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POLYPHASE DUPLEX DECITRIG

TRADE MARK

Slide Rule

No. 4071

A MANUAL

BY

LYMAN M. KELLS, PH.D.
Associate Professor of Mathematics

WILLIS F. KERN
Assistant Professor of Mathematics

AND

JAMES R. BLAND
Assistant Professor of Mathematics

All at the United States Naval Academy

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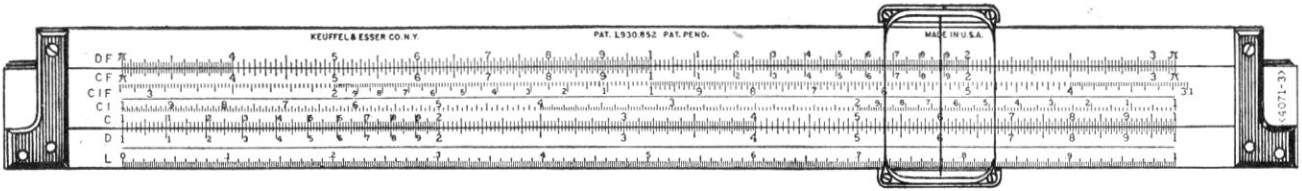

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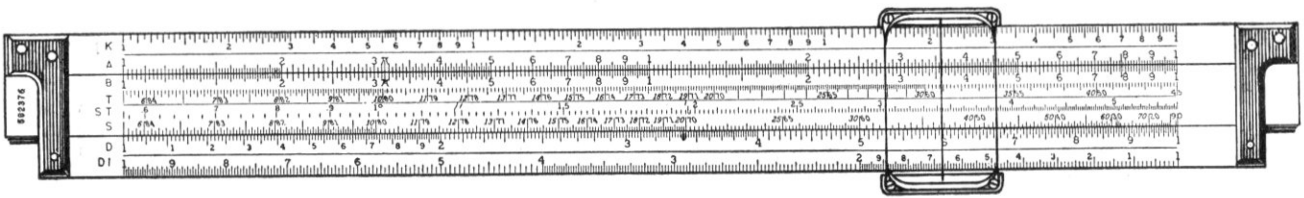

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PREFACE

This slide rule manual has been written for study without the aid of a teacher. For this reason one might suspect that the treatment is superficial. On the contrary, however, the subject matter is so presented that the beginner uses two general principles while he is learning to read the scales and perform the simpler operations. The mastery of these two principles gives the power to devise the best settings for any particular purpose, and to recall settings which have been forgotten.

These principles are so simple and so carefully explained and illustrated both by diagram and by example that they are easily mastered. In Chapter II, they are applied to simple problems in multiplication and division; in Chapters III, IV, V they are used to solve problems involving multiplication, division, square and cube root, trigonometry, logarithms, and powers of numbers. Chapter V explains the slide rule from the logarithmic standpoint. Those who desire a theoretical treatment are likely to be surprised to find that the principles of the slide rule are so easily understood in terms of logarithms.

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CHAPTER I

MULTIPLICATION AND DIVISION

1. Introduction. This pamphlet is designed to enable any interested person to learn to use the slide rule efficiently. The beginner should keep his slide rule before him while reading the pamphlet, should make all settings indicated in the illustrative examples, and should compute answers for a large number of the exercises. The principles involved are easily understood but a certain amount of practice is required to enable one to use the slide rule efficiently and with a minimum of error.

2. Reading the scales.* Everyone has read a ruler in measuring a length. The number of inches is shown by a number appearing on the ruler, then small divisions are counted to get the number of 16th's of an inch in the fractional part of the inch, and finally in close measurement, a fraction of a 16th of an inch may be estimated. We first read a primary length, then a secondary length, and finally estimate a tertiary length. Exactly the same method is used in reading the slide rule. The divisions on the slide rule are not uniform in length, but the same principle applies.

Figure 1 represents, in skeleton form, the fundamental scale of the slide rule, namely the *D* scale. An examination of this actual

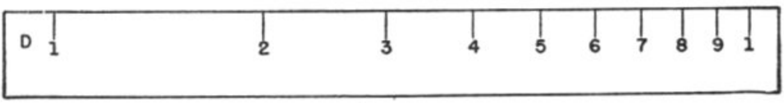


FIG. 1.

scale on the slide rule will show that it is divided into 9 parts by primary marks which are numbered 1, 2, 3, . . . , 9, 1. The space between any two primary marks is divided into ten parts by nine secondary marks. These are not numbered on the actual scale except

*The description here given has reference to the 10″ slide rule. However anyone having a rule of different length will be able to understand his rule in the light of the explanation given.

between the primary marks numbered 1 and 2. Fig. 2 shows the secondary marks lying between the primary marks of the *D* scale. On this scale each italicized number gives the reading to be associated

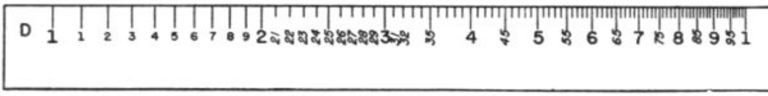


FIG. 2.

with its corresponding secondary mark. Thus, the first secondary mark after 2 is numbered 21, the second 22, the third 23, etc.; the first secondary mark after 3 is numbered 31, the second 32, etc. Between the primary marks numbered 1 and 2 the secondary marks are numbered 1, 2, . . . , 9. Evidently the readings associated with these marks are 11, 12, 13, . . . , 19. Finally between the secondary marks, see Fig. 3, appear smaller or tertiary marks which aid in

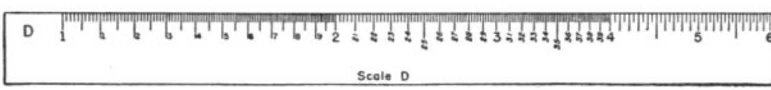


FIG. 3.

obtaining the third digit of a reading. Thus between the secondary marks numbered 22 and 23 there are 4 tertiary marks. If we think of the end marks as representing 220 and 230, the four tertiary marks divide the interval into five parts each representing 2 units. Hence with these marks we associate the numbers 222, 224, 226, and 228; similarly the tertiary marks between the secondary marks numbered 32 and 33 are read 322, 324, 326, and 328, and the tertiary marks between the primary marks numbered 3 and the first succeeding secondary mark are read 302, 304, 306, and 308. Between any pair of secondary marks to the right of the primary mark numbered 4, there is only one tertiary mark. Hence, each smallest space represents five units. Thus the tertiary mark between the secondary marks representing 41 and 42 is read 415, that between the secondary marks representing 55 and 56 is read 555, and the first tertiary mark to the right of the primary mark numbered 4 is read 405. The reading of any position between a pair of successive tertiary marks must be based on an estimate. Thus a position half way between the tertiary marks associated with 222 and 224 is read 223 and a position two fifths of the way from the tertiary mark numbered 415 to the next mark is read 417. The principle illustrated by these readings applies in all cases.

Consider the process of finding on the *D* scale the position representing 246. The first figure on the left, namely 2, tells us that the position lies between the primary marks numbered 2 and 3. This region is indicated by the brace in figure (*a*). The second figure from the left, namely 4, tells us that the position lies

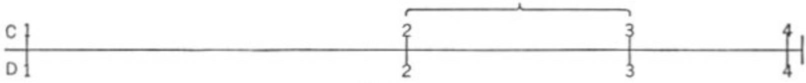


FIG. *a*.

between the secondary marks associated with 24 and 25. This region is indicated by the brace in Fig. (*b*). Now there are 4 marks be-

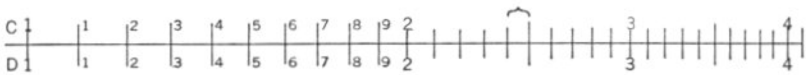


FIG. *b*.

tween the secondary marks associated with 24 and 25. With these are associated the numbers 242, 244, 246, and 248 respectively.

FIG. *c*.

Thus the position representing 246 is indicated by the arrow in Fig. (*c*). Fig. (*abc*) gives a condensed summary of the process.

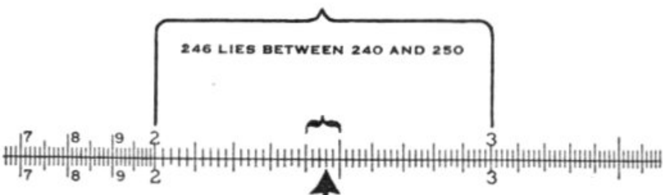


FIG. *abc*.

It is important to note that the decimal point has no bearing upon the position associated with a number on the *C* and *D* scales. Consequently, the arrow in Fig. (*abc*) may represent 246, 2.46, 0.000246, 24,600, or any other number whose principal digits are 2, 4, 6. The placing of the decimal point will be explained later in this chapter.

For a position between the primary marks numbered 1 and 2, four digits should be read; the first three will be exact and the last one estimated. No attempt should be made to read more than three digits for positions to the right of the primary mark numbered 4.

While making a reading, the learner should have definitely in mind the number associated with the smallest space under consideration. Thus between 1 and 2, the smallest division is associated with 10 in the fourth place; between 2 and 3, the smallest division has a value 2 in the third place; while to the right of 4, the smallest division has a value 5 in the third place.

The learner should read from Fig. 4 the numbers associated with the marks lettered A, B, C, . . . and compare his readings with the

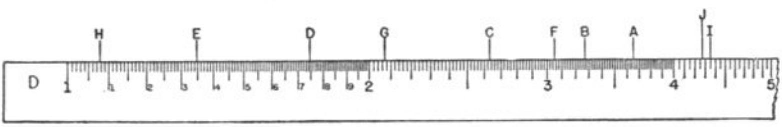


FIG 4.

following numbers: A 365, B 327, C 263, D 1745, E 1347, F 305, G 207, H 1078, I 435, J 427.

3. Accuracy of the slide rule. From the discussion of §2, it appears that we read four figures of a result on one part of the scale and three figures on the remaining part. This means an attainable accuracy of roughly 1 part in 1000 or one tenth of one per cent. The accuracy is nearly proportional to the length of the scale. Hence we associate with the 20 inch scale an accuracy of about one part in 2000, and with the Thacher Cylindrical slide rule, an accuracy of about one part in 10,000. The accuracy obtainable with the 10 inch slide rule is sufficient for most practical purposes; in any case the slide rule result serves as a check.

4. Definitions. The central sliding part of the rule is called the *slide*, the other part the *body*. The glass runner is called the *indicator* and the line on the indicator is referred to as the *hairline*.

The mark associated with the primary number 1 on any scale is called the *index* of the scale. An examination of the D scale shows that it has two indices, one at the left end and the other at the right end.

Two positions on different scales are said to be *opposite* if, without moving the slide, the hairline may be brought to cover both positions at the same time.

5. Multiplication. The process of multiplication may be performed by using scales C and D. The C scale is on the slide, but in other respects it is like the D scale and is read in the same manner.

To multiply 2 by 4,

> to 2 on D set index of C,
> push hairline to 4 on C,
> at the hairline read **8** on D.

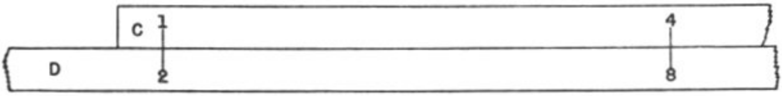


FIG. 5 (*a*).

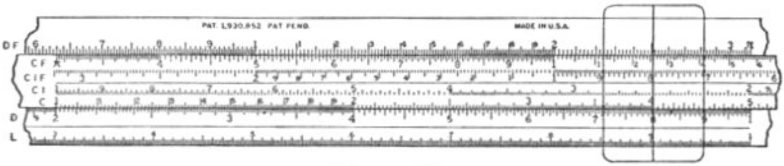


FIG. 5 (*b*).

Fig. 5(*b*) shows the rule set for multiplying 2 by 4 and Fig. 5(*a*) shows the same setting in skeleton form. To multiply 3×3,

> to 3 on D set index of C,
> push hairline to 3 on C,
> at the hairline read **9** on D.

See Fig. 6(*a*) for the setting in skeleton form and Fig. 6(*b*) for a photograph of the setting.

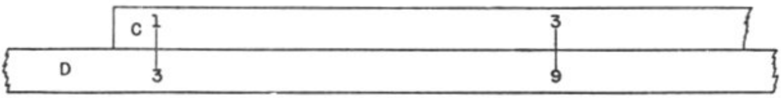


FIG. 6 (*a*).

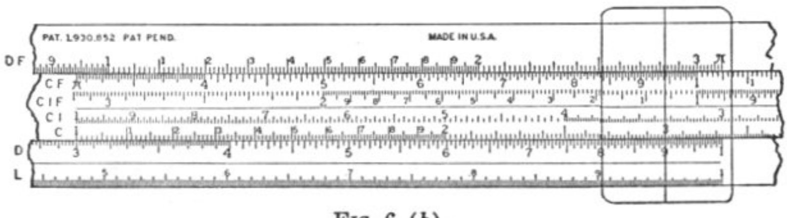


FIG. 6 (*b*).

To multiply 1.5×3.5, disregard the decimal point and
to 15 on D set index of C,
push hairline to 35 on C,
at the hairline read **525** on D.

By inspection we know that the answer is near to 5 and is therefore **5.25**.

To find the value of 16.75×2.83 (see Fig. 7(*a*) and Fig. 7(*b*))

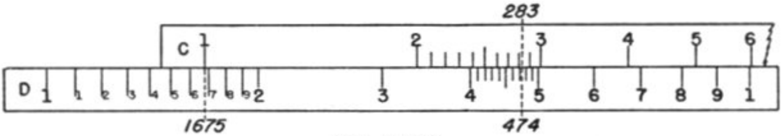


FIG. 7 (*a*).

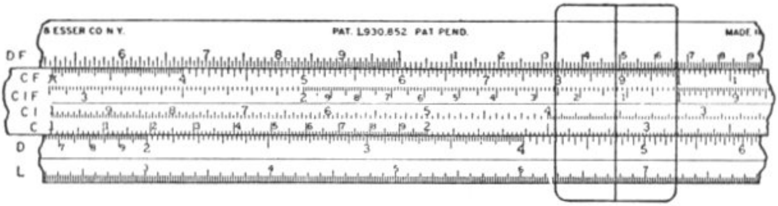


FIG. 7 (*b*).

disregard the decimal point and
to 1675 on D set index of C,
push hairline to 283 on C,
at the hairline read **474** on D.

To place the decimal point we approximate the answer by noting that it is near to $3 \times 16 = 48$. Hence the answer is **47.4**.

These examples illustrate the use of the following rule.

Rule. *To find the product of two numbers, disregard the decimal points, opposite either of the numbers on the D scale set the index of the C scale, push the hairline of the indicator to the second number on the C scale, and read the answer under the hairline on the D scale. The decimal point is placed in accordance with the result of a rough calculation.*

EXERCISES

1. 3×2.
2. 3.5×2.
3. 5×2.
4. 2×4.55.
5. 4.5×1.5.
6. 1.75×5.5.
7. 4.33×11.5.
8. 2.03×167.3.
9. 1.536×30.6.
10. 0.0756×1.093.
11. 1.047×3080.
12. 0.00205×408.
13. $(3.142)^2$
14. $(1.756)^2$

6. Either index may be used. It may happen that a product cannot be read when the left index of the C scale is used in the rule of §5. This will be due to the fact that the second number of the product is on the part of the slide projecting beyond the body. In this case reset the slide using the right index of the C scale in place of the left, or use the following rule: *when a number is to be read on the D scale opposite a number of the C scale and cannot be read, push the hairline to the index of the C scale inside the body and draw the other index of the C scale under the hairline.** The desired reading can then be made.

If, to find the product of 2 and 6, we set the left index of the C scale opposite 2 on the D scale, we cannot read the answer on the D scale opposite 6 on the C scale. Hence, we set the right index of C opposite 2 on D; opposite 6 on C read the answer, **12**, on D.

Again, to find 0.0314×564,

to 314 on D set the right index of C,
push hairline to 564 on C,
at the hairline read **1771** on D.

A rough approximation is obtained by finding $0.03 \times 600 = 18$. Hence the product is **17.71**.

EXERCISES

Perform the indicated multiplications.

1. 3×5.
2. 3.05×5.17.
3. 5.56×634.
4. 743×0.0567.
5. 0.0495×0.0267.
6. 1.876×926.
7. 1.876×5.32.
8. 42.3×31.7.

7. Division. The process of division is performed by using the C and D scales.

To divide 8 by 4 (see Figs. 8(a) and 8(b)),

push hairline to 8 on D,
draw 4 of C under the hairline,
opposite index of C read **2** on D.

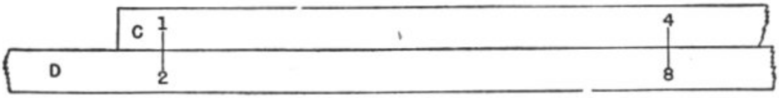


FIG. 8 (a).

*This rule, slightly modified to apply to the scales being used, is generally applicable when an operation calls for setting the hairline to a position on the part of the slide extending beyond the body.

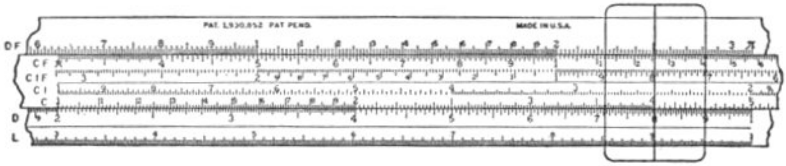


FIG. 8 (*b*).

To divide 876 by 20.4,

push hairline to 876 on D,
draw 204 of C under the hairline,
opposite index of C read **429** on D.

The rough calculation $800 \div 20 = 40$ shows that the decimal point must be placed after the 2. Hence the answer is **42.9.**

These examples illustrate the use of the following rule.

Rule. *To find the quotient of two numbers, disregard the decimal points, opposite the numerator on the D scale set the denominator on the C scale, opposite the index of the C scale read the quotient on the D scale. The position of the decimal point is determined from information gained by making a rough calculation.*

EXERCISES

Perform the indicated operations.

1. $87.5 \div 37.7$.
2. $3.75 \div 0.0227$.
3. $0.685 \div 8.93$.
4. $1029 \div 9.70$.
5. $0.00377 \div 5.29$.
6. $2875 \div 37.1$.
7. $871 \div 0.468$.
8. $0.0385 \div 0.001462$.
9. $3.14 \div 2.72$.
10. $3.42 \div 81.7$.

8. Simple applications, percentage, rates. Many problems involving percentage and rates are easily solved by means of the slide rule.

One per cent (1%) of a number N is $N \times 1/100$; hence 5% of N is $N \times 5/100$, and, in general, p% of N is $pN/100$. Hence to find 83% of 1872

to 1872 on D set right index of C,
push hairline to 83 on C,
at the hairline read **1554** on D.

Since $(83/100) \times 1872$ is approximately $\dfrac{80}{100} \times 2000 = 1600$, the answer is 1554.

To find the answer to the question "M is what per cent of N?" we must find $100\ M \div N$. Thus, to find the answer to the question "87 is what per cent of 184.7?" we must divide $87 \times 100 = 8700$ by 184.7. Hence

push hairline to 87 on D,
draw 1847 of C under the hairline,
opposite index of C read **471** on D.

The rough calculation $\frac{9000}{200} = 45$ shows that the decimal point should be placed after the 7. Hence the answer is **47.1%**.

For a body moving with a constant velocity, distance = rate times time. Hence if we write d for distance, r for rate, and t for time, we have

$$d = rt, \text{ or } r = \frac{d}{t} \text{ or } t = \frac{d}{r}.$$

To find the distance traveled by a car going 33.7 miles per hour for 7.75 hours, write $d = 33.7 \times 7.75$, and

to 337 on D set right index of C,
push hairline to 775 on C,
at hairline read **261** on D.

Since the answer is near to $8 \times 30 = 240$ miles, we have $d =$ **261** miles.

To find the average rate at which a driver must travel to cover 287 miles in 8.75 hours, write $r = 287 \div 8.75$, and

push hairline to 287 on D,
draw 875 of C under the hairline,
opposite the index of C read **328** on D.

Since the rate is near $280 \div 10 = 28$, we have $r =$ **32.8** miles per hour

EXERCISES

1. Find (*a*) 86.3 per cent of 1826.
   (*b*) 75.2 per cent of 3.46.
   (*c*) 18.3 per cent of 28.7.
   (*d*) 0.95 per cent of 483.

2. What per cent of
   (*a*) 69 is 18?
   (*b*) 132 is 85?
   (*c*) 87.6 is 192.8?
   (*d*) 1027 is 28?

3. Find the distance covered by a body moving
   (*a*) 23.7 miles per hour for 7.55 hours.
   (*b*) 68.3 miles per hour for 1.773 hours.
   (*c*) 128.7 miles per hour for 16.65 hours.

4. At what rate must a body move to cover
   (a) 100 yards in 10.85 seconds.
   (b) 386 feet in 25.7 seconds.
   (c) 93,000,000 miles in 8 minutes and 20 seconds.

5. Find the time required to move
   (a) 100 yards at 9.87 yards per second.
   (b) 3800 miles at 128.7 miles per hour.
   (c) 25,000 miles at 77.5 miles per hour.

9. Use of the scales *DF* and *CF* (folded scales). The *DF* and the *CF* scales are the same as the *C* and the *D* scales respectively except in the position of their indices. The fundamental fact concerning the folded scales may be stated as follows: *if for any setting of the slide, a number M of the C scale is opposite a number N on the D scale, then the number M of the CF scale is opposite the number N on the DF scale.* Thus, if the learner will draw 1 of the *CF* scale opposite 1.5 on the *DF* scale, he will find the following opposites on the *CF* and *DF* scales

CF	1	2	4	5	6	6.67	
DF	1.5	3	6	7.5	9	1	

and the same opposites will appear on the *C* and *D* scales.

In accordance with the principle stated above, if the operator wishes to read a number on the *D* scale opposite a number *N* on the *C* scale but cannot do so, he can generally read the required number on the *DF* scale opposite *N* on the *CF* scale. For example to find 2×6,

to 2 on *D* set left index of *C*,
push hairline to 6 on *CF*,
at the hairline read **12** on *DF*.

By using the *CF* and *DF* scales we saved the trouble of moving the slide as well as the attendant source of error. This saving, entering as it does in many ways, is a main reason for using the folded scales.

The folded scales may be used to perform multiplications and divisions just as the *C* and *D* scales are used. Thus to find 6.17×7.34,

to 617 on *DF* set index of *CF*,
push hairline to 734 on *CF*,
at the hairline read **45.3** on *DF*;

or

to 617 on DF set index of CF,
push hairline to 734 on C,
at the hairline read **45.3** on D.

Again to find the quotient 7.68/8.43,

push hairline to 768 on DF,
draw 843 of CF under the hairline,
opposite the index of CF read **0.912** on DF;

or

push hairline to 768 on DF,
draw 843 of CF under the hairline,
opposite the index of C read **0.912** on D.

It now appears that we may perform a multiplication or a division in several ways by using two or more of the scales C, D, CF, and DF. The sentence written in italics near the beginning of the article sets forth the guiding principle. A convenient method of multiplying or dividing a number by π ($= 3.14$ approx.) is based on the statement: any number on DF is π times its opposite on D, and any number on D is $1/\pi$ times its opposite on DF.

EXERCISES

Perform each of the operations indicated in exercises 1 to 11 in four ways; *first* by using the C and D scales only; *second* by using the CF and DF scales only; *third* by using the C and D scales for the initial setting and the CF and DF scales for completing the solution; *fourth* by using the CF and DF scales for the initial setting and the C and D scales for completing the solution.

1. 5.78×6.35.
2. 7.84×1.065.
3. $0.00465 \div 73.6$.
4. $0.0634 \times 53{,}600$.
5. $1.769 \div 496$.
6. $946 \div 0.0677$.
7. 813×1.951.
8. $0.00755 \div 0.338$.
9. $0.0948 \div 7.23$.
10. $149.0 \div 63.3$.
11. $2.718 \div 65.7$.
12. 783π.
13. $783 \div \pi$.
14. 0.0876π.
15. $0.504 \div \pi$.
16. $1.072 \div 10.97$.

CHAPTER II

THE PROPORTION PRINCIPLE AND COMBINED OPERATIONS

10. Introduction. The *ratio* of two numbers a and b is the quotient of a divided by b or a/b. A statement of equality between two ratios is called a proportion. Thus

$$\frac{2}{3}=\frac{6}{9},\quad \frac{x}{5}=\frac{7}{11},\quad \frac{a}{b}=\frac{c}{d}$$

are proportions. We shall at times refer to equations having such forms as

$$\frac{2}{3}=\frac{x}{5}=\frac{9}{y}=\frac{10}{z}, \text{ and } \frac{a}{b}=\frac{c}{d}=\frac{e}{f}$$

as proportions.

An important setting like the one for multiplication, the one for division, and any other one that the operator will use frequently should be practiced until it is made without thought. But, in the process of devising the best settings to obtain a particular result, of making a setting used infrequently, or of recalling a forgotten setting, the application of proportions as explained in the next article is very useful.

11. Use of Proportions. If the slide is drawn to any position, the ratio of any number on the D scale to its opposite on the C scale is, in accordance with the setting for division, equal to the number on the D scale opposite the index on the C scale. In other words, *when the slide is set in any position, the ratio of any number on the D scale to its opposite on the C scale is the same as the ratio of any other number on the D scale to its opposite on the C scale.* For example

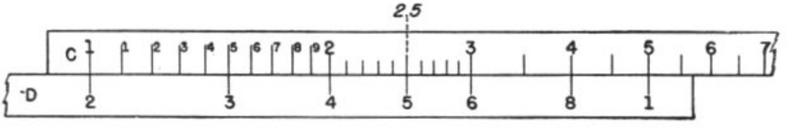


FIG. 1.

draw 1 of C opposite 2 on D (see Fig. 1) and find the opposites indicated in the following table:

C (or CF)	1	1.5	2.5	3	4	5
D (or DF)	2	3	5	6	8	10

,

and draw 2 of C over 1 on D and read the same opposites. The same statement is true if in it we replace C scale by CF scale and D scale by DF scale. Hence, *if both numerator n and denominator d of a ratio in a given proportion are known, we can set n of the C scale opposite d on the D scale and then read, for an equal ratio having one part known, its unknown part opposite the known part.* We could also begin by setting d on the C scale opposite n on the D scale. It is important to observe that *all the numerators of a series of equal ratios must appear on one scale and the denominators on the other.* For example, let it be required to find the value of x satisfying

$$\frac{x}{56} = \frac{9}{7}, \tag{1}$$

Here the known ratio is 9/7. Hence

push hairline to 7 on D,
draw 9 of C under the hairline,
push hairline to 56 on D,
at the hairline read **72** on C.

or

push hairline to 9 on D,
draw 7 of C under the hairline,
push hairline to 56 on C,
at the hairline read **72** on D.

The CF and DF scales could have been used to obtain exactly the same settings and results.

For convenience we shall indicate the settings for solving (1) as follows:

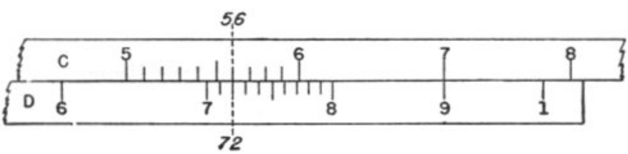


FIG. 2.

C (or CF)	set 7	opposite 56	(2)
D (or DF)	to 9	read x (= 72)	

C (or CF)	set 9	read x (= 72)	(3)
D (or DF)	to 7	opposite 56	

The letter at the beginning of a line indicates that all numbers on that line are to appear on the scale designated by that letter, and any pair of numbers set opposite each other in the frame are to appear as opposites on the slide rule.

To find the values of x, y, and z defined by the equations

$$\frac{C}{D}: \qquad \frac{3.15}{5.29} = \frac{x}{4.35} = \frac{57.6}{y} = \frac{z}{183.4},$$

we observe that 3.15/5.29 is the known ratio, and

push hairline to 529 on D,
draw 315 of C under the hairline;
opposite 435 on D, read x = **2.59** on C;
opposite 576 on C, read y = **96.7** on D;
opposite 1834 on D, read z = **109.2** on C.

The positions of the decimal points were determined by noticing that each denominator had to be somewhat less than twice its associated numerator because 5.29 is somewhat less than twice 3.15. The setting may appear simpler when written in the following form analogous to (2):

C	set 3.15	read x (= 2.59)	opposite 57.6	read z (= 109.2)	(4)
D	to 5.29	opposite 4.35	read y (= 96.7)	opposite 183.4	

When an answer cannot be read, apply the italicized rule of §6. Thus to find the values of x and y satisfying

$$\frac{C}{D}: \qquad \frac{x}{587} = \frac{14.56}{97.6} = \frac{5.78}{y},$$

to 976 on D set 1456 of C; then, since the answers cannot be read, push the hairline to the index on C, draw the right index of C under the hairline and

opposite 587 on D, read $x =$ **87.6** on C;
opposite 578 on C, read $y =$ **38.7** on D.

Here the positions of the decimal points were determined by observing that each denominator had to be about six times the associated numerator.

When a result cannot be read on the C scale nor on the D scale it may be possible to read it on the CF scale or on the DF scale. Thus, to find x and y satisfying the equations

$$\frac{C\ (\text{or } CF)}{D\ (\text{or } DF)}: \qquad \frac{4.92}{x} = \frac{1}{3.23} = \frac{y}{13.08},$$

to 323 on D set left index of C;
opposite 492 on CF, read $x =$ **15.89** on DF;
opposite 1308 on DF, read $y =$ **4.05** on CF.

A slight inspection of the scales will show the value of the statement: If the difference of the first digits of the two numbers of the known ratio is small use the C and D scales for the initial setting; if the difference is large use the CF and DF scales. Since in the next to the last example, the difference between the first digits was great, the CF and DF scales should have been used for the initial setting. This would have eliminated the necessity for shifting the slide.

EXERCISES

Find, in each of the following equations, the values of the unknowns.

1. $\frac{2}{3} = \frac{x}{7.83}$.

2. $\frac{x}{1.804} = \frac{y}{25} = \frac{1}{0.785}$.

3. $\frac{x}{709} = \frac{246}{y} = \frac{28}{384}$.

4. $\frac{x}{0.204} = \frac{y}{0.0506} = \frac{5.28}{z} = \frac{2.01}{0.1034}$.

5. $\frac{x}{2.07} = \frac{3}{61.3} = \frac{z}{1.571}$.

6. $\frac{8.51}{1.5} = \frac{9}{x} = \frac{235}{y}$.

7. $\frac{17}{x} = \frac{1.365}{8.53} = \frac{4.86}{y}$.

8. $\dfrac{x}{y} = \dfrac{y}{7.34} = \dfrac{3.75}{29.7}$.

9. $\dfrac{x}{49.6} = \dfrac{z}{y} = \dfrac{y}{3.58} = \dfrac{1.076}{0.287}$.

12. Forming proportions from equations. Since proportions are algebraic equations, they may be rearranged in accordance with the laws of algebra. For example, if

$$x = \frac{ab}{c}, \tag{1}$$

we may write the proportion

$$\frac{x}{1} = \frac{ab}{c}, \tag{2}$$

or we may divide both sides by a to get

$$\frac{x}{a} = \frac{ab}{ac}, \text{ or } \frac{x}{a} = \frac{b}{c}, \tag{3}$$

or we may multiply both sides by c/x to obtain

$$\frac{cx}{x} = \frac{cab}{xc}, \text{ or } \frac{c}{1} = \frac{ab}{x}. \tag{4}$$

Rule (*A*). *A number may be divided by 1 to form a ratio.* This was done in obtaining proportion (2).

Rule (*B*). *A factor of the numerator of either ratio of a proportion may be replaced by 1 and written as a factor of the denominator of the other ratio, and a factor of the denominator of either ratio may be replaced by 1 and written as a factor of the numerator of the other ratio.* Thus (3) could have been obtained from (1) by transferring a from the numerator of the right hand ratio to the denominator of the left hand ratio.

For example, to find $\dfrac{16 \times 28}{35}$, write $x = \dfrac{16 \times 28}{35}$, apply (*B*) to

obtain $\dfrac{C}{D}$: $\quad \dfrac{x}{16} = \dfrac{28}{35}$,

and push hairline to 35 on *D*,
draw 28 of *C* under the hairline;
opposite 16 on *D*, read $x = \mathbf{12.8}$ on *C*.

To recall the rule for dividing a given number M by a second given number N, write $x = \frac{M}{N}$, apply rule (A) to obtain $\frac{D}{C}$: $\qquad \frac{x}{1} = \frac{M}{N}$,

and push hairline to M on D,
draw N of C under the hairline;
opposite index of C, read x on D.

To recall the rule for multiplication, set $x = \frac{M\ N}{1}$, apply rule (B) to obtain $\frac{D}{C}$: $\qquad \frac{X}{M} = \frac{N}{1}$,

and to N on D set index of C;
opposite M on C, read x on D.

To find x if $\frac{1}{x} = \frac{864}{(7.48)\ (25.5)}$, use rule (B) to get $\frac{7.48}{x} = \frac{864}{25.5}$, make the corresponding setting and read $x =$ **0.221.** The position of the decimal point was determined by observing that x must be about $\frac{1}{40}$ of 8, or 0.2.

EXERCISES

Find in each case the value of the unknown quantity.

1. $y = \frac{86 \times 70.8}{125}$.
2. $y = \frac{147.5 \times 8.76}{3260}$.
3. $y = \frac{0.797 \times 5.96}{0.502}$.
4. $\frac{37 \times 86}{y} = 75.7$.
5. $498 = \frac{89.3x}{0.563}$.
6. $0.874 = \frac{3.95 \times 0.707}{x}$;
7. $2580y = 17.9 \times 587$.
8. $0.695 = \frac{0.0879}{x}$.
9. $\frac{1}{386} = \frac{0.772}{2.85y}$.
10. $3.14y = 0.785 \times 38.7$.

13. Equivalent expressions of quantity.* When the value of a quantity is known in terms of one unit, it is a simple matter to find its value in terms of a second unit. Thus to find the number of square feet in 3210 sq. in., write

$$\frac{1}{144} = \frac{\text{no. of sq. ft.}}{\text{no. of sq. in.}} = \frac{x}{3210},$$

*Tables of equivalents may be found in Engineer's Manuals and in other places.

since there are 144 sq. in. in a square foot; hence

to 144 on D, set index of C;
opposite 3210 on D, read $x = $ **22.3** on C;

that is, there are 22.3 sq. ft. in 3210 sq. in.

Again consider the problem of finding the number of nautical miles in 28.5 ordinary miles. Since there are 5280 ft. in an ordinary mile and 6080 ft. in a nautical mile, write

$$\frac{5280}{6080} = \frac{\text{no. of naut. mi.}}{\text{no. of ord. mi.}} = \frac{x}{28.5},$$

make the corresponding setting and read $x = $ **24.7** naut. mi.

EXERCISES

1. An inch is equivalent to 2.54 cm. Find the length in cm. of a rod 66 in. long.

2. The number of meters in a given length is to the number of yards as 171 is to 187. Find the number of meters in a 300 yd. distance.

3. If 7.5 gal. water weighs 62.4 lbs., find the weight of 86.5 gal. water.

4. 31 sq. in. is approximately 200 sq. cm. How many square centimeters in 36.5 sq. in.?

5. If one horse-power is equivalent to 746 watts, how many watts are equivalent to (a) 34.5 horsepower, (b) 5280 horsepower, (c) 0.832 horsepower?

6. If one gallon is equivalent to 3790 cu. cm., find the number of gallons of water in a bottle which contains (a) 4250 cu. cm., (b) 9.68 cu. cm., (c) 570 cu. cm. of the liquid.

14. The *CI* and *CIF* (reciprocal) scales. The reciprocal of a number is obtained by dividing 1 by the number. Thus, $\frac{1}{2}$ is the reciprocal of 2, $\frac{2}{3}$ $\left(= 1 \div \frac{3}{2}\right)$ is the reciprocal of $\frac{3}{2}$, and $\frac{1}{a}$ is the reciprocal of a.

The reciprocal scales CI, DI and CIF are like the C, D, and CF scales, respectively, with the exception that they are inverted, i.e., the numbers represented by the marks on these scales increase from right to left. The red numbers associated with the reciprocal scales enable the operator to recognize these scales. A very important consideration may be stated as follows: *When the hairline is set to a number on the C scale, the reciprocal (or Inverse) of the number is at the hairline on the CI scale; conversely, when the hairline is set to a number on the CI scale, its reciprocal is at the hairline on the C scale.* A similar relation exists between the D and DI

scales and between the *CF* and *CIF* scales. If the operator will close his rule, he can read the opposites indicated in the diagram

CI (or *DI*)	1	2	4	5	8	9	
C (or *D*)	1	0.5 (= 1/2)	0.25 (= 1/4)	0.2 (= 1/5)	0.125 (= 1/8)	0.1111 (= 1/9)	

By using the facts just mentioned, we can multiply a number or divide it by the reciprocal of another number. Thus to find $\frac{28}{7}$, we may think of it as $28 \times \frac{1}{7}$ and

> to 28 on *D* set index of *C*;
> opposite 7 on *CI* read **4** on *D*.

Again to find 12×3, we may think of it as $12 \div \frac{1}{3}$ and

> push hairline to 12 on *D*,
> draw 3 of *CI* under the hairline;
> opposite index of *C*, read **36** on *D*.

When the *CI* scale is used in multiplication and division, the position of the decimal point is determined in the usual way.

The *DF* and *CIF* scales may be used to perform multiplications and divisions in the same manner as the *D* and *CI* scales; thus to multiply 40.3 by 1/9.04,

> to 403 on *DF* set index of *CF*;
> opposite 904 on *CIF*, read **4.46 on *DF*.**

Again to multiply 40.3 by 1/0.207,

> to 403 on *D* set left index of *C*;
> opposite 207 on *CIF* read **194.7** on *DF*.

EXERCISES

1. Use the *CI* scale to find the reciprocals of 16, 260, 0.72, 0.065, 17.4, 18.5, 67.1.

2. Using the *D* scale and the *CI* scale, multiply 18 by 1/9 and divide 18 by 1/9.

3. Using the *D* scale and the *CI* scale multiply 28.5 by 1/0.385 and divide 28.5 by 1/0.385. Also find 28.5/0.385 and 28.5×0.385 by using the *C* scale and the *D* scale.

4. Using the *D* scale and the *CI* scale multiply 41.3 by 1/0.207 and divide 41.3 by 1/0.207.

5. Perform the operations of Exercises 2, 3, and 4 by using the *CIF* scale and the *DF* scale.

15. Proportions involving the *CI* scale. The CI scale may be used in connection with proportions containing reciprocals. Since any number $a = 1 \div \frac{1}{a}$ and since $\frac{1}{a} = \frac{1}{a} \div 1$, we have

Rule C. *The value of any ratio is not changed if any factor of its numerator be replaced by 1 and its reciprocal be written in the denominator, or if any factor of its denominator be replaced by 1 and its reciprocal be written in the numerator.* Thus $\frac{a}{b} = a\left(\frac{1}{b}\right) = \frac{1}{b\ (1/a)}$. Hence if $\frac{x}{a} = bc$, we may write $\frac{x}{a} = \frac{b}{(1/c)} = \frac{c}{(1/b)}$; if $ax = bc$, we may write $\frac{x}{(1/a)} = \frac{b}{(1/c)} = \frac{c}{(1/b)}$. A few examples will indicate the method of applying these ideas in computations.

To find the value of y which satisfies $\frac{y}{4.27} = 0.785 \times 3.76$, apply Rule C to get $\frac{D}{C}$:
$$\frac{y}{4.27} = \frac{0.785}{(1/3.76)}.$$

Since when 3.76 of CI is under the hairline, $1/3.76$ of C is also under the hairline

push hairline to 785 on D,
draw 376 of CI under the hairline;
opposite 427 on CF, read $y =$ **12.60** on DF.

The position of the decimal point was determined by observing that y was near to $4 \times 1 \times 4 = 16$.

To find the value of y which satisfies $7.89\ y = \frac{0.0645}{0.381}$, use Rule ($C$) to obtain $\frac{D}{C}$:
$$\frac{y}{(1/7.89)} = \frac{0.0645}{0.381},$$

and push hairline to 645 on D,
draw 381 of C under the hairline;
opposite 789 on CI, read $y =$ **0.0215** on D.

The position of the decimal point was determined by observing that .06 is about $\frac{1}{6}$ of 0.38, that y is therefore about $\frac{1}{6}$ of $\frac{1}{8}$, or about 0.02.

To find the values of x and y which satisfy $57.6x = 0.846y = 7$, use Rule (C) to obtain

$$\frac{D}{CI}: \qquad \frac{x}{(1/57.6)} = \frac{y}{(1/0.846)} = \frac{7}{1}, \tag{a}$$

and to 7 on D set index of CI;
opposite 576 on CI, read $x =$ **0.1215** on D;
opposite 846 on CIF, read $y =$ **8.27** on DF.

The folded scales may also be used in this connection. Thus to solve equations (a),

to 7 on DF set index of CIF;
opposite 576 on CIF, read $x =$ **0.1215** on DF;
opposite 846 on CIF, read $y =$ **8.27** on DF.

EXERCISES

In each of the following equations find the values of the unknown numbers:

1. $3.3x = 4.4y = \dfrac{75.2}{1.342}$.
2. $76.1x = 3.44y = \dfrac{111}{22.8}$.
3. $1.83x = \dfrac{y}{24.5} = (162)\ (1.75)$.
4. $\dfrac{0.342}{x} = \dfrac{y}{4.65} = (189)\ (0.734)$.
5. $5.83x = 6.44y = \dfrac{12.6}{z} = 0.2804$.
6. $3.42x = \dfrac{1.83}{y} = \dfrac{17.6}{z} = (2.78)\ (13.62)$.

16. Combined multiplication and division.

Example 1. Find the value of $\dfrac{7.36 \times 8.44}{92}$.

Solution. Reason as follows: first divide 7.36 by 92 and then multiply the result by 8.44. This would suggest that we

push hairline to 736 on D,
draw 92 of C under the hairline;
opposite 8.44 on C, read **0.675** on D.

Example 2. Find the value of $\dfrac{18 \times 45 \times 37}{23 \times 29}$.

Solution. Reason as follows: (a) divide 18 by 23, (b) multiply the result by 45, (c) divide this second result by 29, (d) multiply this third result by 37. This argument suggests that we

push hairline to 18 on D,
draw 23 of C under the hairline.
push hairline to 45 on C,
draw 29 of C under the hairline,
push hairline to 37 on C,
at the hairline read **449** on D.

To determine the position of the decimal point write $\dfrac{20 \times 40 \times 40}{20 \times 30}$ = about 50. Hence the answer is 44.9.

A little reflection on the procedure of Example 2 will enable the operator to evaluate by the shortest method expressions similar to the one just considered. He should observe that: the D scale was used only twice, once at the beginning of the process and once at its end; *the process for each number of the denominator consisted in drawing that number, located on the C scale, under the hairline; the process for each number of the numerator consisted in pushing the hairline to that number located on the C scale.*

If at any time the indicator cannot be placed because of the projection of the slide, apply the rule of §6, *or carry on the operations using the folded scales.*

Example 3. Find the value of $1.843 \times 92 \times 2.45 \times 0.584 \times 365$.

Solution. By using Rule C of §15, write the given expression in the form

$$\frac{1.843 \times 2.45 \times 365}{(1/92)\ (1/0.584)}$$

and reason as follows: (a) divide 1.843 by (1/92), (b) multiply the result by 2.45, (c) divide this second result by (1/0.584), (d) multiply the third result by 365. This argument suggests that we

push hairline to 1843 on D,
draw 92 of CI under the hairline,
push hairline to 245 on C,
draw 584 of CI under the hairline,
push hairline to 365 on C,
at the hairline read **886** on D.

To approximate the answer we write 2(90) (5/2) (6/10) 300 = 81,000 Hence the answer is **88,600.**

Example 4. Find the value of $\dfrac{0.873 \times 46.5 \times 6.25 \times 0.75}{7.12}$.

Solution. The following arrangement in which the difference between the number of factors in the numerator and the number in the denominator is no greater than 1 is obtained by applying Rule (*C*) of §15:

$$\frac{0.873 \times 46.5 \times 0.75}{7.12 \times (1/6.25)}$$

This may be evaluated by (*a*) dividing 0.873 by 7.12, (*b*) multiplying the result by 46.5, (*c*) dividing the second result by (1/6.25), (*d*) multiplying the third result by 0.75. Hence

push hairline to 873 on *D*,
draw 712 of *C* under the hairline,
push hairline to 465 on *C*,
draw 625 of *CI* under the hairline,
push hairline to 75 on *CF*,
at the hairline read **267** on *DF*.

To approximate the answer write $\dfrac{1 \times 42 \times 6 \times 1}{7} = 36$. Hence the answer is 26.7.

A consideration of the examples of this article indicates that the operator should rewrite the expression to be evaluated so that its numerator shall contain the same number of factors, or one more factor than its denominator.

EXERCISES

1. $\dfrac{1375 \times 0.0642}{76{,}400}$.

2. $\dfrac{45.2 \times 11.24}{336}$.

3. $\dfrac{218}{4.23 \times 50.8}$.

4. $\dfrac{235}{3.86 \times 3.54}$.

5. $2.84 \times 6.52 \times 5.19$.

6. $9.21 \times 0.1795 \times 0.0672$.

7. $37.7 \times 4.82 \times 830$.

8. $\dfrac{65.7 \times 0.835}{3.58}$.

9. $\dfrac{362}{3.86 \times 9.61}$.

10. $\dfrac{24.1}{261 \times 32.1}$.

11. $\dfrac{75.5 \times 63.4 \times 95}{3.14}$.

12. $\dfrac{3.97}{51.2 \times 0.925 \times 3.14}$.

13. $\dfrac{47.3 \times 3.14}{32.5 \times 16.4}$.

14 $\dfrac{3.82 \times 6.95 \times 7.85 \times 436}{79.8 \times 0.0317 \times 870}$.

15. $187 \times 0.00236 \times 0.0768 \times 1047 \times 3.14$.

16. $\dfrac{0.917 \times 8.65 \times 1076 \times 3152}{7840}$.

CHAPTER III

SQUARES AND SQUARE ROOTS, CUBES AND CUBE ROOTS

17. Squares. The square of a number is the result of multiplying the number by itself. Thus $2^2 = 2 \times 2 = 4$.

The A scale is so designed that *when the hairline is set to a number on the D scale, the square of the number is found under the hairline on the A scale.* Similarly, if the hairline be set to a number on the C scale its square may be read at the hairline on the B scale. Note that the rule can be turned at will to enable the user to refer from one face to the other. For example, if one hairline of the indicator is set to 2 on C, the number $4 = 2^2$ will be found at the other hairline on scale B.

To gain familiarity with the relations between these scales the operator should set the hairline to 3 on the D scale, and read 9 at the hairline on the A scale; set the hairline to 4 on D, read 16 at the hairline on A; etc. To find 278^2, set the hairline to 278 on D, read 773 at the hairline on A. Since $300^2 = 90{,}000$, we write 77,300 as the answer. Actually $278^2 = 77{,}284$. The answer obtained on the slide rule is accurate to three figures.

The area of a circle may conveniently be found when its radius is known by using the A, B, C, and D scales. If π represents a mathematical constant whose value is approximately 3.14, and d represents the diameter of a circle, then the area A of the circle is given by the formula $A = (\pi/4)\, d^2 = 0.785\, d^2$ nearly. The area is also given by $A = \pi r^2$ where r represents the radius of the circle. Hence to find the area of a circle,

to $\pi/4$ ($= 0.785$ approx.) on A set index of B;
opposite diameter on C, read area on A.

Note that a special mark toward the right end of the A scale gives the exact position of $\pi/4$. Thus to find the area of a circle of diameter 17.5,

to $\pi/4$ on A right set index of B;
opposite 175 on C, read **241** on A.

Therefore the area is **241** sq. ft.

EXERCISES

1. Use the slide rule to find, accurate to three figures, the square of each of the following numbers: 25, 32, 61, 75, 89, 733, 452, 2.08, 1.753, 0.334, 0.00356, 0.953, 5270, 4.73×10^6.

2. Find the area of a circle having diameter (*a*) 2.75 ft.; (*b*) 66.8 ft.; (*c*) 0.753 ft.; (*d*) 1.876 ft.

3. Find the area of a circle having radius (*a*) 3.46 ft.; (*b*) 0.0436 ft.; (*c*) 17.53 ft.; (*d*) 8650 ft.

18. Square roots. The square root of a given number is a second number whose square is the given number. Thus the square root of 4 is 2 and the square root of 9 is 3, or, using the symbol for square root, $\sqrt{4} = 2$, and $\sqrt{9} = 3$.

The *A* scale consists of two parts which differ only in slight details. We shall refer to the left hand part as *A left* and to the right hand part as *A right*. Similar reference will be made to the *B* scale.

Rule. *To find the square root of a number between* 1 *and* 10, *set the hairline to the number on scale A left; and read its square root at the hairline on the D scale. To find the square root of a number between* 10 *and* 100, *set the hairline to the number on scale A right and read its square root at the hairline on the D scale.* In either case place the decimal point after the first digit. *A* similar statement relating to the *B* scale and the *C* scale holds true. For example, set the hairline to 9 on scale *A* left, read 3 $(= \sqrt{9})$ at the hairline on *D*, set the hairline to 25 on scale *B* right, read 5 $(= \sqrt{25})$ at the hairline on *C*.

To obtain the square root of any number, *move the decimal point an even number of places to obtain a number between* 1 *and* 100; *then apply the rule written above in italics; finally move the decimal point one half as many places as it was moved in the original number but in the opposite direction.** The learner may also place the decimal point in accordance with information derived from a rough approximation.

For example, to find the square root of 23,400, move the decimal point 4 places to the left thus getting 2.34 (a number between 1 and

*The following rule may also be used: If the square root of a number greater than unity is desired, use *A* left when it contains an odd number of digits to the left of the decimal point, otherwise use *A* right. For a number less than unity use *A* left if the number of zeros immediately following the decimal point is odd: otherwise, use *A* right.

10), set the hairline to 2.34 on scale A left, read 1.530 at the hairline on the D scale, finally move the decimal point $\frac{1}{2}$ of 4 or 2 places to the right to obtain the answer 153.0. The decimal point could have been placed after observing that $\sqrt{10{,}000} = 100$ or that $\sqrt{40{,}000} = 200$. Also the left B scale and the C scale could have been used instead of the left A scale and the D scale.

To find $\sqrt{3850}$, move the decimal point 2 places to the left to obtain $\sqrt{38.50}$, set the hairline to 38.50 on scale A right, read 6.20 at the hairline on the D scale, move the decimal point one place to the right to obtain the answer **62.0**. The decimal point could have been placed by observing that $\sqrt{3600} = 60$.

To find $\sqrt{0.000585}$, move the decimal point 4 places to the right to obtain $\sqrt{5.85}$, find $\sqrt{5.85} = 2.42$, move the decimal point two places to the left to obtain the answer **0.0242**.

EXERCISES

1. Find the square root of each of the following numbers: 8, 12, 17, 89, 8.90, 890, 0.89, 7280, 0.0635, 0.0000635, 63,500, 100,000.

2. Find the length of the side of a square whose area is (*a*) 53,500 ft.^{2}; (*b*) 0.0776 ft.^{2}; (*c*) 3.27×10^7 ft.^{2}

3. Find the diameter of a circle having area (*a*) 256 ft.^{2}; (*b*) 0.773 ft.^{2}; (*c*) 1950 ft.^{2}

19. Evaluation of simple expressions containing square roots and squares. When the hairline is set to a number on the proper one of the two B scales, its square root is automatically set to the hairline on the C scale. Consequently we may multiply and divide numbers by square roots of other numbers or we may find the value of the unknown in a proportion involving square roots.

For example, to find $3\sqrt{3.24}$ set index of C to 3 on D, push hairline to 3.24 on left B scale, read 540 at the hairline on D. Since $3\sqrt{3.24}$ is nearly equal to $3\sqrt{4} = 6$, the answer is **5.40**.

It will be convenient at times to indicate solutions by forms like the following which indicates a setting for evaluating $3\sqrt{3.240}$:

B		opposite 3.24 (left)
C	set 1	
D	to 3	read **5.40**

Note that each number is written on the same line as the letter naming *the scale on which it is to be set, and that numbers in the same vertical column are to be set opposite each other.*

The same plan is used below to indicate a setting for evaluating $85 \div \sqrt{4290}$:

B	set 4290 (right)	
C		opposite 1
D	to 85	read **1.297**

Computations involving square roots may be considered from the standpoint of multiplication and division or from that of proportions. Thus to find the value of $\frac{28\sqrt{375}}{369}$, we may compute it directly by the multiplication and division principle or we may write $x = \frac{28\sqrt{375}}{369}$, apply Rule B §12 to obtain

$$\frac{C}{D}: \qquad \frac{x}{28} = \frac{\sqrt{375} \rightarrow (B\,\text{left})}{369},$$

and solve the proportion to obtain the answer $x = \mathbf{1.470}$.

To find $x = \frac{347}{7.92 \times \sqrt{0.0465}}$, write the equation in the form $x = \frac{347\ (1/7.92)}{\sqrt{0.0465}}$ and perform the operations indicated in the following diagram:

B	set 0.0465 (left)	
CI		opposite 7.92
D	to 347	read $x =$ **203**

The position of the decimal point was obtained from the fact that $320 \div (0.2)\ 8 = 200$.

A second method consists in applying Rule B §12 to get $7.92x = \frac{347}{\sqrt{0.0465}}$, then applying Rule C §15 to obtain

$$\frac{x}{(1/7.92)} = \frac{347}{\sqrt{0.0465}},$$

and solving this proportion to find $x = \mathbf{203}$.

When the hairline is set to a number on the D scale it is automatically set to the square of the number on the A scale, and when set to a number on the C scale it is automatically set to the square of the number on the B scale. Hence by using the A and B scales as fundamental scales, many expressions involving squares can be evaluated conveniently. Thus to find $x = \dfrac{(24.6)^2 \times 0.785}{4.39}$, write

$$\frac{A}{B}: \qquad \frac{x}{0.785} = \frac{(24.6)^2}{4.39},$$

and

push hairline to 246 on D,
draw 439 of B (either left or right) under the hairline,
push hairline to 785 on B (left or right)
at the hairline read **108.2** on A.

The decimal point was placed in the usual manner. Of course this computation could have been carried out on the C and D scales, but one will find it convenient at times to use the setting just indicated.

EXERCISES

1. $42.2\sqrt{0.328}$.
2. $1.83\sqrt{0.0517}$.
3. $\sqrt{3.28} \div 0.212$.
4. $\sqrt{51.7} \div 103$.
5. $0.763 \div \sqrt{0.0296}$.
6. $\dfrac{5.66 \times (7.48)^2}{79}$.
7. $\dfrac{2.56 \times 4.86}{(1.365)^2}$.
8. $\dfrac{(2.38)^2 \times 19.7}{18.14}$.
9. $\dfrac{6.76}{2.17\ (2.7)^2}$.
10. $\dfrac{\sqrt{277}}{5.34 \times \sqrt{7.02}}$.
11. $\dfrac{645}{5.34\sqrt{13.6}}$.
12. $14.3 \times 47.5\sqrt{0.344}$.
13. $20.6 \times \sqrt{7.89} \times \sqrt{0.571}$.
14. $\dfrac{7.92\sqrt{7.89}}{\sqrt{0.571}}$.

20. Combined operations involving square roots and squares. The principle of Example 2 §16 may be applied to evaluate a fraction containing indicated square roots as well as numbers and reciprocals of numbers. If the learner will recall that when the hairline is set to a number on the CI scale it is automatically set to the reciprocal of the number on the C scale and when set to a number on the B scale it is automatically set to the square root of the number on the C scale, he will easily understand that the method used in this article is essentially the same as that used in §16. The principle of determining whether B left or B right should be used is the same whether we are merely extracting the square root of a number or whether the square root is involved with other numbers.

Example 1. Evaluate $\dfrac{\sqrt{365} \times 915}{804}$.

Solution. To evaluate this expression, we may think "divide $\sqrt{365}$ by 804 and multiply the result by 915." To set $\sqrt{365}$ on D, set the hairline to 365 on A left. Hence

push hairline to 365 on A left,
draw 804 of C under the hairline,
push hairline to 915 on C,
at the hairline read **21.7** on D.

The following diagram indicates the setting:

A	opposite 365 (left)	
C	set 804	opposite 915
D		read **21.7**

Example 2. Evaluate $\dfrac{\sqrt{832} \times \sqrt{365} \times 1863}{(1/736) \times 89{,}400}$.

Solution. Before making the setting indicated in this solution, the learner should read the italicized statement in §16, p. 24.

Push hairline to 832 on A left,
draw 736 of CI under the hairline,
push hairline to 365 on B left,
draw 894 of C under the hairline,
push hairline to 1863 on CF,
at the hairline read **8450** on DF.

Example 3. Evaluate $\dfrac{0.286 \times 652 \times \sqrt{2350} \times \sqrt{5.53}}{785\sqrt{1288}}$.

Solution. Write the expression in the form

$$\frac{0.286 \times \sqrt{2350} \times \sqrt{5.53}}{(1/652) \times 785 \times \sqrt{1288}} \quad \text{and}$$

push hairline to 286 on D,
draw 652 of CI under the hairline,

push hairline to 235 on B right,
draw 785 of C under the hairline,
push hairline to 553 on B left,
draw 1288 of B right under hairline,
opposite the index of C read **0.755** on D.

Example 4. Evaluate $\dfrac{\pi^2 \times 875 \times 278}{(72.2)^2\ (0.317)^2}$.

Solution. Using the A and B scales as fundamental scales,
push hairline to 3.142 on D,
draw 722 of C under the hairline,
push hairline to 875 on B,
draw 317 of C under the hairline,
push hairline to 278 on B,
at the hairline read **4580** on A.

EXERCISES

1. $\dfrac{7.87 \times \sqrt{377}}{2.38}$.
2. $\dfrac{86 \times \sqrt{734} \times \pi}{775 \times \sqrt{0.685}}$.
3. $\dfrac{4.25 \times \sqrt{63.5} \times \sqrt{7.75}}{0.275 \times \pi}$.
4. $\dfrac{(2.60)^2}{2.17 \times 7.28}$.
5. $\dfrac{20.6 \times 7.89^2 \times 6.79^2}{4.67^2 \times 281}$;
6. $\dfrac{189.7 \times \sqrt{0.00296} \times \sqrt{347} \times 0.274}{\sqrt{2.85} \times 165 \times \pi}$.
7. $\sqrt{285} \times 667 \times \sqrt{6.65} \times 78.4 \times \sqrt{0.00449}$.
8. $\dfrac{239 \times \sqrt{0.677} \times 374 \times 9.45 \times \pi}{84.3 \times \sqrt{9350} \times \sqrt{28400}}$.

21. Cubes. The cube of a number is the result of using the number three times as a factor. Thus the cube of 3 (written 3^3) is $3 \times 3 \times 3 = 27$.

The K scale is so constructed that *when the hairline is set to a number on the D scale, the cube of the number is at the hairline on the K scale.* To convince himself of this the operator should set the hairline to 2 on D, read 8 at the hairline on K, set the hairline to 3 on D, read 27 at the hairline on K, etc. To find 21.7^3, set the hairline to 217 on D and read 102 on K. Since $20^3 = 8000$, the answer is near 8000. Hence we write 10,200 as the answer. To obtain this answer otherwise, write

$$21.7^3 = \frac{21.7 \times 21.7}{(1/21.7)}$$

and use the general method of combined operations. This latter method is more accurate as it is carried out on the full length scales.

EXERCISES

1. Cube each of the following numbers by using the K scale and also by using the method of combined operations: 2.1, 3.2, 62, 75, 89, 733, 0.452, 3.08, 1.753, 0.0334, 0.943, 5270, 3.85×10^6.

2. How many gallons will a cubical tank hold that measures 26 inches in depth? (1 gal. = 231 cu. in.)

22. Cube roots. There are three parts to the K scale, each the same as the others except in position. We shall refer to the left hand part, the middle part, and the right hand part as K left, K middle, and K right respectively.

The cube root of a given number is a second number whose cube is the given number.

Rule. *To find the cube root of a number between* 1 *and* 10 *set the hairline to the number on K left, read its cube root at the hairline on D. To find the cube root of a number between* 10 *and* 100, *set the hairline to the number on K middle, and read its cube root at the hairline on D. The cube root of a number between* 100 *and* 1000 *is found on the D scale opposite the number on K right.* In each of the three cases the decimal point is placed after the first digit. To see how this rule is used, set the hairline to 8 on K left, read 2 at the hairline on D; set the hairline to 27 on K middle, read 3 at the hairline on D; set the hairline to 343 on K right, read 7 at the hairline on D.

To obtain the cube root of any number, move the decimal point over three places (or digits) *at a time until a number between* 1 *and* 1000 *is obtained, then apply the rule written above in italics; finally move the decimal point one third as many places as it was moved in the original number but in the opposite direction.* The learner may also place the decimal point in accordance with information derived from a rough approximation.

For example, to find the cube root of 23,400,000, move the decimal point 6 places to the left thus obtaining 23.4. Since this is between 10 and 100, set the hairline to 234 on K middle, read 2.86 at the hairline on D. Move the decimal point $\frac{1}{3}(6) = 2$ places to the right to obtain the answer 286. The decimal point could have been placed after observing that $\sqrt[3]{27{,}000{,}000} = 300$.

To obtain $\sqrt[3]{0.000585}$, move the decimal point 6 places to the right to obtain $\sqrt[3]{585}$, set the hairline to 585 on K right, and read $\sqrt[3]{585} = 8.36$. Then move the decimal point $\frac{1}{3}(6) = 2$ places to the left to obtain the answer 0.0836.

EXERCISES

1. Find the cube root of each of the following numbers: 8.72, 30, 729, 850, 7630, 0.00763, 0.0763, 0.763, 89,600, 0.625, 75×10^7, 10, 100, 100,000.

23. Combined Operations. By setting the hairline to numbers on various scales we may set square roots, cube roots and reciprocals of numbers on the D scale and on the C scale. Hence we can use the slide rule to evaluate expressions involving such quantities, and we can solve proportions involving them. The position of the decimal point is determined by a rough calculation.

Example 1. Find the value of $\frac{\sqrt[3]{385}}{2.36}$.

Solution. We may think of this as a division or write the proportion $\frac{x}{1} = \frac{\sqrt[3]{385}}{2.36}$, and then make the setting indicated in the following diagram:

C	set 2.36	opposite 1
D		read x = **3.08**
K	opposite 385 (right)	

Example 2. Find the value of $\frac{5.37\sqrt[3]{0.0835}}{\sqrt{52.5}}$.

Solution. Equating the given expression to x and applying Rule B §12, we write

$$\frac{x}{5.37} = \frac{\sqrt[3]{0.0835}}{\sqrt{52.5}}.$$

This proportion suggests the setting indicated in the following diagram:

C		opposite 537
D		read x = **0.324**
K	to 835 (middle)	
B	set 52.5 (right)	

Example 3. Evaluate

$$\frac{(1.736)\ (6.45)\ \sqrt{8590}\ \sqrt[3]{581}}{\sqrt{27.8}}.$$

Solution. By using rule (C) of §15, write the given expression in the form

$$\frac{\sqrt[3]{581}\ (6.45)\ \sqrt{8590}}{(1/1.736)\ \sqrt{27.8}},$$

and

set the hairline to 581 on K right,
draw 1736 of CI under the hairline,
push hairline to 645 on C,
draw 278 of B right under the hairline,
push hairline to 8590 on B right,
at the hairline read **1643** on D.

Note that Examples 1 and 2 were attacked by the proportion principle whereas Example 3 was considered as a series of multiplications and divisions. When no confusion results, the student should always think of an exercise as a series of multiplications and divisions. The proportion principle should be used in case of doubt.

EXERCISES

1. $\sqrt[3]{73.2}\ (0.523)$.

2. $24.3\ \sqrt[3]{0.0661}\pi$.

3. $489 \div \sqrt[3]{732}$.

4. $27\pi \div \sqrt[3]{661{,}000}$.

5. $\sqrt[3]{531} \div \sqrt{28.4}$.

6. $\sqrt{9.80} \div \sqrt[3]{160{,}000}$.

7. $(72.3)^2 \times 8.25$.

8. $\dfrac{\pi(0.213)^2}{0.0817}$.

9. $\dfrac{\sqrt[3]{19.2^2}}{(7.13)^2 \times 0.122}$.

10. $\dfrac{\pi\sqrt[3]{740}}{4.46 \times \sqrt{28.5}}$.

11. $3.83 \times 6.26 \times \sqrt[3]{54.2}$.

12. $0.437 \times \sqrt{564} \times \sqrt[3]{1.86}$.

13. $675 \times \sqrt{0.346} \times \sqrt[3]{0.00711}$.

14. $\dfrac{\sqrt[3]{32.1}\ (0.0585)\pi}{(1/3.63)}$.

15. $\dfrac{3.57 \times \sqrt{643} \times 4250}{0.0346\ \sqrt{0.00753}}$.

16. $\dfrac{\sqrt[3]{0.00335}\ \sqrt{273}}{787\ \sqrt{0.723}}$.

17. $\dfrac{0.0872 \times 36.8 \times \sqrt{2.85}}{0.343\pi}$.

18. $76.2\ \sqrt[3]{56.1}\ \sqrt{877}\ (1/3.78)$.

19. $\dfrac{\sqrt{1.735}}{0.0276\sqrt{58{,}300} \times 7.63 \times 0.476}$.

20. $\dfrac{68.7\sqrt[3]{3160}\sqrt{0.0317} \times 89.3}{17.6 \times 277}$.

21. $\dfrac{\sqrt[3]{0.0645} \times 1834 \times \sqrt{21.6}}{89.6 \times 748 \times \sqrt{3460}}$.

22. $\sqrt{(27.5)^2 - (3.483)^2}$.

24. The L scale. The problems of this chapter could well be solved by means of logarithms. The following statements indicate how the L scale is used to find the logarithms of numbers to the base 10.

(A) When the hairline is set to a number on the D scale it is at the same time set to the mantissa (fractional part) of the common logarithm of the number on the L scale, and conversely, when the hairline is set to a number on the L scale it is set **on the** D scale to the antilogarithm of that number.

(B) The characteristic (integral part) of the common logarithm of a number greater than 1 is positive and is one less than the number of digits to the left of the decimal point; the characteristic of a number less than 1 is negative and is numerically one greater than the number of zeros immediately following the decimal point.

Example. Find the logarithm of (a) 50; (b) 1.6; (c) 0.35; (d) 0.00905.

Solution. (a) To find the mantissa of log 50,
push hairline to 50 on D,
at hairline on L read 699.

Hence the mantissa is .699. Since 50 has two digits to the left of the decimal point, its characteristic is 1.

Therefore $\log 50 = \mathbf{1.699}.$

(b) Push hairline to 16 on D,
at hairline on L read 204.

Supplying the characteristic in accordance with (B), we have

$$\log 1.6 = \mathbf{0.204}.$$

(c) Push hairline to 35 on D,
at hairline on L read 544.

Hence, in accordance with (B), we have

$$\log 0.35 = \mathbf{9.544 - 10}.$$

(d) Push hairline to 905 on D,
at hairline on L read 956.

Hence, in accordance with (B), we have

$$\log 0.00905 = \mathbf{7.956 - 10}.$$

EXERCISE

Find the logarithms of the following numbers: 32.7, 6.51, 980,000, 0.676 0.01052. 0.000412. 72.6, 0.267. 0.00802, 432.

CHAPTER IV

PLANE TRIGONOMETRY*

25. Some important formulas from plane trigonometry. The following formulas from plane trigonometry, given for the convenience of the student, will be employed in the slide rule solution of trigonometric problems considered in this chapter.

In the right triangle ABC of Fig. 1, the side opposite the angle A is designated by a, the side opposite B by b, and the hypotenuse by c. Referring to this figure, we write the following definitions and relations.

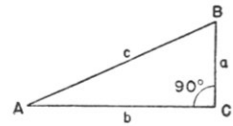


FIG. 1.

Definitions of the sine, cosine, and tangent:

$$\text{sine } A \text{ (written } \sin A) = \frac{a}{c} = \frac{\text{opposite side}}{\text{hypotenuse}},$$

$$\text{cosine } A \text{ (written } \cos A) = \frac{b}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}},$$

$$\text{tangent } A \text{ (written } \tan A) = \frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}}.$$

Reciprocal relations:

$$\text{cosecant } A \text{ (written } \csc A) = \frac{c}{a} = \frac{1}{\sin A},$$

$$\text{secant } A \text{ (written } \sec A) = \frac{c}{b} = \frac{1}{\cos A},$$

$$\text{cotangent } A \text{ (written } \cot A) = \frac{b}{a} = \frac{1}{\tan A}.$$

Relations between complementary angles:

$$\begin{aligned} \cos A &= \sin\,(90° - A), \\ \tan A &= \cot\,(90° - A), \\ \cot A &= \tan\,(90° - A). \end{aligned}$$

*See the authors' "Plane and Spherical Trigonometry," McGraw-Hill Book Co., New York, N.Y., for a thorough treatment of the solution of triangles both by logarithmic computation and by means of the slide rule.

Relations between supplementary angles:

$$\sin (180° - A) = \sin A,$$
$$\cos (180° - A) = - \cos A,$$
$$\tan (180° - A) = - \tan A.$$

Relation between angles in a right triangle:

$$A + B = 90°.$$

If in any triangle such as ABC of Fig. 2, A, B, and C represent the angles and a, b, and c, represent, respectively, the lengths of the sides opposite these angles, the following relations hold true:

Law of sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}\cdot$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos A.$

$$A + B + C = 180°.$$

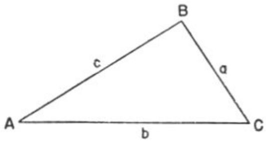


FIG. 2.

26. The *S* (Sine) and *ST* (Sine Tangent) scales. The numbers on the sine scales S and ST* represent angles. In order to set the indicator to an angle on the sine scales it is necessary to determine the value of the angles represented by the subdivisions. Thus since there are ten primary intervals between 4° and 5°, each represents 0.1°; since each of the primary intervals is subdivided into five secondary intervals, each of the latter represents 0.02°. Again since there are five primary intervals between 20° and 25°, each represents 1°; since each primary interval here is subdivided into 5 secondary intervals, each of the latter represent 0.2°. These illustrations indicate the manner in which the learner should analyze the part of the scale involved to find the value of the smallest interval to be considered. In general when setting the hairline to an angle the student should always have in mind the value of the smallest interval on the part of the slide rule under consideration.

When the indicator is set to a black number (angle) on scale S or ST, the sine of the angle is on scale C at the hairline and hence on scale D when the indices on scales C and D coincide.

When scale C is used to read sines of angles on ST, the left index of C is taken as 0.01, the right index as 0.1. When reading sines of angles on S, the left index of C is taken as 0.1, the right index as 1.

*The ST scale is a sine scale, but since it is also used as a tangent scale it is designated ST.

Thus to find sin 36.4°, opposite 36.4° on scale S, read 0.593 on scale C; to find sin 3.40°, opposite 3.40° on scale ST, read 0.0593 on scale C. Fig. 3 shows scales ST, S, *and* C on which certain angles and their sines are indicated. As an exercise, read from your slide rule the sines of the angles shown in the figure and compare your results with those given.

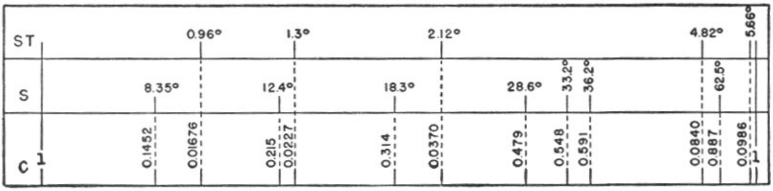


FIG. 3.

EXERCISES

1. By examination of the slide rule verify that on the S scale from the left index to 10° the smallest subdivision represents 0.05°; from 10° to 20° it represents 0.1°, from 20° to 30° it represents 0.2°; from 30° to 60° it represents 0.5°; from 60° to 80° it represents 1°; and from 80° to 90° it represents 5°.

2. Find the sine of each of the following angles:

(*a*) 30°. (*b*) 38°. (*c*) 3.33°. (*d*) 90°. (*e*) 88°.
(*f*) 1.583.° (*g*) 14.63°. (*h*) 22.4°. (*i*) 11.80°. (*j*) 51.5°.

3. Find the cosine of each of the angles in Exercise 2 by using the relation $\cos \varphi = \sin (90° - \varphi)$.

4. For each of the following values of x,

(*a*) 0.5. (*b*) 0.875. (*c*) 0.375. (*d*) 0.1. (*e*) 0.015.
(*f*) 0.62. (*g*) 0.062. (*h*) 0.031. (*i*) 0.92. (*j*) 0.885.

find the value of φ less than 90°, (A) if $\varphi = \sin^{-1}x$, where $\sin^{-1}x$ means "the angle whose sine is x"; (B) if $\varphi = \cos^{-1}x$.

5. Find the cosecant of each of the angles in Exercise 2, by using the relation

$$\csc \varphi = \frac{1}{\sin \varphi}.$$

Hint. Set the angle on S, read the cosecant on CI (or on DI when the rule is closed).

6. Find the secant of each of the angles in Exercise 2, by using the relation

$$\sec \varphi = \frac{1}{\cos \varphi}.$$

7. For each of the following values of x,

(*a*) 2. (*b*) 2.4. (*c*) 1.7. (*d*) 6.12. (*e*) 80.2. (*f*) 4.72.

find the value of φ less than 90°, (A) if $\varphi = \csc^{-1}x$; (B) if $\varphi = \sec^{-1}x$.

27. The T (Tangent) scale. *When the indicator is set to a black angle on scale T, the tangent of the angle is on scale C at the hairline* and hence on scale D when the indices of scales T and D coincide. Also *when the indicator is set to a black angle on scale T, the cotangent of the angle is on scale CI at the hairline.* Thus to find tan 36° push the hairline to 36° on T, at the hairline read **0.727** on C. To find cot 27.2°, push the hairline to 27.2° on T, at the hairline read **1.946** on CI.

When scale C is used to read tangents, the left index is taken as 0.1 and the right index as 1.0. Only those angles which range from 5.711° to 45° appear on scale T. It is shown in trigonometry that for angles less than 5.711°, the sine and tangent are approximately equal. Hence, so far as the slide rule is concerned, the tangent of an angle less than 5.711° may be replaced by the sine of the angle. Thus to find tan 2.25°, push the hairline to 2.25° on ST*, at the hairline read **0.0393** on C. To find the tangent of an angle greater than 45°, use the relation $\cot\theta = \tan(90° - \theta)$. To find tan 56°, push the hairline to 34° ($= 90° - 56°$) on T, at the hairline read **1.483** on CI. The student should observe that he could have set the hairline to 56° in red on the T scale and thus have avoided subtracting 34° from 90°. The method governing the use of red numbers is treated in the next article.

EXERCISES

1. Fill out the following table:

φ	8.10°	27.2°	62.3°	1.117°	74.2°	87°	47.5°
tan φ							
cot φ							

2. The following numbers are tangents of angles. Find the angles.

(*a*) 0.24. (*b*) 0.785. (*c*) 0.92. (*d*) 0.54. (*e*) 0.059.
(*f*) 0.082. (*g*) 0.432. (*h*) 0.043. (*i*) 0.0149. (*j*) 0.374.
(*k*) 3.72. (*l*) 4.67. (*m*) 17.01. (*n*) 1.03. (*o*) 1.232.

3. The numbers in Exercise 2 are cotangents of angles. Find the angles.

28. The red numbers on the S and T scales. The red numbers on the S and T scales express the complements of the angles represented by the corresponding black numbers on these scales.

*The ST scale is a sine scale, but since it is also used as a tangent scale it is designated ST.

When the hairline is set to an angle θ on scale S red, cos θ $[= \sin (90° - \theta)]$ is on scale C at the hairline and hence on scale D when the rule is closed. Also when the hairline is set to an angle θ on scale S red, sec θ $\left(= \dfrac{1}{\cos \theta}\right)$ is on scale CI at the hairline. Thus to find cos 60°, push the hairline to 60° on S red, read at the hairline **0.5** on C. To find sec 60°, push the hairline to 60° on S red, at the hairline read **2** on CI.

When the hairline is set to an angle θ on scale T red, cot θ $(= \tan (90° - \theta))$ is on scale C at the hairline. Also when the hairline is set to an angle θ on scale T red, tan θ $\left(= \dfrac{1}{\tan (90° - \theta)}\right)$ is on scale CI at the hairline. Thus to find cot 54°, push the hairline to 54° on T red, read at the hairline **0.727** on C. To find tan 62.8°, push the hairline to **62.8°** on T red, at the hairline read **1.946** on CI.

In this connection the following statement will be found suggestive. If the hairline be set to an angle on a trigonometric scale, it is automatically set to the complement of this angle. One of these angles is expressed in black type, the other in red. From what has been said it appears that we read at the hairline on the C scale or on the CI scale, a figure expressing a direct function, (sine, tangent, secant) by reading a figure of the same color as that representing the angle, a co-function (cosine, cosecant, cotangent) by reading a figure of the opposite color. Accordingly the student may find it advantageous to observe that: *Direct functions* (sin, tan, sec) *are read on like colors* (black to black, or red to red) *co-functions* (cos, cot, csc) *are read on opposite colors* (black to red, or red to black).

By using the reciprocal relations and the relations cos $\theta = \sin (90° - \theta)$ and cot $\theta = \tan (90° - \theta)$, any of the six trigonometric functions of an angle can be replaced by a sine or tangent of an angle. Hence by using these relations the red scales may be avoided. The learner should always use the red numbers to avoid subtracting an angle from 90°. However, if he uses the trigonometric scales infrequently, it is advisable that he employ mainly the sine and tangent. Anyone dealing with all six trigonometric functions should master §28.

In what follows any reference to an angle on a trigonometric scale will be to the angle in black unless otherwise stated.

EXERCISES

Using the red numbers on the trigonometric scales, solve Exercises 3, 4*B*, 5, 6, and 7 of §26, and Exercises 1 to 3 of §27.

29. Combined operations. The method for evaluating expressions involving combined operations as stated in §§16 and 23 applies without change when some of the numbers are trigonometric functions. This is illustrated in the following examples.

Example 1. Evaluate $\dfrac{4 \sin 38°}{\tan 42°}$.

Solution. Push hairline to 4 on D,
draw 42° of T under the hairline,
push hairline to 38° on S,
at the hairline read **2.73** on D.

Example 2. Evaluate $\dfrac{6.1\sqrt{17} \sin 72° \tan 20°}{2.2}$.

Solution. Use rule C §15 to write

$$\frac{\sqrt{17} \sin 72° \tan 20°}{2.2\left(\dfrac{1}{6.1}\right)}.$$

Push hairline to 17 on A right,
draw 2.2 of C under the hairline,
push hairline to 20° on T,
draw **6.1** of CI under the hairline,
push hairline to 72° on S,
at the hairline read **3.96** on D.

Example 3. Evaluate $\dfrac{7.9 \csc 17° \cot 31° \cos 41°}{18 \tan 48° \sqrt{3.8}}$.

Solution. Replacing csc 17° by $\dfrac{1}{\sin 17°}$, cot 31° by $\dfrac{1}{\tan 31°}$, and tan 48° by $\dfrac{1}{\tan 42°}$ and using rule C §15, we obtain

$$\frac{\left(\dfrac{1}{18}\right) 7.9 \tan 42° \cos 41°}{\sqrt{3.8} \sin 17° \tan 31°}.$$

Push hairline to 18 on DI,
draw 38 of B left under the hairline,
push hairline to 79 on C,
draw 17° of S under the hairline,
push hairline to 42° on T,
draw 31° of T under the hairline,
push hairline to 41° on S red,
at the hairline read **0.870** on D.

The student could have avoided the use of red numbers by replacing in the given expression cos 41° by sin 49°.

The *CF* scale may often be used to avoid shifting the slide. In the process of evaluating a fraction consisting of a number of factors in the numerator over a number of factors in the denominator, the hairline may be pushed to a number of the numerator on the *CF* scale provided that a number of the denominator on the *CF* scale is drawn under the hairline later in the process, and conversely. In other words the *CF* scale may be used at any time for a multiplication (or division) if it is later used for a division (or multiplication).

Example. Evaluate $\dfrac{2.10 \times 2.54 \times \sqrt{45}}{\sin 70° \times \tan 35° \times 3.06}$

Solution. Push hairline to 2.10 on *D*,
draw 70° of *S* under the hairline,
push hairline to 2.54 on *CF*,
draw 35 of *T* under the hairline,
push hairline to 45 on *B* right,
draw 3.06 of *CF* under the hairline,
under index of *C* read **17.77** on *D*.

Note that the folded scale was used twice, once in the third setting and once in the sixth.

EXERCISES

Evaluate the following:

1. $\dfrac{18.6 \sin 36°}{\sin 21°}$.
2. $\dfrac{32 \sin 18°}{27.5}$.
3. $\dfrac{4.2 \tan 38°}{\sin 45.5°}$.
4. $\dfrac{34.3 \sin 17°}{\tan 22.5°}$.
5. $\dfrac{13.1 \cos 40°}{\tan 35.2°}$.
6. $\dfrac{17.2 \cos 35°}{\cot 50°}$.
7. $\dfrac{7.8 \csc 35.5°}{\cot 21.4°}$.
8. $\dfrac{63.1 \sec 80°}{\tan 55°}$.
9. $\dfrac{\sin 18° \tan 20°}{3.7 \tan 41° \sin 31°}$.
10. $\dfrac{\sin 62.4°}{8.1 \tan 22.3°}$.
11. $3.14 \sin 13.17° \csc 32°$.
12. $7.1\pi \sin 47.6°$.
13. $\dfrac{0.61 \csc 12.25°}{\cot 35.3°}$.
14. $\dfrac{1 \sin 22.7°}{\tan 28.2°}$.
15. $\dfrac{3.1 \sin 61.6° \csc 15.30°}{\cos 27.7° \cot 20°}$.
16. $\dfrac{13.1 \sin 3.12°}{\tan 30.2°}$.
17. $\dfrac{0.0037 \sin 49.8°}{\tan 2.10°}$.
18. $\dfrac{\sqrt{16.5} \sin 45.5°}{\sqrt{4.6}\ 41.2 \cot 71.2°}$.
19. $\dfrac{\sqrt[3]{6.1}\ 4.91}{\tan 13.23°}$.
20. $\dfrac{\sin 51.5°}{(39.1)\ (6.28)}$.
21. $\dfrac{\csc 49.5°}{(19.1)\ (7.61)\ \sqrt{69.4}}$.
22. $(48.1)\ (1.68) \sin 39°$.
23. $0.0121 \sin 81° \cot 41°$.
24. $\dfrac{1.01 \cos 71.2° \sin 15°}{\sqrt{4.81} \cos 27.2°}$.

30. The use of the trigonometric scales in solving proportions. Scales S and D can be used together in the same way as other scales have been used in previous chapters for making a setting indicated by a proportion involving sines and numbers. For example let us find the values of x and φ in the proportion

$$\frac{\sin 36^\circ}{270} = \frac{\sin \varphi}{320} = \frac{\sin 10.03^\circ}{x}.$$

Since in the ratio $\frac{\sin 36^\circ}{270}$ both numerator and denominator are known, opposite 270 on scale D, set 36° on scale S, push hairline to 320 on D, at the hairline read φ = **44.2°** on S; push hairline to 10.03° on S, and at the hairline read x = **80** on D. The following form gives a diagramatic setting for the proportion:

S	set 36°	read φ = **44.2°**	opposite 10.03°
D	opposite 270	opposite 320	read x = **80.0**

The learner may well write such a form for the first few exercises but he should, as soon as possible, make the setting directly from the proportion. In any case the decimal point can be placed by means of a rough calculation. For this purpose the student may sometimes find it necessary to replace the symbols in the proportion by rough values found from the slide rule.

To find the value of a product involving sines, we may use the S and D scales together to perform the multiplication in the usual way, or we may write the given expression in the form of a proportion. Thus treating 8 sin 40° as a product, to 8 on D, set the right index of S, push the hairline to 40° on S, and at the hairline read the product **5.14** on D.

To find this product by means of a proportion we write

$$x = 8 \sin 40^\circ,$$

use Rule B §12 to write

$$\frac{x}{\sin 40^\circ} = \frac{8}{1\ (= \sin 90^\circ)},$$

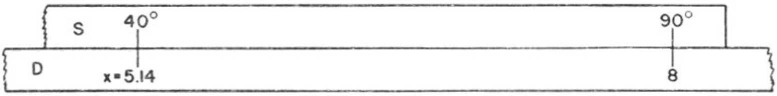


FIG. 4.

opposite 8 on D set 90° on S, and opposite 40° on S read $x = $ **5.14** on D (see Fig. 4).

Example 1. Find θ if $\sin\theta = \dfrac{3}{5}\cdot$

Solution. We write the given equation in the form

$$\frac{\sin\theta}{3} = \frac{1\ (= \sin 90^\circ)}{5},$$

to 5 on D set right index of scale S, push the hairline to 3 on D, and at the hairline read $\theta = $ **36.9°** on S.

Example 2. Find θ if $\cos\theta = 2/3$.

Solution. Since $\cos\theta = \sin(90^\circ - \theta)$, write the given equation in the form

$$\frac{\sin(90^\circ - \theta)}{2} = \frac{1\ (= \sin 90^\circ)}{3},$$

to 3 on D set right index of S, opposite 2 on D, read $90^\circ - \theta = 41.8^\circ$ on S. Hence $\theta = $ **48.2°.**

Scales T and D can be used together in the usual way to make the setting indicated by a proportion. Thus to find

$$y = \frac{16\tan 37^\circ}{0.017}, \text{ apply Rule } B, \text{ §12 to obtain } \frac{y}{16} = \frac{\tan 37^\circ}{0.017},$$

opposite 170 on D set 37° on T, and opposite 160 on D read $y = $ **710** on C. This setting is also explained by the following diagram:

T	set 37°	
C		read $y = $ **710**
D	opposite 170	opposite 160

Again to find $\theta = \tan^{-1}\dfrac{4.23}{6.72}$*, write $\dfrac{\tan\theta}{1} = \dfrac{4.23}{6.72}$ and from this proportion obtain $\theta = $ **32.2°.**

It is worthy of attention that the CF and DF scales may be used in place of the C and D scales in this process of solving proportions. For example find the values of x and φ in the proportion

$$\frac{11}{\sin 56.6^\circ} = \frac{x}{\sin 43^\circ} = \frac{13}{\sin\varphi},$$

by using the setting explained by the following diagram:

DF	opposite 11	read **9**	opposite 13
S	set 56.6°	opposite 43°	read **80.4°**

*The symbol $\tan^{-1} y$ is read "*the angle whose tangent is y.*"

EXERCISES

1. In each of the following proportions find the unknowns:

(a) $\dfrac{\sin 50.4°}{7} = \dfrac{\sin 42.2°}{x} = \dfrac{\sin \theta}{8}$.

(b) $\dfrac{\sin \theta}{30.5} = \dfrac{\sin 35°}{x} = \dfrac{\sin 60.5°}{32.8}$.

(c) $\dfrac{\sin 25°}{20} = \dfrac{\sin 40°}{x} = \dfrac{\sin 70°}{y}$.

(d) $\dfrac{\sin \theta}{15.6} = \dfrac{\sin \varphi}{25.6} = \dfrac{\sin 12.92°}{40.7}$.

2. Find the value of each of the following:

(a) $5 \sin 30°$.
(b) $12 \sin 60°$.
(c) $22/\sin 30°$.
(d) $15/\sin 20°$.
(e) $28 \cos 25°$.
(f) $35 \csc 52.3°$.
(g) $17 \sec. 16°$.
(h) $55 \sin 32° \sin 18°$.

3. Find the value of θ in the following:

(a) $\sin \theta = \dfrac{307 \sin 42.5°}{2030}$.

(b) $\sin \theta = \dfrac{413 \sin 77.7°}{488}$.

(c) $\sin \theta = \dfrac{433 \sin 18.17°}{136}$.

(d) $\sin \theta = \dfrac{156 \sin 12.92°}{40.7}$.

4. Find the value of each of the following:

(a) $\dfrac{179.5 \sin 6.42°}{\sin 34.5°}$

(b) $\dfrac{3.27 \sin 73°}{\sin 2.22°}$.

(c) $\dfrac{123.4 \sin 8.20°}{\sin 33.5°}$.

(d) $\dfrac{375 \sin 18.67°}{\cos 62.7°}$.

5. Find the value of each of the following:

(a) $\dfrac{4 \sin 35° - 5.4 \sin 17°}{\sin 47°}$.

(b) $\dfrac{8 - 6 \sin 70°}{\sin 37° - 0.21}$.

(c) $\dfrac{18 \sin 52.5° - 23.4 \cos 42.2°}{\sin 22° \sin 63°}$.

(d) $\dfrac{(27.7 \sin 39.2°)^2 - 16 \cos 12.67°}{46.2 \sin 10.17° + 32.1 \sin 17.27°}$.

6. Solve for the unknowns in the following equations:

(a) $\dfrac{\tan \theta}{27} = \dfrac{\tan \alpha}{49} = \dfrac{\tan 33.2°}{38}$.

(b) $y = \dfrac{\tan 24.2°}{6.15} = \dfrac{\tan \theta}{1.07}$.

(c) $y = (407 \cot. 82.88°)^2$.

(d) $y = \dfrac{17.2}{\tan 34.2°}$.

(e) $y = \dfrac{84.1 \tan 75°}{27.4}$.

(f) $y = \dfrac{9.32 \tan 17°}{32.2}$.

(g) $y = \dfrac{10.7}{15.1 \cot 42°}$.

(h) $\tan \theta = \dfrac{4.77 \tan 21.2°}{25.7}$.

(i) $\tan \theta = \dfrac{472 \tan 11.75°}{333}$.

31. Law of sines applied to right triangles. We have just seen how a proportion may be solved with the slide rule. A method of writing a proportion from a triangle is given by the law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c},$$

which applies to all triangles.

Consider its application to a right triangle in which $C = 90°$ and write

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin 90°}{c}.$$

By inspection we note that if any two parts, other than the two legs a and b, are known, one of the ratios in the proportion is known. Hence we can find all the unknowns by the principle explained in the preceding article.

Example 1. Given a right triangle in which $A = 36°$ and $a = 520$, find b, c, and B (see Fig. 5).

Solution. Since $A + B = 90°$, $B = 90° - A = 90° - 36° = \mathbf{54°}$.

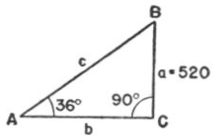


FIG. 5.

Application of the law of sines to the right triangle of Fig. 5 gives

$$\frac{\sin 36°}{520} = \frac{\sin 54°}{b} = \frac{\sin 90°}{c}.$$

The solution of this proportion for b and c is accomplished by using the setting (see Fig. 6) explained by the following diagram:

S	set 36°	opposite 54°	opposite 90°
D	opposite 520	read $b =$ **716**	read $c =$ **885**

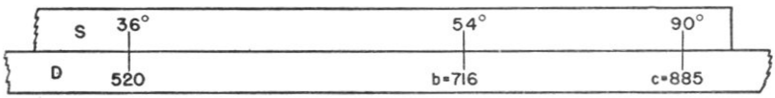


FIG. 6.

Example 2. Given a right triangle in which $A = 44°$, $c = 51$, find a, B, b.

Solution. $B = 90° - A = 90° - 44° = \mathbf{46°}$. Application of the law of sines to the triangle of Fig. 7, gives

$$\frac{\sin 44°}{a} = \frac{\sin 46°}{b} = \frac{\sin 90°}{51}.$$

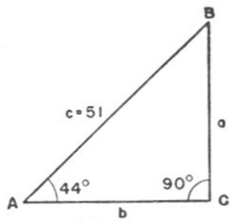


FIG. 7.

The solution of this proportion for a and b is accomplished by using the setting explained by the following diagram:

S	set 90°	opposite 44°	opposite 46°
D	opposite 51	read $a =$ **35.4**	read $b =$ **36.7**

Example 3. Given a right triangle in which $c = 49$ and $a = 22$, find A, B, and b (see Fig. 8).

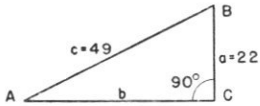


FIG. 8.

Solution. Application of the law of sines to the triangle of Fig. 8, gives

$$\frac{\sin A}{22} = \frac{\sin 90^\circ}{49} = \frac{\sin B}{b}.$$

The solution of this proportion for b and c is accomplished by using the settıng explained by the following diagram:

S	set 90°	read $A =$ **26.7°**	opposite $B =$ **63.3°** $(= 90° - A)$
D	opposite 49	opposite 22	read $b =$ **43.8**

The student should note that he will save time by using the red numbers to read 63.3° directly instead of subtracting 26.7° from 90°.

Whenever an angle less than 5.739°, that is, an angle whose sine or tangent is less than 0.1, is involved in the solution of a triangle, it is necessary to use the ST scale, instead of the S or T scale, as illustrated in the following example.

Example 4. Given the right triangle in which $c = 4.81$ and $a = 0.31$, find A, B, and b (see Fig. 9).

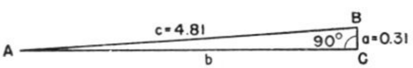


FIG. 9.

Solution. Application of the law of sines to the right triangle of Fig. 9, gives

$$\frac{\sin A}{0.31} = \frac{\sin B}{b} = \frac{\sin 90^\circ}{4.81}.$$

The solution of this proportion for b and A is accomplished by using the setting explained by the following diagram:

ST		read $A =$ **3.70°**	
S	set 90°		opposite **86.3°** $(= 90° - 3.70°)$
D	opposite 481	opposite 31	read $b =$ **4.8**

Note that the *DF* scale may be used in place of the *D* scale in making a setting based on the law of sines.

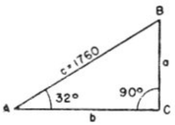


FIG. 10.

Example 5. Given the right triangle of Fig. 10 in which $c = 1760$ and $A = 32°$, find a, b, and B.

Solution. Application of the law of sines to the right triangle of Fig. 10 gives

$$\frac{1760}{\sin 90°} = \frac{a}{\sin 32°} = \frac{b}{\sin B}.$$

The solution of this proportion for a and b is accomplished by using the following setting explained by the following diagram:

DF	opposite 1760	read a = **933**	read b = **1490**
S	set 90°	opposite 32°	opposite 58° (= 90° − 32°

In the foregoing examples the law of sines was written in each case. After the student has solved a few exercises, he should, to save time, make the setting on the slide rule directly from the figure. To do this *select from the figure* (see Fig. 11) *a known side* **a** *opposite a known angle* **A**, *push the hairline to* **a** *on scale D, draw* **A** *of scale S under the hairline, push the hairline to a known part, and at the hairline read the opposite unknown part.*

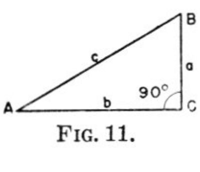


FIG. 11.

In other words it is unnecessary to write a proportion. *As soon as a side and an angle opposite are known, set them opposite on the slide rule,* thus making the setting necessary for solving the triangle.

EXERCISES

Solve the following right triangles. In each case draw a figure and write a proportion from the law of sines.

1. $a = 60$, $c = 100$.

2. $a = 50.6$, $A = 38.7°$.

3. $a = 729$, $B = 68.8°$.

4. $b = 200$, $A = 64°$.

5. $c = 37.2$, $B = 6.20°$.

6. $c = 11.2$, $A = 43.5°$.

7. $b = 47.7$, $B = 62.9°$.

8. $a = 0.624$, $c = 0.910$.

9. $a = 83.4$, $A = 72.1°$.

Solve the following right triangles. In each case draw a figure, from it make the setting directly, and write the results without first writing the law of sines.

10. $b = 4247$, $A = 52.7°$.

11. $b = 2.89$, $c = 5.11$.

12. $b = 512$, $c = 900$.

13. $c = 35.7$, $A = 58.6°$.

14. $c = 0.726$, $B = 10.85°$.

15. $a = 0.821$, $B = 21.6°$.

16. $a = 52$, $c = 60$.

17. $a = 1875$, $B = 2.33°$.

18. $b = 9$, $A = 88.433°$.

19. The length of a kite string is 250 yds., and the angle of elevation of the kite is 40°. If the line of the kite string is straight, find the height of the kite.

20. A vector is directed due N.E. and its magnitude is 10. Find the component in the direction of north.

21. Find the angle made by the diagonal of a cube with the diagonal of a face of the cube drawn from the same vertex.

32. The law of sines applied to right triangles with two legs given. When the two legs of a right triangle are the given parts, we may first find the *smaller* acute angle from its tangent and then apply the law of sines to complete the solution.

Example. Given the right triangle of Fig. 12 in which $a = 3$, $b = 4$; solve the triangle.

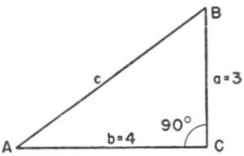


FIG. 12.

Solution. From the triangle we read $\tan A = \frac{3}{4}$. This equation when written in the form

$$\frac{\tan A}{3} = \frac{1}{4}$$

indicates the setting explained by the following diagram:

T	set index	read $A = \mathbf{36.9}°$ and $B = \mathbf{53.1}°$ (red)
D	opposite 4	opposite 3

Since an angle and its opposite side are now known, we may apply the law of sines to the triangle to obtain

$$\frac{\sin 36.9°}{3} = \frac{\sin 90° \text{ (either index)}}{c}$$

and make the setting explained in the following diagram:

S	set $A = 36.9°$	opposite 90°
D	opposite 3	read $c = \mathbf{5}$

The student should notice that after reading $A = 36.9°$ on T, the setting indicated in the second diagram is made by drawing 36.9° on S under the hairline.

Those operators who have occasion to solve many right triangles of the type under consideration should use the following rule:

Rule. *To solve a right triangle for which two legs are given,*
to larger leg on D set proper index of slide,
push hairline to smaller leg on D,
at the hairline read smaller acute angle of triangle on T,
draw this angle on S under the hairline,
at index of slide read hypotenuse on D.

EXERCISES

Solve the following right triangles:

1. $a = 12.3$, $b = 20.2$.	**4.** $a = 273$, $b = 418$.	**7.** $a = 13.2$, $b = 13.2$.
2. $a = 101$, $b = 116$.	**5.** $a = 28$, $b = 34$.	**8.** $a = 42$, $b = 71$.
3. $a = 50$, $b = 23.3$.	**6.** $a = 12$, $b = 5$.	**9.** $a = 0.31$, $b = 4.8$.

10. The length of the shadow cast by a 10-ft. vertical stick on a horizontal plane is 8.39 ft. Find the angle of elevation of the sun.

11. From right triangle ABC of Fig. 13 we have

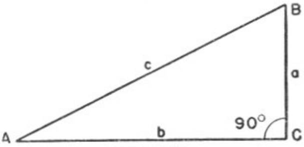


FIG. 13.

$$\frac{a}{c} = \sin A \text{ or } a = c \sin A,$$

and

$$\frac{a}{b} = \tan A \text{ or } a = b \tan A.$$

or, by using Rule C §15,

$$\frac{(1 = \sin 90°)}{1/a} = \frac{\tan A}{1/b} = \frac{\sin A}{1/c}.$$

By means of this proportion solve Exs. 1, 2, 4, 5.

33. Law of sines applied to oblique triangles when two opposite parts are known. The same process used to solve right triangles may be used to solve any triangle when a side and angle opposite are given, since the law of sines which we have been using applies to any triangle.

Example 1. Given an oblique triangle (see Fig. 14) in which $a = 50$, $A = 65°$, and $B = 40°$. Find b, c, and C.

Solution. Since $A + B + C = 180°$, $C = 180° - (A + B) = \mathbf{75°}$.

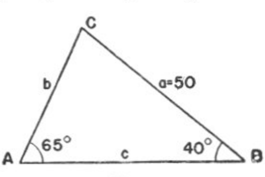


FIG. 14.

Application of the law of sines to the triangle gives

$$\frac{\sin 65°}{50} = \frac{\sin 40°}{b} = \frac{\sin 75°}{c}.$$

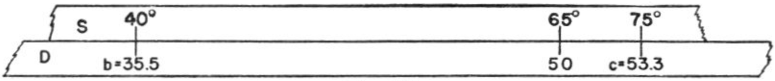


FIG. 15.

The solution of this proportion for b and c is accomplished by using the setting (see Fig. 15) explained in the following diagram:

S	set 65°	opposite 40°	opposite 75°
D	opposite 50	read b = **35.5**	read c = **53.3**

Example 2. Given an oblique triangle in which $A = 75°$, $a = 40$, and $b = 30$, find c, B, and C (see Fig. 16).

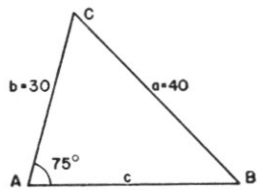


FIG. 16.

Solution. From the figure we observe that $a = 40$ and $A = 75°$ are the known parts which are opposite. Hence push hairline to 40 on D, draw 75° on S under the hairline, push hairline to 30 of D, and at the hairline read B = **46.4**° on S, push hairline to C = **58.6**° ($= 180° - A -$ B) on S and at the hairline read c = **35.3** on D.

Example 3. Given the oblique triangle of Fig. 17 in which $A = 75°$, $a = 40$, $b = 3$, find c, B, and C.

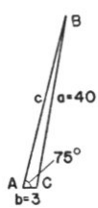


FIG. 17.

Solution. From the figure we observe that $a = 40$ and $A = 75°$ are the known parts which are opposite. Hence by pushing the hairline to 40 on D and drawing 75° of S under the hairline, we make the setting used in solving the triangle. The solution is explained in the following diagram:

ST		read B = **4.15**°	
S	set 75°		opposite **79.2**° (= $A + B$)
D	opposite 40	opposite 3	read c = **40.7.**

To obtain C = **100.85**° we use the relation $C = 180° - (A + B)$.

EXERCISES

Solve the following oblique triangles.

1. $a = 50$, $A = 65°$, $B = 40°$.	**5.** $a = 120$, $b = 80$, $A = 60°$.	**9.** $b = 91.1$, $c = 77$, $B = 51.1°$.
2. $c = 60$, $A = 50°$, $B = 75°$.	**6.** $b = 0.234$, $c = 0.198$, $B = 109°$.	**10.** $a = 50$, $c = 66$, $A = 123.2°$.
3. $a = 550$, $A = 10.20°$. $B = 46.6°$.	**7.** $a = 795$, $A = 80°$, $B = 44.7°$	**11.** $a = 8.66$, $c = 10$, $A = 59.9°$.
4.* $a = 222$, $b = 4570$, $C = 90°$.	**8.*** $a = 21$, $A = 4.17°$. $B = 75°$.	**12.*** $b = 8$, $a = 120$, $A = 60°$.

13. A ship at point S can be seen from each of two points, A and B, on the shore. If $AB = 800$ ft., angle $SAB = 67.7°$, and angle $SBA = 74.4°$, find the distance of the ship from A.

14. To determine the distance of an inaccessible tower A from a point B, a line BC and the angles ABC and BCA were measured and found to be 1000 yd., 44°, and 70°, respectively. Find the distance AB.

34. Law of sines applied to oblique triangles, continued. The ambiguous case. When the given parts of a triangle are two sides and an angle opposite one of them, and when the side opposite the given angle is less than the other given side, there may be two triangles which have the given parts. We have already solved triangles in which the side opposite the given angle is greater than the other side. In this case there is always only one solution. Let us now consider a case where there are two solutions.

Example. Given $a = 175$, $b = 215$, and $A = 35.5°$; solve the triangle.

Solution. Fig. 18 shows the two possible triangles having the given parts. Application of the law of sines to triangle ABC, gives

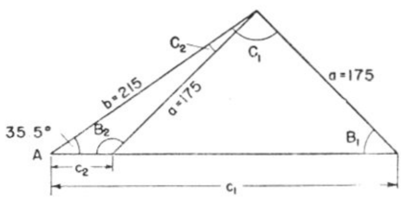


FIG. 18.

$$\frac{\sin 35.5°}{175} = \frac{\sin B_1}{215} = \frac{\sin C_1}{c_1}.$$

*The ST scale must be used in the solution.

The setting and results are shown in the following diagram.

S	set 35.5°	read $B_1 =$ **45.5°**	opposite **81°** ($= A + B$)
D	opposite 175	opposite 215	read $c_1 =$ **298.**

It appears from the Fig. 18 that $B_2 = \mathbf{180° - B_1}$. Hence to solve triangle $A\ B_2\ C_2$ we write

$$B_2 = 180° - 45.5° = \mathbf{134.5°},$$
$$C_2 = 180° - (A + B_2) = 180° - \mathbf{170°} = \mathbf{10°}$$

and
$$\frac{\sin 35.5°}{175} = \frac{\sin 10°}{c_2}.$$

The setting and results are shown in the following diagram.

S	set 35.5°	opposite 10°
D	opposite 175	read $c_2 =$ **52.3.**

The student should notice that the solution of both triangles was made with a single setting of the slide rule, since each of the two proportions used contained the same known ratio

$$\frac{175}{\sin 35.5°}.$$

When the known ratio $\left(\frac{\sin A}{a}\right)$ in the ambiguous case is set on the slide rule, we find opposite the index of scale S on scale D a number equal to or greater than the largest value that leg b may have consistent with the proportion $\frac{\sin A}{a} = \frac{\sin B}{b}$. Therefore the following rule applies:

Set angle A on S opposite a on D and read the value on D opposite the index of S. If this value is greater than **b**, *there are two solutions; if it is equal to* **b**, *there is one solution, a right triangle; if it is less than* **b**, *there is no solution.*

EXERCISES

Solve the following oblique triangles.

1. $a = 18$, $b = 20$, $A = 55.4°$.

2. $b = 19$, $c = 18$, $C = 15.82°$.

3. $a = 32.2$, $c = 27.1$, $C = 52.4°$.

4. $b = 5.16$, $c = 6.84$, $B = 44°$.

5. $a = 177$, $b = 216$, $A = 35.6°$.

6. $a = 17{,}060$, $b = 14{,}050$, $B = 40°$.

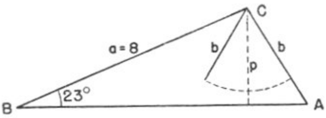


FIG. 19.

7. Find the length of the perpendicular p for the triangle of Fig. 19. How many solutions will there be for triangle ABC if (a) $b = 3$? (b) $b = 4$? (c) $b = p$?

35. Law of sines applied to an oblique triangle in which two sides and the included angle are given. To solve an oblique triangle in which two sides and the included angle are given, it is convenient to divide the triangle into two right triangles. The method is illustrated in the following example.

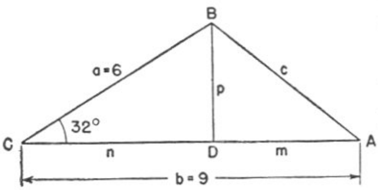


FIG. 20.

Example. Given an oblique triangle in which $a = 6$, $b = 9$, and $C = 32°$, solve the triangle.

Solution. From B of Fig. 20, drop the perpendicular p to side b. Applying the law of sines to the right triangle CBD, we obtain

$$\frac{\sin 90°}{6} = \frac{\sin 32°}{p} = \frac{\sin 58°}{n}.$$

Solving this proportion, we find $p = 3.18$ and $n = 5.09$. From the figure $m = 9 - 5.09 = 3.91$. Hence, in triangle ABD, we have

$$\tan A = \frac{p}{m} = \frac{3.18}{3.91},$$

or

$$\frac{\tan A}{3.18} = \frac{1}{3.91}.$$

Solving this proportion, we get $A = 39.1.°$ Now applying the law of sines to triangle ABD, we obtain

$$\frac{\sin 39.1°}{3.18} = \frac{\sin 90°}{c}.$$

Solving this proportion, we find $c =$ **5.04.** Finally, using the relation, $A + B + C = 180°$, we obtain $B =$ **108.9°.** The italicized rule of §32 could have been used in place of the last two proportions.

If the given angle is obtuse, the perpendicular falls outside the triangle, but the method of solution is essentially the same as that used in the above example.

EXERCISES

Solve the following triangles:

1. $a = 94$, $b = 56$, $C = 29°$.	**4.** $b = 2.30$, $c = 3.57$, $A = 62°$.	**7.** $a = 0.085$, $c = 0.0042$, $B = 56.5°$
2. $a = 100$, $c = 130$, $B = 51.8°$	**5.** $a = 27$, $c = 15$, $B = 46°$.	**8.** $a = 17$, $b = 12$, $C = 59.3°$
3. $a = 235$, $b = 185$, $C = 84.6°$	**6.** $a = 6.75$, $c = 1.04$, $B = 127.2°$	**9.** $b = 2580$, $c = 5290$, $A = 138.3°$

10. The two diagonals of a parallelogram are 10 and 12 and they form an angle of 49.3° Find the length of each side.

11. Two ships start from the same point at the same instant. One sails due north at the rate of 10.44 mi. per hr., and the other due northeast at the rate of 7.71 mi. per hr. How far apart are they at the end of 40 minutes?

12. In a land survey find the latitude and departure of a course whose length is 525 ft. and bearing N 65.5° E. See Fig. 21.

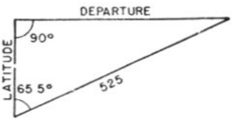


FIG. 21.

13. Solve Ex. 1 by means of the law of tangents

$$\frac{a - b}{\tan \frac{1}{2}(A - B)} = \frac{a + b}{\tan \frac{1}{2}(A + B)}.$$

36. Law of cosines applied to oblique triangles in which three sides are given. When the three sides are the given parts of an oblique triangle, we may find one angle by means of the law of cosines $a^2 = b^2 + c^2 - 2bc \cos A$ and then complete the solution by using the law of sines.

Example. Given the oblique triangle of Fig. 22, in which $a = 15$, $b = 18$, and $c = 20$, find A, B, and C.

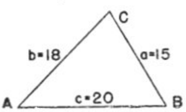


FIG. 22.

Solution. From the law of cosines we write $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

or
$$\cos A = \sin (90° - A) = \frac{18^2 + 20^2 - 15^2}{2 \times 18 \times 20} = \frac{499}{720}.$$

The ratio indicates the setting from which we find $90° - A = 43.9°$ or $A = 46.1°$. Since we now know an angle and the side opposite, we may apply the law of sines to obtain

$$\frac{\sin 46.1°}{15} = \frac{\sin B}{18} = \frac{\sin C}{20}.$$

Making the setting indicated by the proportion, we find

$$B = 59.9°, \ C = 74°.$$

We may use the relation $A + B + C = 180°$ as a check. Thus: $46.1° + 59.9° + 74° = 180°$ *check.*

EXERCISES

Solve the following triangles:

1. $a = 3.41$, $b = 2.60$, $c = 1.58$.	**4.** $a = 61.0$, $b = 49.2$, $c = 80.5$.	**7.** $a = 97.9$, $b = 106$, $c = 139$.
2. $a = 111$, $b = 145$, $c = 40$.	**5.** $a = 7.93$, $b = 5.08$, $c = 4.83$.	**8.** $a = 57.9$, $b = 50.1$, $c = 35.0$.
3. $a = 35$, $b = 38$, $c = 41$.	**6.** $a = 21$, $b = 24$, $c = 27$.	**9.** $a = 13$, $b = 14$, $c = 15$.

10. The sides of a triangular field measure 224 ft., 245 ft., and 265 ft. Find the angles at the vertices.

11. Find the largest angle of the triangle whose sides are 13, 14, 16.

12. Solve Ex. 11 by means of the following formula:

$$\tan \frac{A}{2} = \sqrt{\frac{(s-b)\ (s-c)}{s\ (s-a)}} \quad \text{where } s = \frac{1}{2}(a + b + c).$$

13. In Ex. 4, §35, find the side opposite the known angle by means of the law of cosines and then complete the solution by means of the law of sines.

14. In triangle ABC of Fig. 23

$p^2 = b^2 - m^2 = a^2 - n^2.$
Hence $b^2 - a^2 = m^2 - n^2,$
Factoring and replacing $(m + n)$ by c, we have

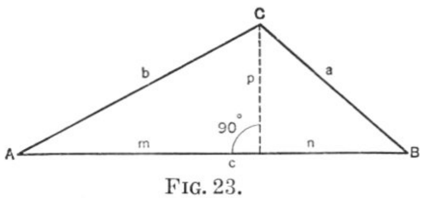


FIG. 23.

$$(b + a)\ (b - a) = (m + n)\ (m - n) = c\ (m - n),$$

or

$$\frac{b + a}{c} = \frac{m - n}{b - a}. \tag{a}$$

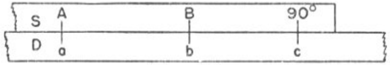
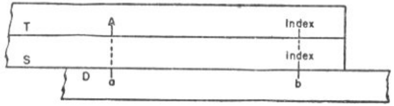
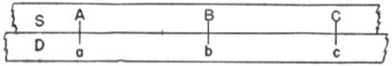
To solve the triangle ABC, find $m - n$ by using proportion (a). Combine this result with

$$m + n = c,$$

to find m and n. Then solve each of the right triangles of triangle ABC and use the results to find the angles A, B, and C.

Apply this method to solve Exs. 1, 2, 3.

37. Summary. The following tables summarize the methods of solution of triangles.

	Known	Solve by	In general the setting will be
RIGHT TRIANGLES	Any two parts other than two legs §31	Law of sines $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin 90^\circ}{c}$	 FIG. 24.
	Two legs §32	Rule of §32 or The proportion $\frac{\tan A}{a} = \frac{1}{b}$, and the law of sines	 FIG. 25.
OBLIQUE TRIANGLES	Three parts, two of which are a side and angle opposite §§33–34	Law of sines $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	 FIG. 26.
	Two sides and the included angle §35	Dropping a perpendicular and solving the two right triangles thus formed	
	Three sides §36	Law of cosines $a^2 = b^2 + c^2 - 2bc \cos A$ and law of sines	

38. MISCELLANEOUS EXERCISES

Solve the following triangles:

1. $c = 80$, $A = 20°$, $C = 90°$.

2. $b = 30$, $A = 10°$, $C = 90°$.

3. $a = 80$, $A = 75°$, $C = 90°$.

4. $a = 10.11$, $b = 17.3$, $C = 90°$.

5. $a = 2$, $b = 3$, $c = 4$.

6. $a = 5.6$, $b = 4.3$, $c = 4.9$.

7. $c = 1$, $A = 36°$, $C = 90°$.

8. $a = 795$, $A = 80°$, $B = 44.7°$.

9. $a = 500$, $A = 10.2°$, $B = 46.6°$.

10. $b = 29.0$, $A = 87.7°$. $C = 33.2°$.

11. $a = 55.6$, $c = 66.7$, $C = 77.7°$.

12. $a = 51.38$, $c = 67.94$, $B = 79.2°$.

13. $a = 0.321$, $b = 0.361$, $c = 0.402$.

14. $a = 4$, $b = 7$, $c = 6$.

15. $a = 78$, $b = 83.4$, $B = 56.5°$.

16. $b = 8000$, $A = 24.5°$, $B = 86.5°$.

17. $a = 42{,}930$, $c = 73{,}480$, $C = 127.6°$.

18. $a = 61.3$, $b = 84.7$, $c = 47.6$.

19. If the sides of a triangular field are 70 ft., 110 ft., and 96 ft. long, find the angle opposite the longest side.

20. The diagonals of a parallelogram are 5 ft. and 6 ft. in length. If the angle they form is 49.3°, find the sides of the parallelogram.

21. A car is traveling at a rate of 44 ft. per second up a grade which makes an angle of 10° with the horizontal. Find how long it takes for the car to rise 200 ft.

22. A lighthouse is 16 mi. in the direction 29.5° east of north from a cliff. Another lighthouse is 12 mi. in the direction 72.8° west of south from the cliff. What is the direction of the first lighthouse from the second?

23. A 52-ft. ladder is placed 20 ft. from the foot of an inclined buttress, and reaches 46 ft. up its face. What is the inclination of the buttress?

24. If in a circle a chord of 41.36 ft. subtends an arc of 145.6°, find the radius of the circle.

39. To change radians to degrees or degrees to radians. In the next article we shall find it convenient to use the angular unit called the radian. To change radians to degrees or degrees to radians we use the proportion

$$\frac{\pi}{180} = \frac{r \text{ (number of radians)}}{d \text{ (number of degrees)}}.$$

The setting is as follows:

Opposite π on DF set 180 on CF,

Opposite radians on D (or DF) read degrees on C (or CF),

or

Opposite degrees on C (or CF) read radians on D (or DF).

Thus to convert 1.5 radians to degrees, we make the following setting:

Opposite π on DF set 180 on CF,

Opposite 1.5 on DF read **85.9°** on CF.

To convert 45° to radians, we make the following setting:

Opposite π on DF set 180 on CF,

Opposite 45 on C (or CF) read **0.785** on D (or DF).

EXERCISES

1. Express the following angles in radians:

(a) 45°. (d) 180°. (g) 22.5°.
(b) 60°. (e) 120°. (h) 200°.
(c) 90°. (f) 135°. (i) 3000°.

2. Express the following angles in degrees:

(a) $\pi/3$ radians. (c) $\pi/72$ radian. (e) $20\pi/3$ radians
(b) $3\pi/4$ radians. (d) $7\pi/6$ radians. (f) 0.98π radians.

3. Express in radians the following angles:

(a) 1°. (c) 1″. (e) 180.572°.
(b) 1′. (d) 10.18°. (f) 300.4°.

4. Find the following angles in degrees and minutes:

(a) $\frac{1}{10}$ radian; (b) $2\frac{1}{2}$ radians; (c) 1.6 radians; (d) 6 radians.

40. Sines and tangents of small angles. To find $\sin\theta$ or $\tan\theta$ for an angle smaller than those given on the ST scale, we may use the approximate relation

$$\sin\theta = \tan\theta = \theta \text{ (in radians)}, \qquad \text{(approximately)}.$$

Since $1° = \frac{\pi}{180}$ radians, set 180 on CF to π on DF; opposite the angles expressed in degrees on C or CF, read the same angles expressed in radians on D or DF respectively. Thus $\sin 0.3° = 0.3\frac{\pi}{180} = 0.00524$.

Since $1' = \frac{\pi}{180 \times 60}$ radian, and since $1'' = \frac{\pi}{180 \times 60 \times 60}$ radian, we can find the sine or tangent of any small angle expressed in minutes (or seconds) by multiplying it by the value of $1'$ (or $1''$) in radians. Thus $\sin 18' = 18\frac{\pi}{180 \times 60} = 0.00524$; $\sin 35'' = 35\frac{\pi}{180 \times 60 \times 60} =$ 0.0001697.

For convenience the value of $\frac{180 \times 60}{\pi}$ has been marked by a "minute" gauge point on scale ST near the 2° division, and the value of $\frac{180 \times 60 \times 60}{\pi}$ has been marked by a "second" gauge point near the 1.167° division. To approximate an answer for the purpose of placing the decimal point, it is convenient to remember that **sin 0.1°** = **.002** (**2 zeros, 2**) nearly, that **sin 1′** = **.0003** (**3 zeros, 3**) nearly, and that **sin 1″** = **.000005** (**5 zeros, 5**) nearly.

To find sin 35″ or tan 35″ set the slide rule as indicated in the following diagram:

ST	set "second" gauge point	opposite left index
D	opposite 35	read **0.0001697**

To find 540 sin 28′ set the rule as indicated in the following diagram:

ST	set "minute" gauge point	
C		opposite 540
D	opposite 28	read **4.4**

To find 540 sin 0.467°, set the rule as indicated in the following diagram:

CF	set 180	
C		opposite 540
D	opposite 0.467	read **4.4**

To find tan 89.75°, write $\tan 89.75° = \cot 0.25° = \frac{1}{\tan 0.25°} = \frac{180}{0.25\pi}$, and then set the slide rule as indicated in the following diagram:

DF	to π	
CF	set 180	
C		opposite 0.25
DI		read **229**

EXERCISES

Find the values of the following:

1. $\sin 5'$.
2. $\sin 5''$.
3. $\sin 21'$.
4. $\sin 32''$.
5. $\tan 7'$.
6. $\tan 52''$.
7. $\cos 89.75°$.
8. $\cos 89.98861°$.
9. $\sec. 89° 40'$.
10. $\csc 16''$.
11. $\csc 2'$.
12. $\cot 0.5°$.
13. $250 \sin 23'$.
14. $42 \tan 19'$.
15. $150 \cos 89.667°$.
16. $38 \sin 52''$.
17. $500 \tan 35''$.
18. $432 \sin 33'$.
19. $\dfrac{\tan 0.2°}{0.0001745}$.
20. $\dfrac{\sin 0.3°}{0.131}$.

41. Applications involving vectors. Since vectors are used in the solution of a great number of the problems of science, a few applications involving vectors will be considered at this time.

A vector AB (see Fig. 27) is a segment of a straight line containing an arrowhead pointed toward B to indicate a direction from its initial point A to its terminal point B. The length of the segment indicates the *magnitude* of the vector and the line with attached arrowhead indicates direction. If from the ends A and B of the vector, perpendiculars be dropped to the line of a vector $A'B'$ and meet it in the points A'' and B'', respectively, then the vector $A''B''$ directed from A″ to B″ is called the component of vector AB in the direction of $A'B'$.

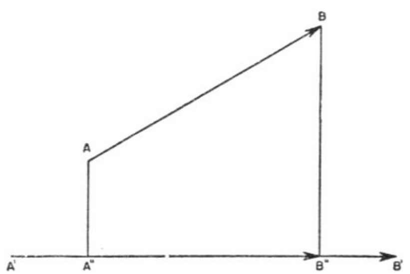


FIG. 27.

A force may be represented by a vector, the length of the vector representing the magnitude of the force, and the direction of the vector the direction of the force. In fact, many quantities defined by a magnitude and a direction can be represented by vectors.

In each of the following applications, two mutually perpendicular components of a vector are considered. Evidently these components may be thought of as the legs of a right triangle having as hypotenuse the vector itself.

For convenience the rule for solving a right triangle when two legs are given is repeated here.

Rule. *To solve a right triangle for which two legs are given,*
to larger leg on D set proper index of slide,
push hairline to smaller leg on D,
at the hairline read smaller acute angle of triangle on T,
draw this angle on S under the hairline,
at index of slide read hypotenuse on D.

Example 1. Find the magnitude and the angle of the vector representing the complex number $3.6 + j\,1.63$ where $j = \sqrt{-1}$.

Solution. If the numbers x and y be regarded as the rectangular coordinates of a point, the complex number $x + jy$ is represented by the vector from the origin to the point (x, y). Hence we must find R and θ in Fig. 28. Therefore, in accordance with the italicized rule stated above,

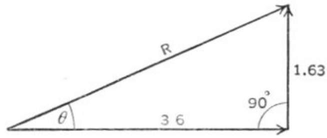


FIG. 28.

To 3.6 on D set right index of slide,
push hairline to 1.63 on D,
at the hairline read $\theta =$ **24.4°** on T,
draw 24°22′ of S under the hairline,
at index of slide read $\mathbf{R} =$ **3.95** on D.

Example 2. A force of 26.8 lb. acts at an angle of 38° with a given direction. Find the component of the force in the given direction, and also the component in a direction perpendicular to the given one.

Solution. Denoting the required components by x and y (see Fig. 29), we write

$$\frac{26.8}{\sin 90^\circ} = \frac{y}{\sin 38^\circ} = \frac{x}{\sin 52^\circ},$$

make the corresponding setting, and read $x =$ **21.1**, $y =$ **16.5.**

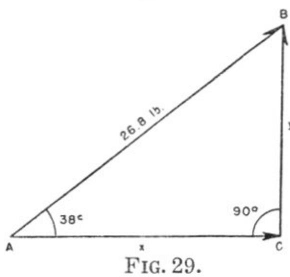


FIG. 29.

Example 3. A certain circuit consists of a resistance $R = 3.6$ and an inductive reactance $X = 2.7$ in series. Find the impedance z, the susceptance B, and the conductance G.

Solution. The quantities R, X and z have relations which may be read from Fig. 30. Conductance G and

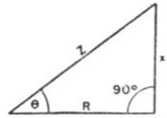


FIG. 30.

susceptance B are found from the relations

$$G = \frac{R}{R^2 + X^2}, \quad B = \frac{X}{R^2 + X^2},$$

or, since $z = \sqrt{R^2 + X^2}$,

$$\left.\begin{aligned} G &= \frac{R}{\sqrt{R^2+X^2}\sqrt{R^2+X^2}} = \frac{\cos\theta}{z}, \\ B &= \frac{x}{\sqrt{R^2+X^2}\sqrt{R^2+X^2}} = \frac{\sin\theta}{z}. \end{aligned}\right\} \qquad (a)$$

From equations (a), we obtain

$$\frac{z}{1} = \frac{\sin\theta}{B} = \frac{\cos\theta}{G}. \qquad (b)$$

First apply the italicized rule stated above to find z and θ of Fig. 30, and then use the proportion principle to find B and G from (b). Hence

To 3.6 on D set right index of slide,
push hairline to 2.7 on D,
at the hairline read $\theta = 36.9°$ on T,
draw 36.9° on S under the hairline,
at index of slide read $\mathbf{z} = \mathbf{4.5}$ on D,
draw 4.5 of C opposite left index of D,
push hairline to 36.9° on S,
at the hairline read $\mathbf{B} = \mathbf{0.133}$ on D,
push hairline to 36.9° *red* on S,
at the hairline read $\mathbf{G} = \mathbf{0.178}$ on D.

EXERCISES

1. Find the unknown angles θ and the unknown magnitudes of the vectors in Figs. 31, 32, and 33.

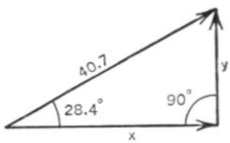


FIG. 31.

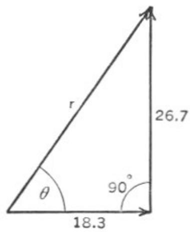


FIG. 32.

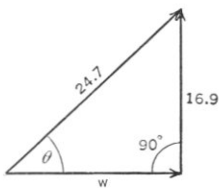


FIG. 33.

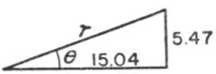


FIG. 34.

2. The rectangular components of a vector are 15.04 and 5.47 (see Fig. 34). Find the magnitude r and direction angle θ of the vector.

3. Find the magnitude and direction of a vector ha ing as the horizontal and vertical components 18.12 and 8.45, respectively.

4. Find the horizontal and vertical components of a vector having magnitude 2.5 and making an angle of 16.25° with the horizontal.

5. A force of magnitude 28.8 lb. acts at an angle of 68° with the horizontal. Find its horizontal component, and its vertical component.

6. A 12-inch vector and an unknown vector r have as resultant a 16-inch vector which makes an angle of 28° with the 12-inch vector as shown in Fig. 35. Find the unknown vector r.

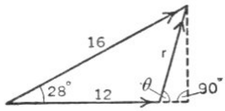


FIG. 35.

7. Find the magnitude and the angle of the vector representing the imaginary number $-2.7 + j3.6$. Hint. Use Fig. 36.

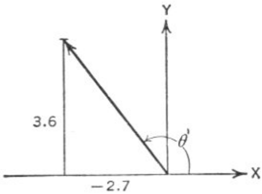


FIG. 36.

8. Through what angle θ measured counter-clockwise must a vector whose complex expression is $-10 - j5$ be rotated to bring it into coincidence with the vector whose complex expression is $3 + j4$. (See Fig. 37).

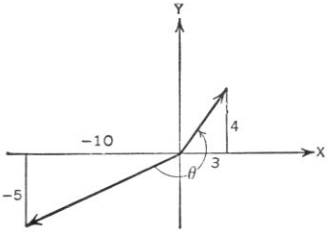


FIG. 37.

9. The complex expressions for two vectors (see Fig. 38) are $v_1 = 7 - j14$ and $v_2 = -6 - j8$. From the tip of v_2 a line is drawn perpendicular to v_1. Find the length m of this perpendicular, and the length n of the line from the origin to the foot of the perpendicular.

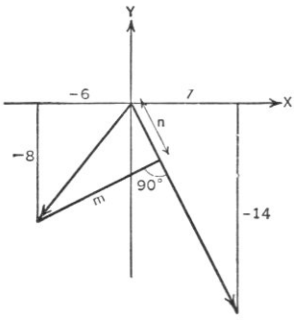


FIG. 38.

10. A certain circuit consists of a resistance of 8.24 ohms and an inductive reactance of 4.2 ohms, in series. Find the impedance, the susceptance, and the conductance. (See Example 3.)

11. Find the impedance, the susceptance, and the conductance of a circuit which consists of a resistance of 8.76 ohms and an inductive reactance of 11.45 ohms in series.

42. Applications. The solutions of many practical problems are obtained by dealing with rectilinear figures. In finding the length of a specified line segment of a rectilinear figure, the beginner is likely to read a number of lengths which are not needed. This may be well at first, but the efficient operator reads and tabulates only useful numbers. The following examples and solutions indicate efficient methods of finding desired parts of rectilinear figures.

Example 1. Find the line segment marked x in Fig. 39.

Solution. By using the law of sines, we write

$$\frac{368}{\sin 39^\circ} = \frac{y}{\sin 65^\circ}, \quad \frac{y}{\sin 50^\circ} = \frac{x}{\sin 28^\circ}$$

and then find x by making the following settings:
push hairline to 368 on D,
draw 39° of S under the hairline,
push hairline to 65° on S,
draw 50° of S under the hairline,
push hairline to 28° on S,
at the hairline read x = **325** on D.

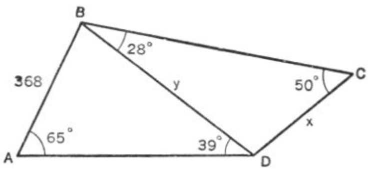


FIG. 39.

The value of y was not tabulated, but it could have been read at the hairline on scale D when the hairline was set to 65° of scale S. Also it was not necessary to write the ratios; for, when one remembers that each ratio is that of a side of a triangle to the sine of the opposite angle, he has no difficulty in perceiving, from an inspection of the figure, the settings to be made.

Generally it is necessary to compute the magnitudes of a number of angles before the slide rule computation can be carried out. This process is illustrated in Example 2.

Example 2. Find the length of the side marked z in Fig. 40(a).

Solution. To find the length of the side marked z in Fig. 40(a), first draw Fig. 40(b), compute the angles shown in the figure, and push the hairline to 289 on D,
draw 77° (= 180° − 103°) of S under the hairline,
push hairline to 32° on S,
draw 38° of S under the hairline,
push hairline to 65° on S,

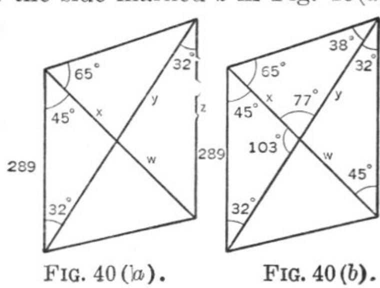


FIG. 40 (a). FIG. 40 (b).

draw 45° of S under the hairline,
push hairline to 77° on S,
at the hairline read $z =$ **319** on D.

In some problems it is necessary to perform some of the earlier settings in a chain of settings, compute some parts on the basis of the results, make some more settings, compute more parts, etc. This process is illustrated in Example 3.

Example 3. Find the side x of the inscribed quadrilateral shown in Fig. 41 (a).

Solution. Angles Q and S are right angles because each is inscribed in a semicircle. Knowing two legs of right triangle PQR we first find its hypotenuse and then deal with triangle PSR. Accordingly

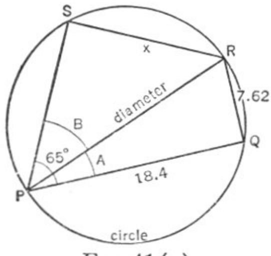


FIG. 41 (a).

To 18.4 on D set left index of slide,
push hairline to 7.81 on D,
at the hairline read A [Fig. 41 (b)] = 23° on T,
draw 23° of S under the hairline,
compute B [Fig. 41 (b)] = 65° − A = 42°,
exchange indices (see § 6),
push hairline to 42° on S,
at the hairline read **x** = **13.37** on D.

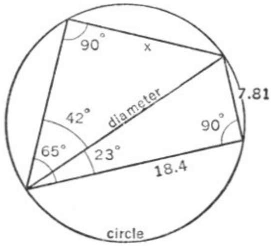


FIG. 41 (b).

The following example illustrates more in detail the same method of procedure.

Example 4. An engineer in a level country wishes to find the distance between two inaccessible points C and D and the direction of the line connecting them. He runs the line AB Fig. 42 (a) due north and measures the side and angles as indicated. Using his data solve his problem.

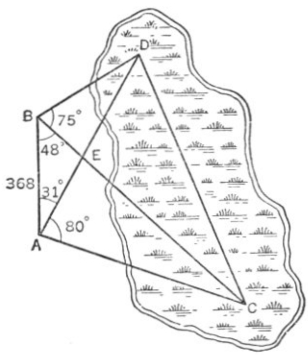


FIG. 42 (a).

Solution. First find EA (but do not write it), and then find $EC = 766$; afterwards find BE (but do not write it) and then $ED = 425$. In the triangle DEC [see Fig. 42 (b)] two sides and the included angle are now known; hence the method of §35 may be applied to it to find $DC =$ **944** and angle $ECD = 26\frac{1}{4}°$. Therefore the angle $NCD = 48° - 26\frac{1}{4}° = 21\frac{3}{4}°$, and line CD makes an angle of $\mathbf{21\frac{3}{4}°}$ with a line directed due north. The operator may check these answers by making the suggested settings.

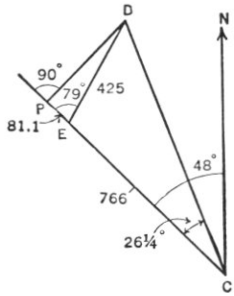


FIG. 42 (b).

EXERCISES

1. Find the length of the line segment BC in Fig. 39.

2. Find the length of the line segment marked w in Fig. 40a.

3. In Fig. 43 find the length of the line segment marked x.

4. Line segment AB in Fig. 44 is horizontal and CD is vertical. Find the length of CD.

5. In the statement of Ex. 4, replace "Fig. 44" by "Fig. 45" and solve the resulting problem.

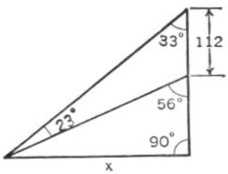


FIG. 43.

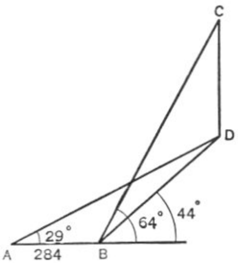


FIG. 44.

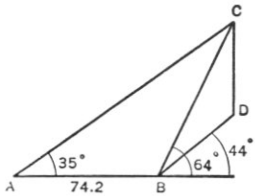


FIG. 45.

6. Find the length of the line segment marked x in Fig. 46.

7. If in Fig. 47 line segment BD is perpendicular to plane ABC, find its length.

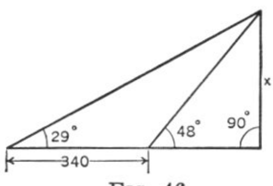


FIG. 46.

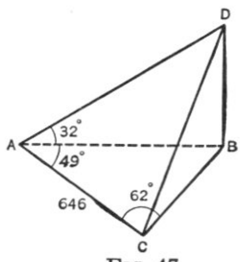


FIG. 47.

8. A tower and a monument stand on a level plane. (See Fig. 48). The angles of depression of the top and bottom of the monument viewed from the top of the tower are 13° and 31° respectively; the height of the tower is 145 ft. Find the height of the monument.

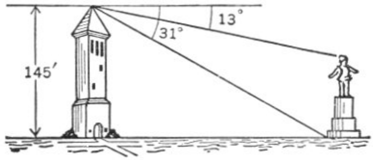


FIG. 48.

9. The captive balloon C shown in Fig. 49 is connected to a ground station A by a cable of length 842 ft. inclined 65° to the horizontal. In a vertical plane with the balloon and its station and on the opposite side of the balloon from A a target B was sighted from the balloon on a level with A. If the angle of depression of the target from the balloon is 4° find the distance from the target to a point C directly under the balloon.

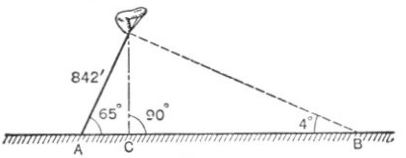


FIG. 49.

10. A light house standing on the top of the cliff shown in Fig. 50 is observed from two boats A and B in a vertical plane through the light house. The angle of elevation of the top of the light house viewed from B is 16° and the angles of elevation of the top and bottom viewed from A are 40° and 23°, respectively. If the boats are 1320 ft. apart find the height of the light house and the height of the cliff.

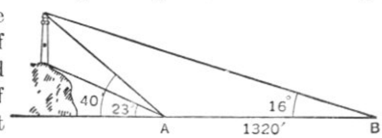


FIG. 50.

11. Fig. 51 represents a 600 ft. radio tower. AC and AD are two cables in the same vertical plane anchored at two points C and D on a level with the base of the tower. The angles made by the cables with the horizontal are 44° and 58° as indicated. Find the lengths of the cables and the distance between their anchor points.

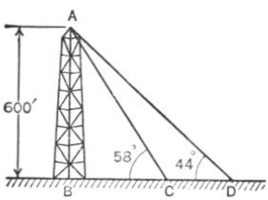


FIG. 51.

12. Two fixed objects, A and B of Fig. 52, were observed from a ship at point S to be on a straight line passing through S and bearing N 15° E. After sailing 5 miles on a course N 42° W the captain of the ship found that A bore due east and B bore N 40° E. Find the distance from A to B.

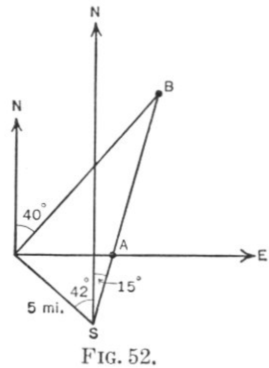


FIG. 52.

CHAPTER V

LOGARITHMS AND THE SLIDE RULE

43. Construction of the *D* scale. Perhaps the simplest explanation of the construction of the scales of the slide rule can be made in terms of logarithms. Since nearly all the scales are constructed by the same method, a detailed consideration of the construction of the *D* scale will indicate how most of the other scales are made.

The *D* scale is ten inches long.* To construct a *D* scale, draw a line 10 inches long, make a mark at its left end and letter it 1, see Fig. 1. This mark will be referred to as the left index. Taking

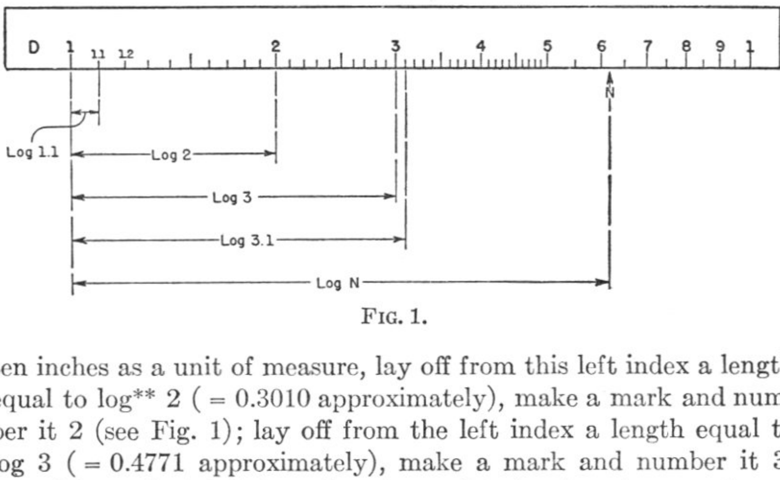


FIG. 1.

ten inches as a unit of measure, lay off from this left index a length equal to log** 2 (= 0.3010 approximately), make a mark and number it 2 (see Fig. 1); lay off from the left index a length equal to log 3 (= 0.4771 approximately), make a mark and number it 3; lay off from the left index a length equal to log 4, make a mark and letter it 4, etc.; until a mark has been made for each of the digits from 1 to 9. Instead of marking the right index 10 as we should expect, since log 10 = 1.0, number it 1. This gives the ten primary divisions. The other division marks are located in a similar manner. Thus *to each division mark is associated a number and this mark is*

*Nominally the *D* scale is 10 inches long. Its exact length however is 25 centimeters.

**The symbol log N will be understood to mean the mantissa of $\log_{10} N$ unless otherwise specified.

situated at a distance from the left index equal to the mantissa of the logarithm of that number.

It is interesting to note that the distance on this scale between a number N and a number M, $N>M$, is equal to $\log \frac{N}{M}$.

The mantissa, or fractional part of the logarithm of a number, is independent of the position of the decimal point. Hence if we think of the distances from the left index as the mantissas of the logarithms of the numbers represented by the divisions, it appears that we can think of the primary divisions as representing the range of numbers 1, 2, 3, . . . 10, the range 10, 20, 30, . . . 100, the range 100, 200, 300, . . . 1000, etc. Naturally, in each of these cases, we think of the secondary divisions as representing appropriate numbers lying between the numbers represented by adjacent primary divisions.

44. Accuracy. From §43 we write

$$\log_{10} N = d \tag{1}$$

where N represents the number associated with any specified mark on the D scale and d is the distance of the mark from the left index. By applying calculus to equation (1) we easily prove that for small errors in d

$$\text{Relative error in } N = \frac{(\text{error in } N)}{N} = 2.3026 \text{ (error in } d). \tag{2}$$

Now the error in d is the error made in making the reading. The right hand member is independent of N. Therefore the relative error in the number read does not depend on its size and hence is the same for all parts of the scale. Near the left end of the D scale a careful reading should be in error by no more than 1 in the fourth place i.e. the relative error should be no greater than 1 in 1000. Hence the accuracy of any part of the D scale is roughly 1 in 1000 or one tenth of one percent.

45. Multiplication and division. The middle part of the rule which may be moved back and forth relative to the other part is referred to as the *slide*; the outer or fixed part of the rule is called the *body*. The D scale is located on the body and the C scale is the same as the D scale except that it is located on the slide. Hence the C scale may be moved relative to the D scale, and we are able to add distances as indicated in Fig. 2.

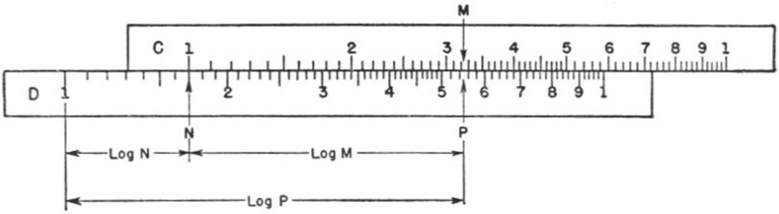


FIG. 2.

From this figure and the considerations of §43, it appears that

$$\log P = \log N + \log M. \tag{3}$$

But the sum of two logarithms is equal to the logarithm of a product. Hence from (3) we have

$$\log P = \log MN, \text{ or } P = MN. \tag{4}$$

Thus it appears that Fig. 2 shows the setting to be used for mul-

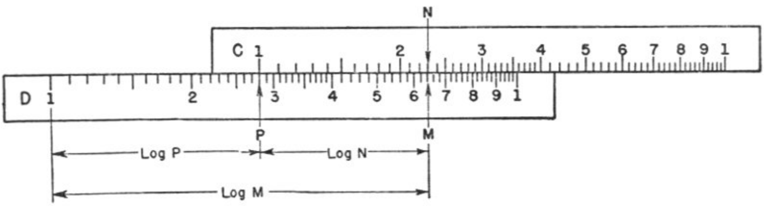


FIG. 3.

tiplying numbers. From Fig. 3 and the considerations of §43 it appears that

$$\log P = \log M - \log N, \tag{5}$$

or since

$$\log M - \log N = \log (M/N),$$

we have

$$\log P = \log \frac{M}{N}, \text{ and } P = \frac{M}{N}. \tag{6}$$

Thus Fig. 3 shows the setting to be used for dividing numbers.

The rule for multiplication §5 and the rule for division §7 are justified by the principles set forth above.

46. The inverted scales. The CI and DI scales are constructed in the same manner as the D scale except that the distances are measured leftward from the right index, and the numbers associated with the primary division marks are in red.

Let N be the number associated with a position on the C scale and K the number on the CI scale associated with the same position. Then, in accordance with §43,

$$\log N + \log K = 1.$$

Hence we may write

$$\log K = 1 - \log N = \log 10 - \log N = \log \frac{10}{N},$$

or

$$K = \frac{10}{N}.$$

Therefore, except for the position of the decimal point, K is the reciprocal of N. In other words, *when the hairline is set to a number on the CI scale, it is automatically set to the reciprocal of that number on the C scale.*

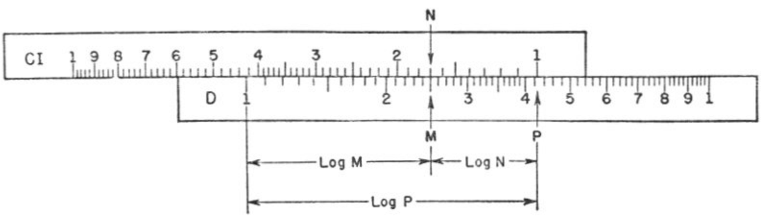


FIG. 4.

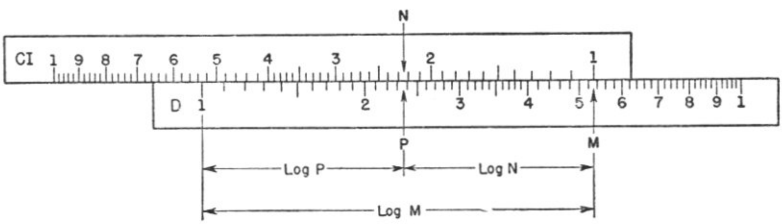


FIG. 5.

Fig. 4 indicates how multiplication may be accomplished by using the CI scale in conjunction with the D scale while Fig. 5 indicates how division may be accomplished. From Fig. 4, we have

$$\log P = \log M + \log N, \text{ or } P = MN,$$

and from Fig. 5, we have

$$\log P = \log M - \log N, \text{ or } P = M/N.$$

Scale DI is the same as scale CI except that scale DI is located on the body. Evidently, then, the CI and DI scales can be used together in the processes of multiplication and division just like the scales C and D.

47. The A scale, the B scale, and the K scale. The A scale is constructed by the method used in the case of the D scale except that the unit of measure employed is 5 inches instead of 10 inches and the scale is repeated. The B scale is the same as the A scale except that it is situated on the slide while the A scale is on the body.

When the hairline is set to a number N on the A scale it is automatically set to a number M on the D scale, see Fig. 6. The two

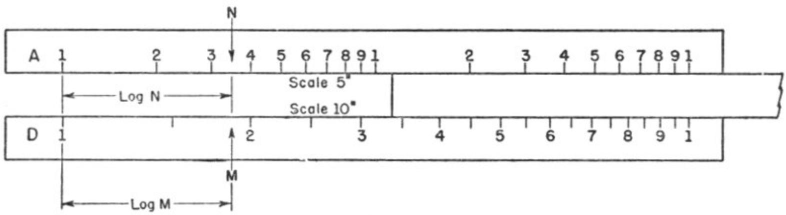


FIG. 6.

lengths marked log N and log M in the figure are equal. However since the unit in the case of log N is half the unit in the case of log M, we have

$$\log M = \frac{1}{2} \log N = \log N^{\frac{1}{2}} = \log \sqrt{N},$$

and

$$M = \sqrt{N}.$$

Hence, *a number on scale D is the square root of the opposite number on scale A.* A similar relation exists between numbers on scales C and B.

The K scale is constructed by the method used in the case of the D scale except that the unit of measure employed is one third of 10 inches instead of 10 inches. The argument used above may be employed to show that *when the hairline is set to a number on the K scale it is automatically set to the cube root of the number on the D scale.*

48. The trigonometric scales. The general plan of constructing the S (sine) scale is the same as that for the D scale. Here again 10 inches is taken as the unit of measure. To each division mark on the S scale is associated an acute angle (in black) such that the distance of the division from the left index is equal to the mantissa of the logarithm of the sine of the angle. Thus Fig. 7 shows the division marked 25 at a distance from the left index of the mantissa of log sin 25°. Hence when the hairline is set to an angle on the sine scale, it is automatically set to the sine of the angle on the C scale. Fig. 8 shows a setting for finding $P = \dfrac{16 \sin 68°}{\sin 27°}$.

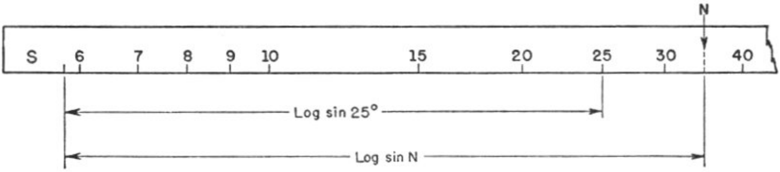


FIG. 7.

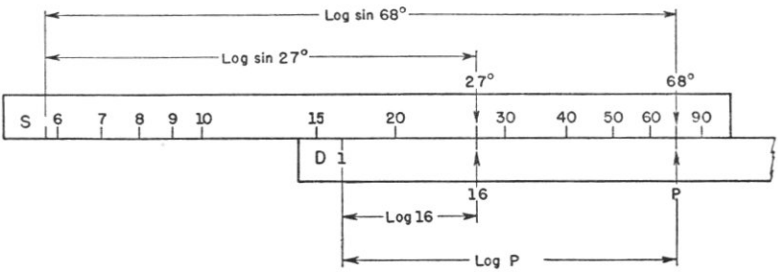


FIG. 8.

From this figure it appears that

$$\log P = \log 16 - \log \sin 27° + \log \sin 68° = \log \frac{16 \sin 68°}{\sin 27°},$$

or

$$P = \frac{16 \sin 68°}{\sin 27°}.$$

Since the slide rule does not take account of the characteristics of the logarithms, the position of the decimal point is determined in accordance with the result of a rough approximation.

If the learner will note that the angles designated by red numbers are the complements of the angles in black, and remember that the distance from a division on the C scale to the right index is the logarithm of the reciprocal of the number represented by the division, and also that

$$\begin{aligned} \sin \theta &= \cos (90° - \theta), \\ \csc \theta &= 1/\sin \theta, \\ \sec \theta &= 1/\cos \theta, \end{aligned}$$

he will easily see the relations indicated in Fig. 9 for the representative angle 25°.

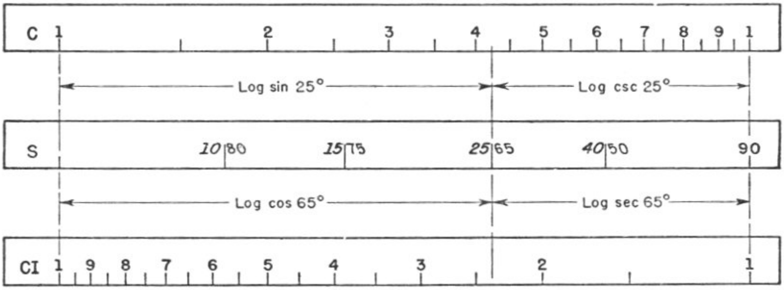


FIG. 9.

The T scale was constructed by taking 10 inches as the unit of measure and associating to each division mark on it an acute angle such that the distance of the mark from the left index is equal to the mantissa of the logarithm of the tangent of the angle. Recalling that

$$\cot(90^\circ - \theta) = \tan\theta = 1/\cot\theta,$$

the student will easily see the relations indicated in Fig. 10 for the representative angle 25°.

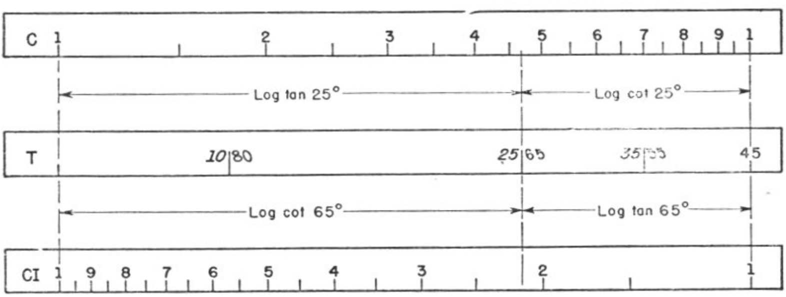


FIG. 10.

The facts illustrated in Figs. 9 and 10 are the basis of the following rule:

If the hairline be set to an angle on a trigonometric scale, it is automatically set to the *complement* of this angle. One of these angles is expressed in black type, the other in red. From what has been said it appears that we read, at the hairline on the C scale or on the CI scale, a figure expressing a direct function (sine, tangent, secant) by reading a figure of the same color as that representing the angle, a co-function (cosine, cosecant, cotangent) by reading a figure of the opposite color. In other words, *associate direct function with like colors, co-function with opposite colors.*

The S scale applies to angles ranging from 5.73° to 90°; the sines of these angles range from 0.1 to 1. Any angle in the range from 0.583° to 5.73° has a sine approximately equal to its tangent. The ST scale is related to the angles ranging from 0.583° to 5.73° just as the S scale is related to the angles ranging from 5.73° to 90°. Since any angle greater than 0.583° but less than 5.73° has its sine approximately equal to its tangent, the ST scale may be used for tangents as well as for sines.

49. Two applications. An interesting setting is one from which may be obtained the solution of a right triangle when two legs are given. Let it be required to find the angle α and the side c of the triangle shown in Fig. 11. From the triangle we write

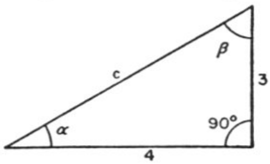


FIG. 11.

$$\cot \alpha = \frac{4}{3}, \; c = 3 \csc \alpha.$$

Fig. 12, showing the setting from which we read $\alpha = 36.9°$ on the tangent scale and $c = 5$ on the DI scale, is self explanatory.

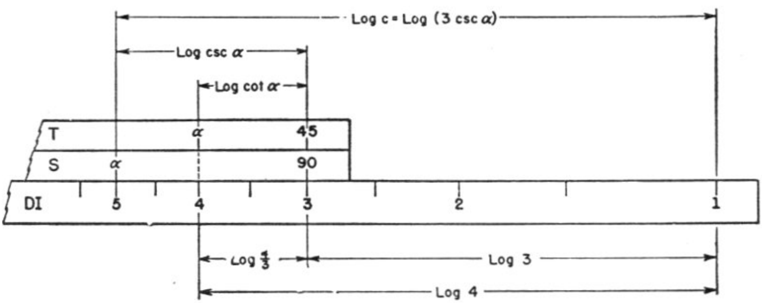


FIG. 12.

Fig. 13 indicates the logarithmic basis of a setting which may be used to evaluate $\dfrac{\sqrt{223} \tan 25°}{\sin 16°}$. From the figure it appears that

$$\log P = \log \sqrt{223} - \log \sin 16° + \log \tan 25° = \log \frac{\sqrt{223} \tan 25°}{\sin 16°},$$

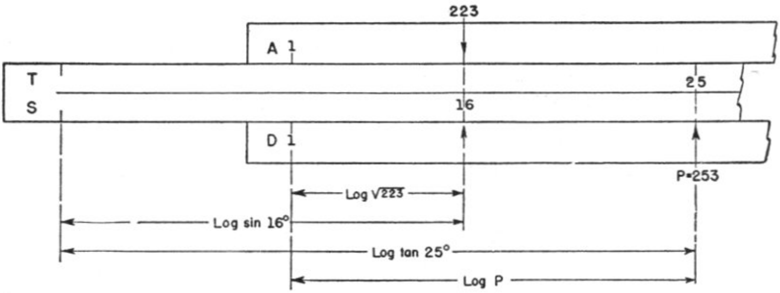


FIG. 13.

or $P = \dfrac{\sqrt{223}\tan 25°}{\sin 16°}$. Since the reading at P is 253, we have $\dfrac{\sqrt{223}\tan 25°}{\sin 16°} = 25.3.$

HISTORICAL NOTE

In 1614 John Napier, of Merchiston, Scotland, first published his "Canon of Logarithms."

Napier concisely sets forth his purpose in presenting to the world his system of Logarithms as follows:

"Seeing there is nothing (right well beloved Students of Mathematics) that is so troublesome to mathematical practice, nor doth more molest and hinder calculators, than the multiplications, divisions, square and cubical extractions of great numbers, which besides the tedious expense of time are for the most part subject to many slippery errors, I began therefore to consider in my mind by what certain and ready art I might remove those hindrances."

Napier's invention of logarithms made possible the modern slide rule, the fruition of his early conception of the importance of abbreviating mathematical calculations.

In 1620 Gunter invented the straight logarithmic scale, and effected calculation with it by the aid of compasses.

In 1630 Wm. Oughtred arranged two Gunter logarithmic scales adapted to slide along each other and kept together by hand. He thus invented the first instrument that could be called a slide rule.

In 1675 Newton solved the cubic equation by means of three parallel logarithmic scales, and made the first suggestion toward the use of an indicator.

In 1722 Warner used square and cube scales.

In 1755 Everard inverted the logarithmic scale and adapted the slide rule to gauging.

In 1815 Roget invented the log-log scale.

In 1859 Lieutenant Amédée Mannheim, of the French Artillery, invented the present form of the rule that bears his name.

In 1881 Edwin Thacher invented the cylindrical form which bears his name.

In 1891 Wm. Cox devised the **Duplex*** Slide Rule. The sole rights to this type of rule were then acquired by Keuffel & Esser Co.

For a complete history of the Logarithmic Slide Rule, the student is referred to "A History of the Logarithmic Slide Rule," by Florian Cajori, published by the Engineering News Publishing Company, New York City. This book traces the growth of the various forms of the rule from the time of its invention to 1909.

*REG. U. S. PAT. OFF.

ANSWERS

§5. Page 8

1. 6
2. 7
3. 10
4. 9.1
5. 6.75
6. 9.62
7. 49.8
8. 340
9. 47.0
10. 0.0826
11. 3220
12. 0.836
13. 9.86
14. 3.08

§6. Page 9

1. 15
2. 15.8
3. 3530
4. 42.1
5. 0.001322
6. 1737
7. 9.98
8. 1,340

§7. Page 10

1. 2.32
2. 165.2
3. 0.0767
4. 106.1
5. 0.000713
6. 77.5
7. 1861
8. 26.3
9. 1.154
10. 0.0419

§8. Pages 11, 12

1. (*a*) 1576
   (*b*) 2.60
   (*c*) 5.25
   (*d*) 4.59
2. (*a*) 26.1%
   (*b*) 64.4%
   (*c*) 220%
   (*d*) 2.73%
3. (*a*) 178.9 mi.
   (*b*) 121.1 mi.
   (*c*) 2140 mi.
4. (*a*) 9.22 yds./sec.
   (*b*) 15.02 ft./sec.
   (*c*) 186,000 mi./sec.
5. (*a*) 10.13 sec.
   (*b*) 29.5 hrs.
   (*c*) 322 hrs.

§9. Page 13

1. 36.7
2. 8.35
3. 0.0000632
4. 3400
5. 0.00357
6. 13,970
7. 1586
8. 0.0223
9. 0.01311
10. 2.36
11. 0.0414
12. 2460
13. 249
14. 0.275
15. 0.1604
16. 0.0977

§11. Pages 17, 18

1. $x = 5.22$
2. $x = 2.30, \quad y = 31.8$
3. $x = 51.7, \quad y = 3370$
4. $\begin{cases} x = 3.97 \\ y = 0.984 \\ z = 0.272 \end{cases}$
5. $\begin{cases} x = 0.1013 \\ z = 0.0769 \end{cases}$
6. $\begin{cases} x = 1.586 \\ y = 41.4 \end{cases}$
7. $\begin{cases} x = 106.2 \\ y = 30.4 \end{cases}$
8. $\begin{cases} x = 0.1170 \\ y = 0.927 \end{cases}$
9. $\begin{cases} x = 186 \\ y = 13.42 \\ z = 50.3 \end{cases}$

§12. Page 19

1. 48.7
2. 0.396
3. 9.46
4. 42.0
5. 3.14
6. 3.20
7. 4.07
8. 0.1264
9. 104.6
10. 9.69

§13. Page 20

1. 167.8 cm.
2. 274 m.
3. 720 lb.
4. 235 sq. cm.
5. (a) 25,700 watts
   (b) 3,940,000 watts
   (c) 621 watts
6. (a) 1.122 gal.
   (b) 0.00255 gal.
   (c) 0.1504 gal.

§14. Page 21

1. 0.0625, 0.00385, 1.389, 15.38
   0.0575, 0.0541, 0.01490
2. 2, 162
3. 74.0, 10.97
4. 200, 8.55

§15. Page 23

1. $x = 16.98$, $y = 12.74$
2. $x = 0.0640$, $y = 1.415$
3. $x = 154.9$, $y = 6940$
4. $x = 0.00247$, $y = 645$
5. $\begin{cases} x = 0.0481 \\ y = 0.0435 \\ z = 44.95 \end{cases}$
6. $\begin{cases} x = 11.07 \\ y = 0.0484 \\ z = 0.465 \end{cases}$

§16. Page 25

1. 0.001156
2. 1.512
3. 1.015
4. 17.2
5. 96.1
6. 0.1111
7. 150,800
8. 15.32
9. 9.76
10. 0.00288
11. 144,700
12. 0.0267
13. 0.279
14. 41.3
15. 111.1
16. 3430

§17. Page 27

1. 625, 1024, 3720, 5620, 7920, 537,000, 204,000, 4.33, 3.07, 0.1116, 0.00001267
   0.908, 27,800,000, 2.24×10^{13}
2. (a) 5.94 ft.^{2} (b) 3500 ft.^{2} (c) 0.445 ft.^{2} (d) 2.76 ft.^{2}
3. (a) 37.6 ft.^{2} (b) 0.00597 ft.^{2} (c) 966 ft.^{2} (d) 2.35×10^8 ft.^{2}

§18. Page 28

1. 2.83, 3.46, 4.12, 9.43, 2.98, 29.8, 0.943, 85.3, 0.252, 0.00797, 252, 316
2. (a) 231 ft. (b) 0.279 ft. (c) 5720 ft.
3. (a) 18.05 ft. (b) 0.992 ft. (c) 49.7 ft.

§19. Page 30

1. 24.2
2. 0.416
3. 8.54
4. 0.0698
5. 4.43
6. 4.01
7. 6.69
8. 6.14
9. 0.428
10. 1.176
11. 32.8
12. 398
13. 43.7
14. 29.4

§20. Page 32

1. 64.2
2. 11.41
3. 109.2
4. 0.428
5. 9.65
6. 0.0602
7. 1.525×10^5
8. 1.589

§21. Page 33

1. 9.260, 32.8, 238,000, 422,000, 705,000, 3.94×10^8, 0.0923, 29.2, 5.39, 0.0000373, 0.839, 1.46×10^{11}, 5.71×10^{19}. **2.** 76.2

§22. Page 34

1. 2.06, 3.11, 9.00, 9.47, 19.69, 0.1969, 0.424, 0.914, 44.8, 0.855, 909, 2.15, 4.64, 46.4

§23. Page 35

1. 2.19	**7.** 43,100	**12.** 12.77	**17.** 5.02
2. 30.9	**8.** 1.745	**13.** 76.4	**18.** 2290
3. 54.2	**9.** 1.156	**14.** 2.12	**19.** 0.0544
4. 0.974	**10.** 1.192	**15.** 1.281×10^8	**20.** 3.29
5. 1.522	**11.** 90.7	**16.** 0.00370	**21.** 0.000867
6. 0.0577			**22.** 27.3

§24. Page 36

1.515, 0.814, 5.991, 9.830–10, 8.022–10, 6.615–10,
1.861, 9.427–10, 7.904–10, 2.636

§26. Page 39

2.		(*a*) 0.5	(*b*) 0.616	(*c*) 0.0581	(*d*) 1	(*e*) 0.999
		(*f*) 0.0276	(*g*) 0.253	(*h*) 0.381	(*i*) 0.204	(*j*) 0.783
3.		(*a*) 0.866	(*b*) 0.788	(*c*) 0.998	(*d*) 0	(*e*) 0.0349
		(*f*) 1.00	(*g*) 0.968	(*h*) 0.924	(*i*) 0.979	(*j*) 0.623
4.	*A*.	(*a*) 30°	(*b*) 61.1°	(*c*) 22°	(*d*) 5.73°	(*e*) 0.859°
		(*f*) 38.3°	(*g*) 3.55°	(*h*) 1.776°	(*i*) 66.9°	(*j*) 62.2°
	B.	(*a*) 60°	(*b*) 28.9°	(*c*) 68.0°	(*d*) 84.27°	(*e*) 89.14°
		(*f*) 51.7°	(*g*) 86.4°	(*h*) 88.224°	(*i*) 23.1°	(*j*) 27.8°
5.		(*a*) 2	(*b*) 1.623	(*c*) 17.21	(*d*) 1	(*e*) 1.001
		(*f*) 36.2	(*g*) 3.95	(*h*) 2.63	(*i*) 4.90	(*j*) 1.277
6.		(*a*) 1.155	(*b*) 1.27	(*c*) 1.002	(*d*) ∞	(*e*) 28.65
		(*f*) 1	(*g*) 1.033	(*h*) 1.082	(*i*) 1.021	(*j*) 1.605
7.	*A*.	(*a*) 30°	(*b*) 24.6°	(*c*) 36°	(*d*) 9.40°	(*e*) 0.717°
		(*f*) 12.23°				
	B.	(*a*) 60°	(*b*) 65.4°	(*c*) 54°	(*d*) 80.6°	(*e*) 89.283°
		(*f*) 77.8°				

§27. Page 40

1. 0.1423, 0.514, 1.905, 0.01949, 3.53, 19.08, 1.091
7.03, 1.946, 0.525, 51.3, 0.283, 0.0524, 0.916

2.	(*a*) 13.50°	(*b*) 38.13°	(*c*) 42.62°	(*d*) 28.37°	(*e*) 3.38°
	(*f*) 4.70°	(*g*) 23.38°	(*h*) 2.47°	(*i*) 0.854°	(*j*) 20.50°
	(*k*) 74.95°	(*l*) 77.92°	(*m*) 86.63°	(*n*) 45.85°	(*o*) 50.93°
3.	(*a*) 76.50°	(*b*) 51.87°	(*c*) 47.38°	(*d*) 61.63°	(*e*) 86.62°
	(*f*) 85.30°	(*g*) 66.63°	(*h*) 87.53°	(*i*) 89.146°	(*j*) 69.50°
	(*k*) 15.05°	(*l*) 12.08°	(*m*) 3.37°	(*n*) 44.15°	(*o*) 39.07°

§29. Page 43

1. 30.5	**7.** 5.26	**13.** 2.033	**19.** 38.18
2. 0.360	**8.** 254	**14.** 0.720	**20.** 0.00319
3. 4.61	**9.** 0.0679	**15.** 4.24	**21.** 0.001086
4. 24.2	**10.** 0.267	**16.** 1.226	**22.** 50.8
5. 14.25	**11.** 1.347	**17.** 0.0771	**23.** 0.01375
6. 16.79	**12.** 16.47	**18.** 0.0963	**24.** 0.0432

§30. Page 46

1. (*a*) $x = 6.10$, $\theta = 61.71°$ (*b*) $\theta = 54.0°$, $x = 21.6$ (*c*) $x = 30.4$, $y = 44.5$ (*d*) $\theta = 4.92°$, $\varphi = 8.08°$
2. (*a*) 2.5 (*b*) 10.39 (*c*) 44 (*d*) 43.9 (*e*) 25.4 (*f*) 44.1 (*g*) 17.68 (*h*) 9.01
3. (*a*) 5.87° (*b*) 55.8° (*c*) 84° (*d*) 59°
4. (*a*) 35.4 (*b*) 80.7 (*c*) 31.9 (*d*) 261.5
5. (*a*) 0.978 (*b*) 6.03 (*c*) − 9.15 (*d*) 16.45
6. (*a*) $\theta = 25°$, $\alpha = 40.2°$ (*b*) $y = 0.0731$, $\theta = 4.47°$ (*c*) $y = 2568$ (*d*) $y = 25.3$ (*e*) $y = 11.45$ (*f*) $y = 0.0885$ (*g*) $y = 0.638$ (*h*) $\theta = 4.13°$ (*i*) $\theta = 16.42°$

§31. Pages 49, 50

1. $A = 36.9°$, $B = 53.1°$, $b = 80$
2. $B = 51.3°$, $c = 80.9$, $b = 63.2$
3. $A = 21.2°$, $b = 1884$, $c = 2020$
4. $B = 26°$, $a = 410$, $c = 457$
5. $A = 83.8°$, $a = 36.98$, $b = 4.02$
6. $B = 46.5°$, $a = 7.71$, $b = 8.12$
7. $A = 27.1°$, $a = 24.37$, $c = 53.56$
8. $A = 43.3°$, $B = 46.7°$, $b = 0.662$
9. $B = 17.92°$, $b = 26.91$, $c = 87.6$
10. $B = 37.3°$, $a = 5570$, $c = 7007$
11. $B = 34.4°$, $A = 55.6°$, $a = 4.22$
12. $A = 55.3°$, $B = 34.7°$, $a = 740$
13. $B = 31.4°$, $a = 30.5$, $b = 18.59$
14. $A = 79.2°$, $a = 0.713$, $b = 0.1367$
15. $A = 68.4°$, $b = 0.3245$, $c = 0.883$
16. $A = 60.1°$, $B = 29.9°$, $b = 29.94$
17. $A = 87.67°$, $b = 76.1$, $c = 1877$
18. $B = 1.567$, $a = 329.1$, $c = 329.2$
19. 160.7 yd.
20. 7.07
21. 35.3°

§32. Page 51

1. $A = 31.3°$, $B = 58.7°$, $c = 23.7$
2. $A = 41.0°$, $B = 49.0°$, $c = 153.8$
3. $A = 65°$, $B = 25°$, $c = 55.2$
4. $A = 33.2°$, $B = 56.8°$, $c = 499$
5. $A = 39.5°$, $B = 50.5°$, $c = 44$
6. $A = 67.4°$, $B = 22.6°$, $c = 13$
7. $A = 45°$, $B = 45°$, $c = 18.67$
8. $A = 30.6°$, $B = 59.4°$, $c = 82.5$
9. $A = 3.7°$, $B = 86.3°$, $c = 4.8$
10. 50°

§33. Page 53

1. $C = 75°$, $b = 35.46$, $c = 53.3$
2. $C = 55°$, $b = 70.7$, $a = 56.1$
3. $C = 123°\,12'$, $b = 2257$, $c = 2599$
4. $A = 2°\,47'$, $B = 87°\,13'$, $c = 4570$
5. $B = 35°\,16'$, $C = 84°\,44'$, $c = 138$
6. $A = 17°\,41'$, $C = 53°\,19'$, $a = 0.0751$
7. $C = 55°\,20'$, $b = 568$, $c = 664$
8. $b = 279$, $c = 284$, $C = 100°\,50'$
9. $A = 87°\,41'$, $C = 41°\,12'$, $a = 116.9$
10. Impossible.
11. $B = 30°\,3'$, $C = 90°$, $b = 5.01$
12. $c = 123.8$, $B = 3°\,18'\,35''$, $C = 116°\,41'\,25''$
13. 1253 ft.
14. 1034.8 yd.

§34. Pages 54, 55

1. $B_1 = 66°\,10'$, $C_1 = 58°\,26'$, $c_1 = 18.6$, $B_2 = 113°\,50'$, $C_2 = 10°\,46'$, $c_2 = 4.08$
2. $B_1 = 16°\,43'$, $A_1 = 147°\,28'$, $a_1 = 35.5$, $B_2 = 163°\,17'$, $A_2 = 0°\,54'$, $a_2 = 1.04$
3. $A_1 = 70°\,12'$, $B_1 = 57°\,24'$, $b_1 = 28.79$, $A_2 = 109°\,48'$, $B_2 = 17°\,48'$, $b_2 = 10.45$
4. $A_1 = 68°\,47'$, $C_1 = 67°\,10'$, $a_1 = 6.92$, $A_2 = 23°\,7'$, $C_2 = 112°\,50'$, $a_2 = 2.91$
5. $B_1 = 45°\,16'$, $C_1 = 99°\,8'$, $c_1 = 300$, $B_2 = 134°\,44'$, $C_2 = 9°\,40'$, $c_2 = 51.1$
6. $A_1 = 51°\,19'$, $C_1 = 88°\,41'$, $c_1 = 21{,}850$, $A_2 = 128°\,41'$, $C_2 = 11°\,19'$, $c_2 = 4290$
7. $p = 3.13$; (*a*) none, (b) 2, (*c*) 1

§35. Page 56

1. $A = 119°\,54'$, $B = 31°\,6'$, $c = 52.6$
2. $A = 49°\,4'$, $C = 79°\,7'$, $b = 104.1$
3. $A = 55°\,2'$, $B = 40°\,21'$, $c = 285$
4. $B = 39°\,16'$, $C = 78°\,44'$, $a = 3.21$
5. $A = 100°\,57'$, $C = 33°\,3'$, $b = 19.8$
6. $A = 46°\,26'$, $C = 6°\,24'$, $b = 7.43$
7. $A = 121°\,4'$, $C = 2°\,26'$, $b = 0.0828$
8. $A = 77°\,12'$, $B = 43°\,30'$, $c = 15$
9. $B = 13°\,22'$, $C = 28°\,17'$, $a = 7420$
10. 10 and 4.68
11. 4.93 mi.
12. Lat. = 218 ft. Dep. = 478 ft.

§36. Page 57

1. $A = 106°\,47'$, $B = 46°\,53'$, $C = 26°\,20'$
2. $A = 27°\,21'$, $B = 143°\,8'$, $C = 9°\,32'$
3. $A = 52°\,26'$, $B = 59°\,23'$, $C = 68°\,12'$
4. $A = 49°\,12'$, $B = 37°\,36'$, $C = 93°\,12'$
5. $A = 106°\,18'$, $B = 37°\,55'$, $C = 35°\,47'$
6. $A = 48°\,11'$, $B = 58°\,25'$, $C = 73°\,24'$
7. $A = 44°\,42'$, $B = 49°\,37'$, $C = 85°\,40'$; $A = 83°\,42'$, $B = 59°\,22'$, $C = 36°\,56'$
9. $A = 53°\,8'$, $B = 59°\,30'$, $C = 67°\,22'$
10. $51°\,53'$, $59°\,32'$, $68°\,35'$
11. $72°\,36'$

§38. Page 59

1. $B = 70°$
$a = 27.4$
$b = 75.2$

2. $B = 80°$
$a = 5.29$
$c = 30.5$

3. $B = 15°$
$b = 21.4$
$c = 82.8$

4. $A = 30.3°$
$B = 59.7°$
$c = 20.04$

5. $A = 29°$
$B = 46.6°$
$C = 104.4°$

6. $A = 74.6°$
$B = 47.8°$
$C = 57.6°$

7. $B = 54°$
$a = 0.589$
$b = 0.809$

8. $C = 55.3°$
$b = 568$
$c = 664$

9. $C = 123.2°$
$b = 2052$
$c = 2363$

10. $B = 59.1°$
$a = 33.78$
$c = 18.54$

11. $A = 54.5°$
$B = 47.8°$
$b = 50.5$

12. $A = 40.9°$
$C = 59.9°$
$b = 77.1$

13. $A = 49.4°$
$B = 58.6°$
$C = 72°$

14. $A = 34.8°$
$B = 86.4°$
$C = 58.8°$

15. $C = 72.3°$
$A = 51.2°$
$c = 95.2$

16. $C = 69°$
$a = 3320$
$c = 7480$

17. $B = 24.8°$
$b = 38{,}900$
$A = 27.6°$

18. $A = 45.2°$
$B = 101.4°$
$C = 33.4°$

19. 81°

20. 5 ft., 2.34 ft.

21. 26.17 sec.

22. 47.9° east of no.

23. 84°

24. 21.7 ft.

§39. Page 60

1. (*a*) 0.785 (*b*) 1.047 (*c*) 1.571 (*d*) 3.14 (*e*) 2.09
(*f*) 2.36 (*g*) 0.393 (*h*) 3.49 (*i*) 52.4

2. (*a*) 60° (*b*) 135° (*c*) 2.5° (*d*) 210° (*e*) 1200° (*f*) 176.4°

3. (*a*) 0.01745 (*b*) 0.0002909 (*c*) 0.00000485 (*d*) 0.1778 (*e*) 3.152 (*f*) 5.24

4. (*a*) 5.73° (*b*) 143.2° (*c*) 91.7° (*d*) 343.8°

§40. Page 62

1. 0.001454
2. 0.0000242
3. 0.00611
4. 0.0001551
5. 0.00204
6. 0.000252
7. 0.00436
8. 0.0001988
9. 171.8
10. 12,890
11. 1718
12. 114.5
13. 1.67
14. 0.232
15. 0.873
16. 0.00961
17. 0.0848
18. 4.15
19. 20
20. 0.04

§41. Pages 64 & 65

1. $x = 35.8$, $y = 19.36$
$r = 32.4$, $\theta = 55.6°$
$W = 18.0$, $\theta = 43.2°$

2. 16, 20°

3. 20, 25°

4. 2.4, 0.7

5. $x = 10.79$ lb.
$y = 26.7$ lb.

6. $r = 7.81$, $\theta = 74.2°$

7. 4.5, 126.9°

8. 206.6°

9. $m = 8.94$
$n = 4.47$

10. $z = 9.25$, $\theta = 27°$
$G = 0.0963$
$B = 0.0491$

11. $z = 14.41$, $B = 0.0551$,
$G = 0.0422$

§42. Pages 68 & 69

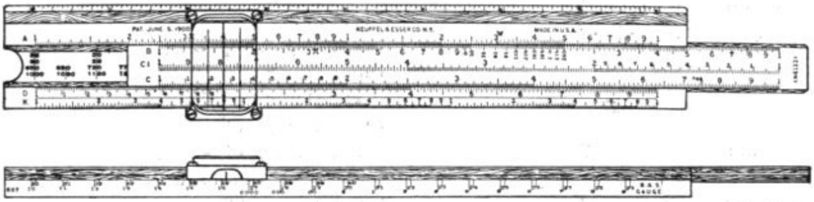
1. 677
2. 173.4
3. 129.4
4. 415
5. 41.7
6. 376
7. 382
8. 89.3 ft.
9. 10,910 ft.
10. 284 ft., 291 ft.
11. 864 ft., 708 ft., 246 ft.
12. 7.87 mi.

INDEX

SLIDE RULES FOR MANY PURPOSES

In addition to our extended line of regular Mannheim and Duplex* Slide Rules, we manufacture a variety of rules for special purposes, of which a few are named below:

THE ROYLANCE ELECTRICAL SLIDE RULE



This rule is a modification of our regular Polyphase* Slide Rule and can be used for all the calculations made with the ordinary Slide Rule. In addition to the usual scales, it carries a series of scales or gauge marks by means of which the different properties of copper wire, such as size, conductivity weight, etc., may be determined without the use of tables.

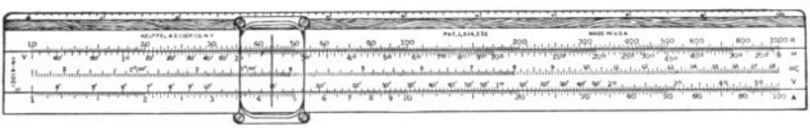
SURVEYOR'S DUPLEX SLIDE RULE

REG. U. S. PAT. OFF.

The fact that all astronomical data essential to surveying, such as azimuth, time, latitude, etc., can be ascertained by means of the usual type of Transit with vertical circle but without solar attachment, while generally known, is rather seldom utilized in this country. The main reason for this condition is the difficulty of computing, in the field, by spherical trigonometry, the results of observations.

The K & E Surveyor's Duplex* Slide Rule entirely eliminates this difficulty by reducing the hitherto complicated calculations to mere mechanical operations, thereby rendering the method of field astronomy with the regular Engineer's Transit extremely simple and practical.

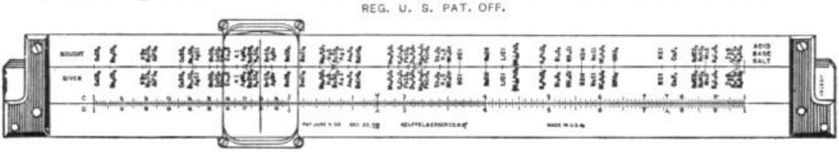
K & E STADIA SLIDE RULE



This form of Stadia Slide Rule is remarkable for its simplicity. By one setting of the slide the horizontal distance and vertical height can be obtained at once when the stadia rod reading and the angle of elevation or depression of the telescope are known.

THE CHEMIST'S DUPLEX SLIDE RULE

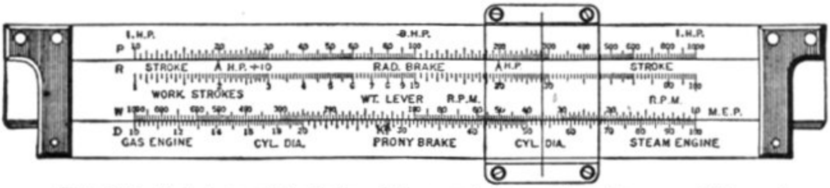
REG. U. S. PAT. OFF.



For the rapid solution of problems in Stoichiometry, such as Gravimetric and Volumetric Analysis, Equivalents, Percentage Composition, Conversion Factors and many others.

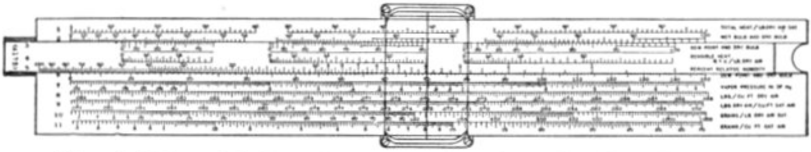
*REG. U. S. PAT. OFF.

POWER COMPUTING SLIDE RULE.



This Slide Rule is specially designed for use in computing Power and Dimensions of Steam, Gas and Oil Engines; since it gives all data for finding speed, length of stroke, dimensions of cylinder, etc.

KURTZ PSYCHROMETRIC SLIDE RULE.



The calculating of all air conditioning problems, heretofore figured by means of the burdensome psychrometric charts or tables, can now be accomplished by the new **Kurtz Psychrometric Slide Rule.** This rule entirely replaces the psychrometric chart, and it affords a simpler and more accurate means of determining any or all of the necessary air conditioning factors.

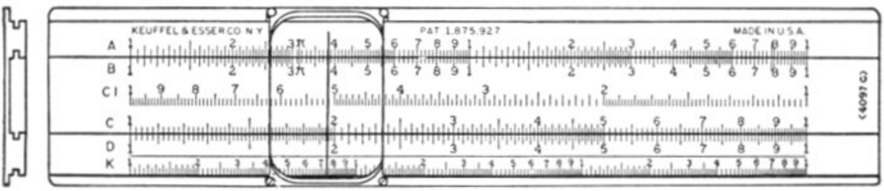
EVER-THERE SLIDE RULES.

REG. U. S. PAT. OFF.

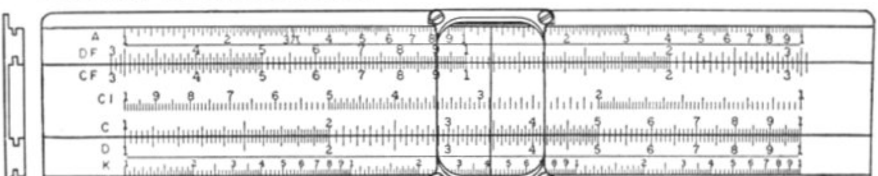
The EVER-THERE Slide Rules are made entirely of white Xylonite, a strong, tough material. On this base the graduations are engine-divided. The handiness of the **EVER-THERE** slide rules is evident from the fact that they weigh no more than a fountain pen, and are much less bulky in the pocket.

The Ever-There Slide Rules Nos. 4097B, C and D are pre-eminently pocket instruments, as the following dimensions will indicate:

Length over all........6 inches. Width over all.................. $1\frac{3}{32}$ inches.
Thickness................ $\frac{1}{8}$ inch. Thickness of indicator........ $\frac{3}{16}$ inch.
Weight..................about $\frac{3}{4}$ ounce.



EVER-THERE Slide Rule No. 4097C has all the scales of the Polyphase* Slide Rule including the Logarithmic and Trigonometrical Scales, as well as inch and centimeter scales on the back. The slide is reversible.



EVER-THERE Slide Rule No. 4097D has all the scales of the Polyphase Duplex* Slide Rule except the CIF scale, together with inch and centimeter scales on the back. The slide is reversible.

* REG. U. S. PAT. OFF

The *SRT* Scale

Supplement to the Manual

on the

POLYPHASE DUPLEX DECITRIG®

Slide Rule

KEUFFEL & ESSER CO.

SRT SCALE REPLACES *ST* SCALE

For the latest model slide rules on which the *SRT* scale has replaced the *ST* scale, wherever *ST* is mentioned in the instruction manual, read *SRT*.

The scale designated *SRT* is, as its name indicates, used to find sines, radian equivalents, and tangents of angles, ranging from 0.573 degrees to 5.73 degrees approximately.

SECTION 38 (REVISED)

The revised section 38 below replaces sections 39 and 40 (pages 59-62) in the POLYPHASE DUPLEX DECITRIG Manual.

38. Radians. Small Angles. A radian is an angular unit equal to $\left(\frac{180}{\pi}\right)^\circ$, or 57.3° accurate to three figures. The *SRT* scale is a *C* scale whose marks represent numbers of degrees ranging from 0.573° to 5.73° approximately. It is so folded that the following rule holds.

Rule. *When the hairline is set to an angle in degrees on the SRT scale, it is also set to the same angle in radians on the C* scale, provided the number on the C scale is prefixed by "0.0" as indicated by the legend* 0.01 *to* 0.1 *at the end of the SRT scale.*

For example, in accordance with the rule,

push hairline to 3.56° on *SRT*,
at hairline read 621 on *C*.

Therefore 3.56° = **0.0621 radian.**

* Of course the *D* scale may be used instead of the *C* scale when the rule is closed.

Observe that if we multiply both members of the equation

$$3.56° = 0.0621 \text{ radian}$$

by 10, 10^2, $\frac{1}{10}$, and $\frac{1}{10^2}$ in succession, we get

$$(10)\,(3.56°) = (10)\,(0.0621), \text{ or } 35.6° = 0.621 \text{ radian},$$
$$(100)\,(3.56°) = (100)\,(0.0621), \text{ or } 356° = 6.21 \text{ radians},$$
$$\left(\frac{1}{10}\right)(3.56°) = \left(\frac{1}{10}\right)(0.0621), \text{ or } 0.356° = 0.00621 \text{ radian},$$
$$\left(\frac{1}{100}\right)(3.56°) = \left(\frac{1}{100}\right)(0.0621), \text{ or } 0.0356° = 0.000621 \text{ radian}.$$

In general for any integer k,
positive or negative $10^k(3.56°) = 10^k(0.0621)$ radian.

Now using the rule in reverse,

push the hairline to 1176 on *C*,
at hairline read 0.674° on *SRT*,

and conclude that

$$0.01176 \text{ radian} = \mathbf{0.674°}.$$

Multiplying this through by 10^2, $\frac{1}{10}$, and 10^k in succession, we get

$$1.176 \text{ radians} = 67.4°,$$
$$0.001176 \text{ radian} = 0.0674°,$$
$$10^k\,(0.01176) \text{ radians} = 10^k\,(0.674°).$$

For angles θ in radians, where θ is less than 0.1 radian (or 5.73°), the following relation holds

$$\boldsymbol{\theta} \textbf{ radians} \cong \textbf{sin } \boldsymbol{\theta} \cong \textbf{tan } \boldsymbol{\theta}, \qquad (20)$$

where the symbol "$\cong$" means "*approximately equals*". In other words, *the value of an angle in radians found by means of the italicized rule is also its sine and its tangent to slide rule accuracy.**

For example:

push hairline to 3.84° on *SRT*
at hairline read 670 on *C*.

Therefore, in accordance with the italicized rule,

$$\sin 3.84° = \tan 3.84° = 0.0670,$$

and, in agreement with equation (20),

$$\sin 0.384° = \tan 0.384° = 0.00670,$$
$$\sin 0.0384° = \tan 0.0384° = 0.000670,$$

and so on.

* The greatest error inherent in formula (20) is at $\theta = 0.1$ radian; it is nearly $+0.00017$ for sin 0.1 and -0.00033 for tan 0.1. These errors are comparable in magnitude with other errors occurring in slide rule computation.

By using the relations of §25, the italicized rule, and (20), we can find the values of other trigonometric functions of small angles. For example

$$\cot 1.352° = 1/\tan 1.352° \cong 1/\sin 1.352° = \csc 1.352°.$$

Hence

push hairline to 1.352° on SRT,
at hairline read on C, $0.0236 \cong \sin 1.352°$,
at hairline read on CI, $40.31 \cong \csc 1.352° \cong \cot 1.352°$.

Also to find cos 88.76°, use (8) §25 to get

$$\cos 88.76° = \sin(90° - 88.76°) = \sin 1.24°.$$

push hairline to 1.24° on SRT
at hairline read 0.0216 on C.

Therefore

$$\cos 88.76° = \sin 1.24° = \mathbf{0.0216}.$$

Then without moving the slide

at hairline read 462 on CI

and conclude that

$$\sec 88.76° = 1/\cos 88.76° = \mathbf{46.3},$$
$$\tan 88.76° = 1/\cot 88.76° = 1/\cos 88.76° = \mathbf{46.3}.$$

Before beginning the exercises, the student should use the slide rule, the italicized rule of this section and (20) to verify the following approximate equations:

(*a*) 1.272° = 0.0222 radian
(*b*) 12.72° = 0.222 radian
(*c*) 0.0531 radian = 3.04°
(*d*) 5.31 radians = 304°
(*e*) sin 2.86° = 0.0499
(*f*) sin 0.286° = 0.00499
(*g*) tan 0.286° = 0.00499
(*h*) csc 0.286° = 20.0
(*i*) cot 0.286° = 20.0
(*j*) sec 87.25° = 20.8

EXERCISES

1. Express in radians:
(*a*) 1.416° (*b*) 0.833° (*c*) 2.5° (*d*) 2.67°.

2. Express in degrees:
(*a*) 0.01823 radian (*b*) 0.0462 radian (*c*) 0.0865 radian

3. Express in radians:
(*a*) 3.59° (*b*) 0.0359° (*c*) 35.9° (*d*) 359°.

4. Express in degrees:
(*a*) 0.0296 radian (*b*) 0.296 radian (*c*) 0.000296 radian

5. Express in radians:
(*a*) 912° (*b*) 435° (*c*) 0.000314° (*d*) 2900°.

6. Find sin 3.42°, tan 3.42°, csc 3.42°, cot 3.42°.

7. Find sin 0.056°, tan 0.056°, csc 0.056°, cot 0.056°.

8. Find cos 89.75°, sec 89.75°, tan 89.75°, cot 89.75°.

9. Express in degrees the following angles expressed in radians:

(a) $\frac{\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{72}$ (d) $\frac{\pi}{180}$ (e) $\frac{5\pi}{6}$.

Hint: Since π radians = 180°, replace π by 180°. However to change $\frac{5\pi}{6}$ radians to degrees

opposite 6 on *DF* set 5 of *C*
opposite index of *D* read 15 on *SRT*, and
$5\pi/6$ radians = 150 degrees.

10. Evaluate the following:

(a) 83 sin 0.0144°

(b) 500 tan 0.0097°

(c) 432 sin 0.716°

(d) $\dfrac{\tan 0.2°}{0.0001745}$

(e) $\dfrac{\sin 0.3°}{0.131}$

(f) $\dfrac{8 \sec 88.25°}{4.72}$

11. The angles in the following exercises are in radians. Change these angles to degrees and then find the values of the functions:

(a) sin 0.345 (b) tan 0.524 (c) sin 1 (d) cos 1
(e) sin 0.628 (f) csc 2 (g) cot 4.17 (h) cos 2.81.

ANSWERS

1. (*a*) 0.0247 (*b*) 0.01454 (*c*) 0.0436 (*d*) 0.0466
2. (*a*) 1.044° (*b*) 2.65° (*c*) 4.96°
3. (*a*) 0.0627 (*b*) 0.000627 (*c*) 0.627 (*d*) 6.27
4. (*a*) 1.696° (*b*) 16.96° (*c*) 0.01696°
5. (*a*) 15.92 (*b*) 7.59 (*c*) 0.00000548 (*d*) 50.6
6. 0.0597, 0.0597, 16.75, 16.75
7. 0.000977, 0.000977, 1023, 1023
8. 0.00436, 229, 229, 0.00436
9. (*a*) 60°, (*b*) 135° (*c*) 2.5° (*d*) 1° (*e*) 150°
10. (*a*) 0.0209 (*b*) 0.0846 (*c*) 5.40 (*d*) 20.0 (*e*) 0.0400 (*f*) 55.5
11. (*a*) 0.338 (*b*) 0.578 (*c*) 0.841 (*d*) 0.540 (*e*) 0.588 (*f*) 1.100 (*g*) 0.603 (*h*) −0.946

Printed in U.S.A.

Explanation of These Scans

The scans of the 4071-3 manual are a mix of scans from a 1939 edition and a 1962 edition

The body of the two manuals are identical. Most of these scans are from the 1962 edition of the manual except for a few pages that had blemishes on them.

The illustration of the rule is from the 1939 version of the manual.

The cover, title page, and copyright page are scanned separately for each edition.

The advertisement pages at the end of the 1939 edition are included but they are not in the 1962 edition.

SRT Scale: In 1955 K&E introduced the SRT scale the replace the ST scale on all decimal trig slide rules. The manual was not updated. A supplement was produced and sent with each manual after 1955. This supplement is included in these scans.

Enjoy
Clark McCoy, 2014