84.1°
24. 21.65 ft.

§41. Page 71

1. $x = 35.8$, $y = 19.36$; $r = 32.4$, $\theta = 55.6°$; $w = 18.0$, $\theta = 43.2°$
2. 16, 20°
3. 20, 25°
4. 2.4, 0.7
5. $x = 10.79$ lb.; $y = 26.7$ lb.
6. $r = 7.81$, $\theta = 74.2°$
7. 4.5, 126.9°
8. 206.6°
9. $m = 8.94$; $n = 4.47$
10. $z = 9.25$, $\theta = 27°$; $G = 0.0963$; $B = 0.0491$
11. $z = 14.42$, $B = 0.0551$; $G = 0.0422$

§43. Page 75

1. 677
2. 173.4
3. 129.4
4. 415
5. 41.7
6. 376
7. 382
8. 89.3 ft.
9. 10.910 ft.
10. 284 ft., 291 ft.
11. 864 ft., 708 ft., 246 ft.
12. 7.87 mi.

§45. Page 78

1. 0.2, 0.8, 0.9615
2. (*a*) 0.0625 (*b*) 0.284 (*c*) 64.5 (*d*) 0.513
   (*e*) 1.330 (*f*) 0.860 (*g*) 0.9860 (*h*) 1.0510

§46. Page 81

1. (*a*) 20.1 (*b*) 0.0498 (*c*) 1.492 (*d*) 0.670 (*e*) 1.03562 (*f*) 0.9656 (*g*) 3.827 (*h*) 0.2613 (*i*) 0.0854 (*j*) 34.8 (*k*) 0.9740 (*l*) 1.0890
2. (*a*) 8.33 (*b*) 0.1200 (*c*) 1.2361 (*d*) 0.8090 (*e*) 1.02143 (*f*) 0.9790
3. (*a*) 54.6 (*b*) 3640 (*c*) 1.537 (*d*) 1.0216 (*e*) 0.0334 (*f*) 0.8496 (*g*) 0.9817 (*h*) 0.00203
4. (*a*) 2.028 (*b*) 0.493 (*c*) 0.6553 (*d*) 1.526 (*e*) 2.314 (*f*) 0.4321 (*g*) 9.36 (*h*) 0.1069 (*i*) 0.686 (*j*) 1.458 (*k*) 1.781 (*l*) 0.561 (*m*) 25.0 (*n*) 25.95 (*o*) 2.201
5. (*a*) 0.680 (*b*) – 1.090 (*c*) 1.878 (*d*) 0.239 (*e*) –1.061 (*f*) – 0.0566
6. 17.33 seconds.
7. (*a*) 1.238 (*b*) 6.25 (*c*) 0.887 (*d*) 0.822 (*e*) 1.155 (*f*) 0.558
8. l = 129.2 ft., s = 53.0 ft.

§47. Page 83

1. 4, 25, 49, 64, 0.25, 0.04, 0.0204, 0.015625
2. 0.7646, 0.626, 0.405, 1.308, 1.598, 2.47
3. 0.3333, 1.1161, 0.8960, 1.01105, 0.98908
4. 9, 0.1111,16, 0.0625, 36, 0.0278, 81, 0.0123, 166.4, 0.00601
5. 0.9434, 1.791, 0.5584, 339, 0.00295
6. 10, 0.7943, 1.259, 0.97724, 1.0233
7. 5, 0.2, 1.1746, 0.8513.
8. 0.5927, 1.687, 0.00535, 187, 0.9490, 1.0537

§48. Page 87

1. 1.0474, 1.589, 0.00975
2. (a) 1.0550, 0.9479, 0.5857
   (b) 48.6, 0.0206, 1.0396
   (c) 0.576, 0.00403, 1.0567
   (d) 0.283, 1.01269, 3.53

3. 1.707	12. 1.01736	21. 3.00
4. 2.534	13. 0.842	22. 0.333
5. 1.475	14. 0.0492	23. 4.14
6. 0.9646	15. 2.380	24. 0.242
7. 0.0273	16. 0.8205	25. 2.23
8. 36.7	17. 0.9437	26. 0.9365
9. 11.4	18. 329	27. 0.0764
10. 0.0879	19. 0.9707	28. 1.2934
11. 1.1874	20. 0.0532	29. 3.893

30. (a) 0.0169, (b) 0.13, (c) 0.542

§49. Page 88

1. 6.21, 3.91, 0.693, 0.3365, 0.0421
2. − 6.21, − 3.91, − 0.693, − 0.3365, − 0.0953, − 0.0198
3. 20.1, 1.350, 1.0305, 0.0498, 0.741, 0.9704

4. (a) 4.33	(d) − 0.1744	(g) − 0.0954
(b) 2.03	(e) − 1.93	(h) 0.358
(c) 2.22	(f) − 0.1132	(i) 0.0421
5. (a) 30.0	(d) 0.698	(g) 0.347
(b) 1.405	(e) 0.577	(h) 9.21
(c) 0.1755	(f) 2.88	(i) 0.893

7. (a) 1.386 (b) 0.25, 1.1488, 0.8705, 1.01396, 0.98623

§50. Page 89

1. 2.365	11. 2.303
2. 1.333	12. 0.1386
3. 5	13. − 4.57
4. − 0.508	14. − 24.3
5. 0.0899	15. − 44.7
6. 0.52	16. 2.29
7. 1.4346	17. 103.2
8. 4.695	18. 0.00647
9. 5.535	19. 3.11, 4.24, − 0.0339, − 1.095, − 2.57
10. 1.762	20. 0.479, − 4.9, − 5.77, 0.0708

§52. Page 92

1. 20	10. 0.856	19. 1.0989	28. 6.35
2. 2.864	11. 0.2119	20. 1.53	29. − 1.988
3. 2.155	12. 0.00435	21. 0.791	30. − 0.3356
4. 2.335	13. 0.5805	22. 0.524	31. − 6.55
5. 1.935	14. 0.9471	23. 49	32. − 70.7
6. 0.7275	15. 9600	24. 0.847	33. 1.253
7. 0.058	16. 2.5	25. − 0.379	34. − 0.467
8. 0.752	17. 1.096	26. 1.541	35. 0.23
9. 0.9846	18. 36	27. 17.54	36. 48

§53. Page 94

1. (a) 5.86×10^3 (b) 6.75×10^5 (c) 6.23×10^{-2} (d) 4.79×10^{-1} (e) 2.8×10^7 (f) 2.76×10^9 (g) 9.1×10^{-7} (h) 4.95×10^3 (i) 8.645×10^{-3}
2. (a) 1.8×10^7 (b) 3.43×10^{-7} (c) 2.7×10^{-5} (d) 2.45×10^4 (e) 8×10^4 (f) 2×10^{-4}
3. (a) 6.91×10^{-7} (b) 9.04×10^9 (c) 8.48×10^4 (d) 2.37×10^3 (e) 1.288×10^1 (f) 1.781
4. (a) 4.31×10^{-1} (b) 1.17×10^3 (c) 5.03×10^4 (d) 3.8 (e) 2.38×10^1 (f) 3.425×10^2
5. 5.88×10^{12} mi.

§54. Page 95

1. 2.49×10^8
2. 2.23×10^8
3. 5.52×10^{-11}
4. 0.81
5. (a) 5.56×10^8 (b) 4.42×10^8 (c) 2.24×10^{-11} (d) 0.797
6. (a) 4.31×10^5 (b) 3.44×10^5 (c) 1.0061 (d) 1.0067
7. 4.467
8. 36.9
9. 5.74
10. 5.76
11. 0.00283

§55. Page 99

1. 1.0920, 0.9157, 2.411, 0.415
2. 1.07, 0.9345, 1.00677, 0.99323
3. 1.0233, 0.9772, 1.00231, 0.99769, 1.000231, 0.000769
4. 1.007, 0.993, 1.0007, 0.9993
5. 1.0305, 0.9705, 1.0325, 0.9685
6. 1.499, 0.667, 1.4966, 0.668
7. 1.049, 1.00478, 1.000478
8. 0.9762, 0.99760, 0.999760
9. (a) 1.3037, 1.0269, 1.002656, 1.0002656
   (b) 0.7652, 0.9736, 0.99733, 0.999733
10. 1.0000417, 0.9999583, 1000417, 0.999583, 1.00416, 0.99583, 1.0427, 0.9592, 1.517, 0.659

§56. Page 101

1. (a) 0.605, 0.07278, 0.9472, 976.9
   (b) 0.001736, 0.00012235, 104.66, 1.0138
   (c) 0.0004077, 33.75, 4.77, 0.044327
   (d) 2.747, 216.46, 0.0002639, 0.00020256
2. 0.001912, 96.53, 0.00010451, 0.0007775, 4032, 115.8, 0.027218, 12.61, 160, 0.00023474.
3. 0.9968, 1.006, 0.992, 1.001, 0.010017, 99.58

§57. Page 102

1. 11.9
2. 1.281
3. 0.1755
4. 0.698
5. 0.285
6. 0.882
7. 2.06
8. 1.783
9. 0.5769
10. 0.561
11. 2.88
12. 0.347
13. 0.914
14. 0.916
15. 0.9607
16. 0.996
17. 1.78
18. 318
19. 2.382
20. 1.254
21. 2.99
22. 2.05
23. 0.45
24. 0.1613

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